Note Title 5/11/2021

$\int_\gamma A$ non varia se si sostituisce a γ una curva equivalente ovvero

γ e σ sono equivalenti se $\exists\, \rho$ strett. monotona tale che

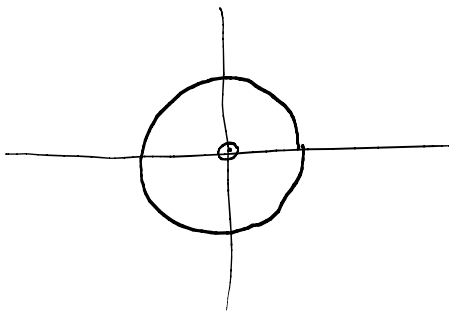
A qualunque $\sigma(t) = \gamma(\rho(t))$

A è C^0 e integrabile $(\exists f : \nabla f = A)$ allora

$\int_\gamma A$ non varia al variare di γ, purché gli estremi restino uguali, nell'ordine.

$A \in C^1$ irrotazionale $((A_i)_{x_j} = (A_j)_{x_i}\ \forall i,j)$

$\int_\gamma A$ se a γ si sostituisce una curva ad essa omotopa.



$$\left(\frac{-y}{x^2+y^2} \;/\; \frac{x}{x^2+y^2} \right)$$

irrotazionale

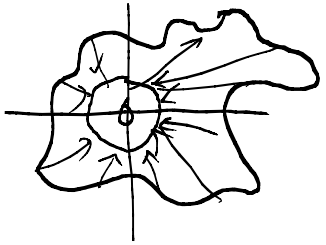
Ma NON integrabile

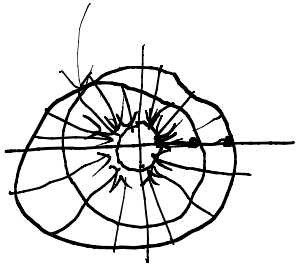
$f(x,y) = \frac{1}{2}\log(x^2+y^2)$ è definita su $\mathbb{R}^2 \setminus \{(0,0)\}$

$$f_x = \frac{x}{x^2+y^2}$$

$$f_y = \frac{y}{x^2+y^2}$$

$$A = \left(\frac{x}{x^2+y^2} \;/\; \frac{y}{x^2+y^2} \right)$$





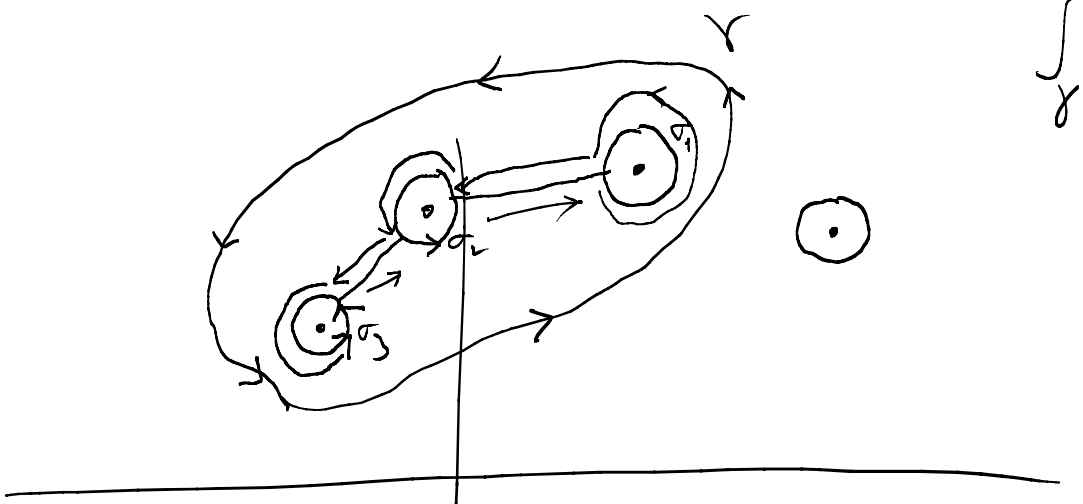
$$\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = \frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right)$$

$\gamma = (\cos t, \sin t)$

$t \in [0, 2\pi]$

$$\int_\gamma A = \int_0^{2\pi} \frac{\cos t(-\sin t)}{1} + \frac{\sin t(\cos t)}{1}\, dt = 0$$

Domnio di A $\quad \mathbb{R}^2 \setminus \{x_1, x_2, \dots x_n\}$

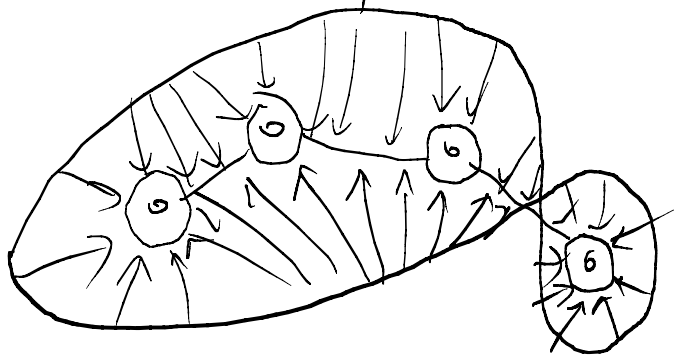


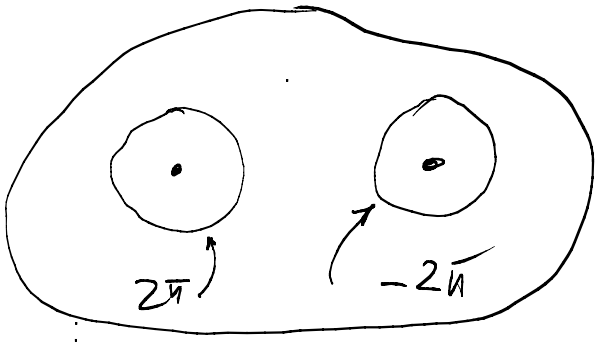
$$\int_\gamma A = \int_{\sigma_1} A + \int_{\sigma_2} A + \int_{\sigma_3} A$$

la 2ª prova che

$$\int_{\sigma_i} A = 0 \quad \forall i = 1 \dots n$$

CAMPI E FORME (III)





$A(x,y) = (y, x)$ ha primitive? Quali sono?
A_1 A_2

$\partial_y A_1 = 1$
$\partial_x A_2 = 1$ sono uguali!! $\Rightarrow$ A è irrotazionale

Il dominio è $\mathbb{R}^2$ (CONVESSO $\Rightarrow$ STELLATO $\Rightarrow$ SEMPLIC. CONNESSO)

$\Rightarrow$ A è integrabile su $\mathbb{R}^2$ (CONNESSO) (*)

$f_x \equiv y$
$f_y \equiv x$

Integrando la I equazione rispetto a x $\boxed{f \equiv xy + c(y)}$ ch,
derivata rispetto a y fornisce $\boxed{f_y \equiv x + c'(y)}$ $\Rightarrow x \equiv x + c'(y)$
$\rightarrow \boxed{c'(y) \equiv 0}$
$\Leftarrow$
$c(y) =$ costante

sostit. in (*) si ha $\boxed{f(x,y) = xy + \text{cost.}}$

$$A(x,y) = \begin{pmatrix} y \\ -x \end{pmatrix}$$

$$\begin{cases} \partial_y A_1 = 1 \\ \partial_x A_2 = -1 \end{cases}$$

DIVERSI

$\operatorname{rot} A \neq 0$

$$\nabla \times A \equiv \operatorname{rot} A$$

$$\begin{pmatrix} \partial_{x_1} \\ \partial_{x_2} \\ \partial_{x_3} \end{pmatrix} \times \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

$f_x = y$

$f = xy + c(y)$

$f_y = x + c'(y)$

$f_y = -x$

$-x = x + c'(y)$

$c'(y) = -2x \qquad c(y) = -2xy + c$

$f(x,y) = xy - 2xy + c$

$= -xy + c$

$f_x = -y$ NO

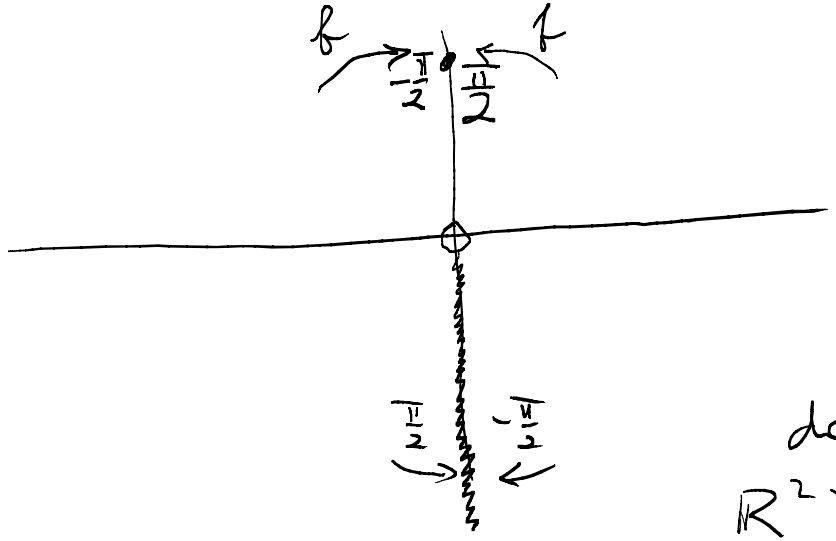
$f_y = -x$ SI

OCCORRE VERIFICARE CHE IL DOMINIO DEL POTENZIALE TROVATO COINCIDA CON QUELLO DEL CAMPO

$$A = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$$

$f = \arctg \frac{y}{x}$ è una primitiva di A su $\{x > 0\}$

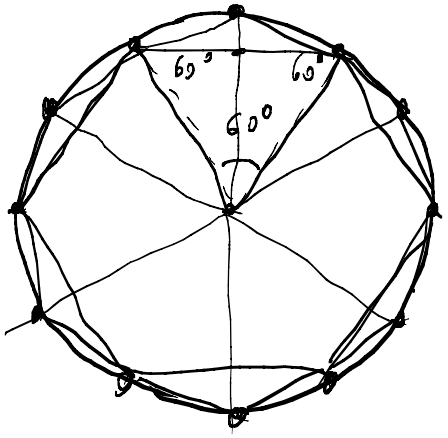
$= \arg(x+iy)$



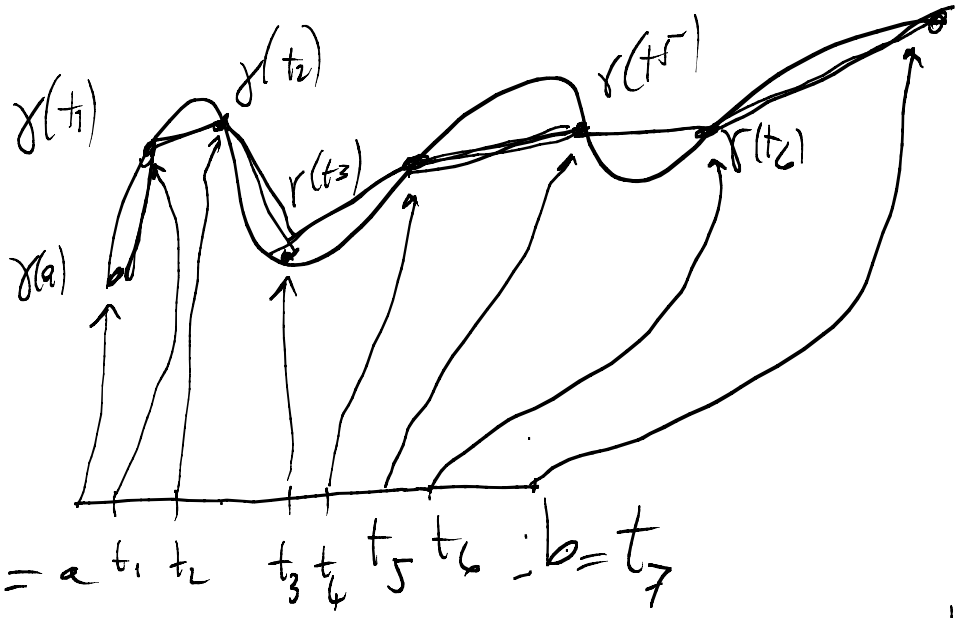
$$g(x,y) = \begin{cases} \arctg \frac{y}{x} & x > 0 \\ \arctg \frac{y}{x} + \pi & x < 0 \\ \frac{\pi}{2} & x = 0 \; y > 0 \end{cases}$$

dom g $\mathbb{R}^2 \setminus \{(0,x) \; x \leq 0\}$? $\subset \mathbb{R}^2 \setminus \{0,0\}$

CURVE RETTIFICABILI



circonf. $> 6r$

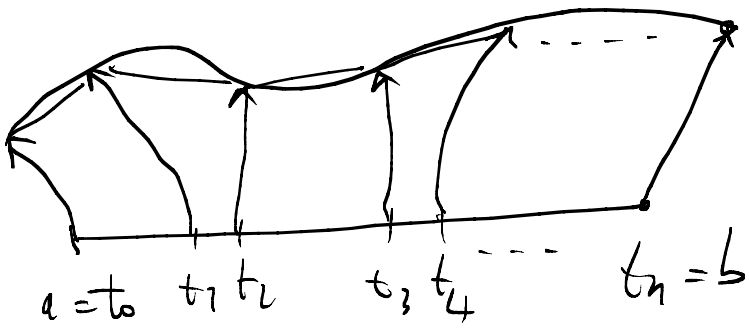


$$|\gamma(t_1)-\gamma(t_0)|+|\gamma(t_2)-\gamma(t_1)|+\dots+|\gamma(t_7)-\gamma(t_6)|$$

$\gamma : [a,b] \to \mathbb{R}^n$ curva parametrica

Dato $[a,b]$ chiamiamo <u>partizione</u> Π di $[a,b]$ una sequenza $t_0 < t_1 < t_2 < \dots < t_{n-1} < t_n$ con $t_0 = a$ $t_n = b$.

La <u>lunghezza</u> della poligonale associata (inscritta) alla partizione $\Lambda(\Pi)$ il numero



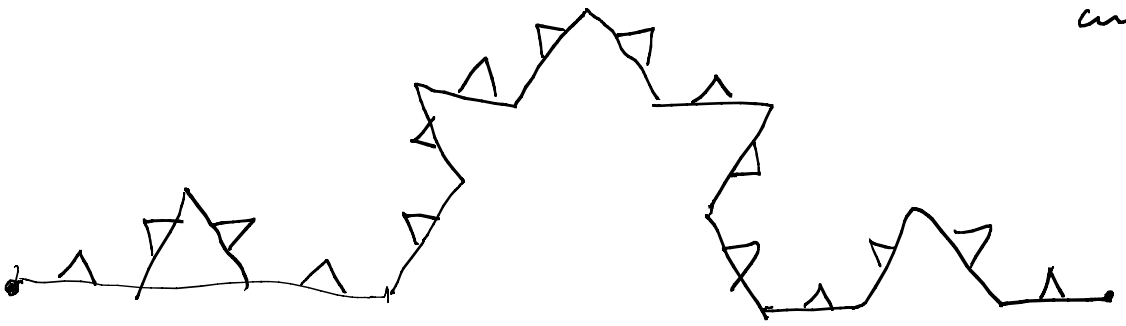
$$\Lambda(\Pi) = \sum_{0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)|$$

La lunghezza di γ è l'estremo superiore delle lunghezze di tutte le poligonali inscritte

$$\Lambda(\gamma) = \sup_{\Pi} \Lambda(\Pi)$$

curve di von KOCH
curva fiocco di neve

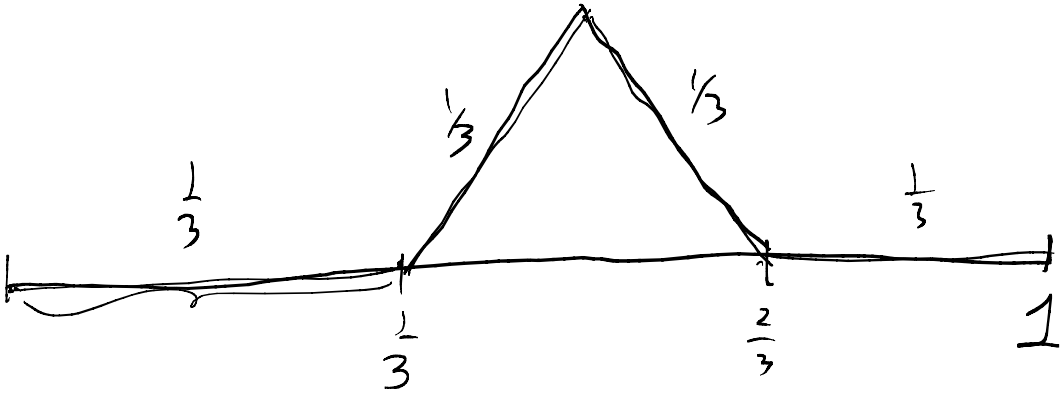
n pezzi $= \frac{4^n}{3^n}$



2 e segmenti

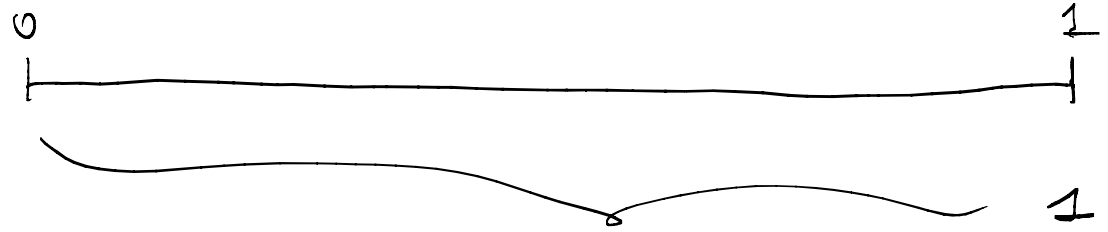
$n=2 \quad \Lambda = \frac{16}{9}$

$\Lambda = \frac{4}{3}$



1

$\Lambda = 1$



0

Se $\Lambda(\gamma) < \infty$ γ sarà detta RETTIFICABILE