Note Title 5/7/2021

(1) $\int_\gamma A = \int_{\gamma'}$

A arbitrario

se γ e γ' sono equivalenti (equivalenti) ρ strett. crescente tale che

$$\gamma'(t) = \gamma(\rho(t))$$

(2) Se $A \in C^0$ e integrabile allora $\int_\gamma A = \int_\sigma A$

per ogni γ, σ con estremi comuni

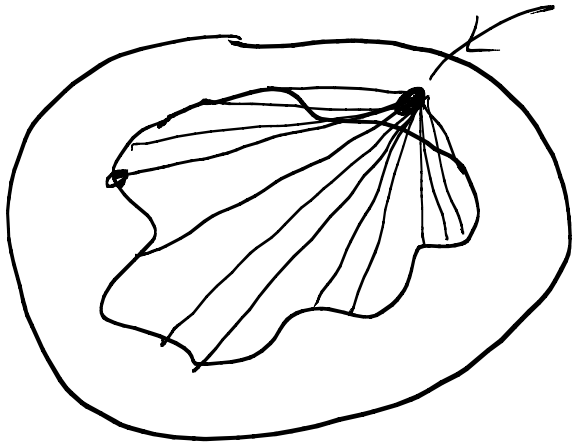
$A \in C^1$ e irrotazionale $\left((A_i)_{x_j} = (A_j)_{x_i} \ \forall i,j \right)$

$\int_\gamma A = \int_\sigma A$ se γ e σ sono omotope

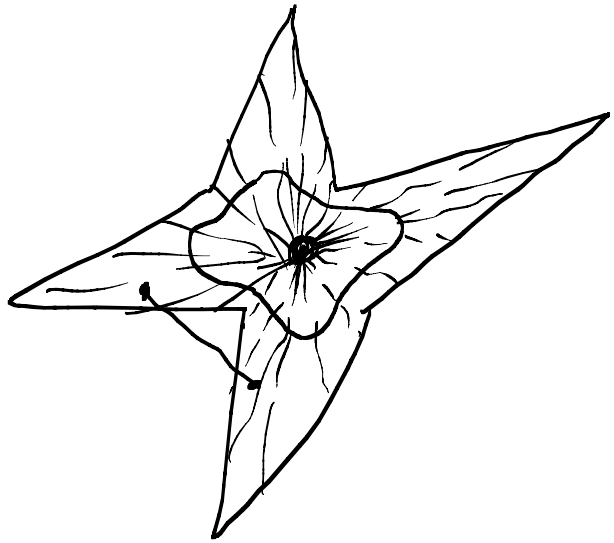
INVARIANZA OMOTOPICA

(PRODI Analisi Matem. II)

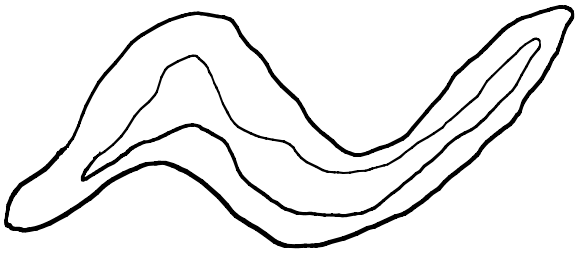
Ω si dice SEMPLICEMENTE CONNESSO se OGNI CURVA CHIUSA IN Ω È OMOTOPA IN Ω AD UNA CURVA COSTANTE



CONVESSO



STELLA



NÉ CONVESSO NÉ STELLA

Ω si dice

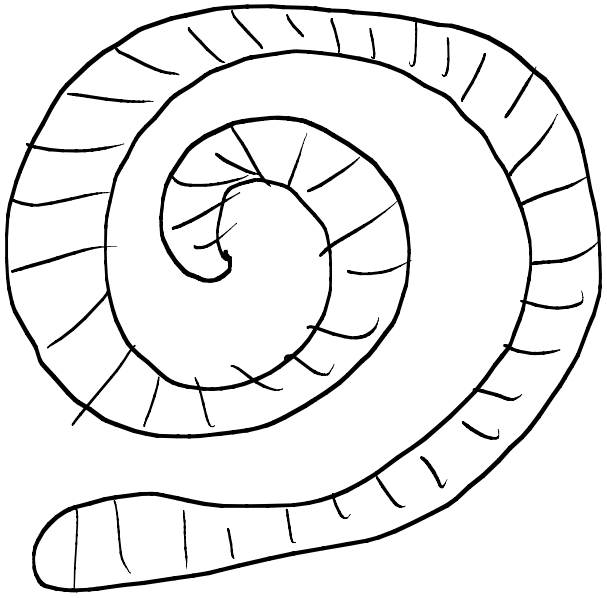
STELLA

se $\exists x_0$: $\overline{x_0 x} \subseteq \Omega$

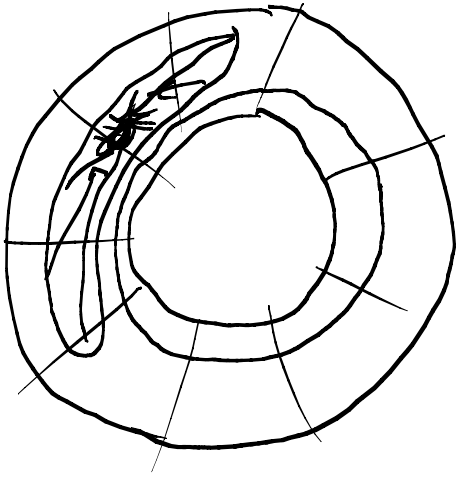
$\forall x \in \Omega$

(una luce posta in x_0 è visibile da qualsiasi punto di Ω

("CONVESSO RISPETTO AL PUNTO x_0, detto POLO)



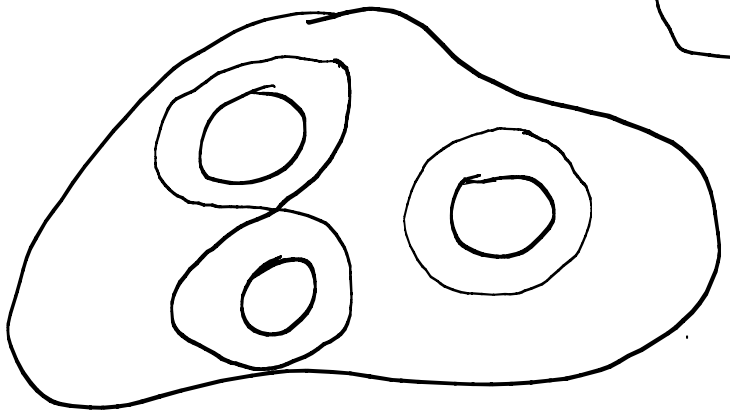
$\partial\Gamma$

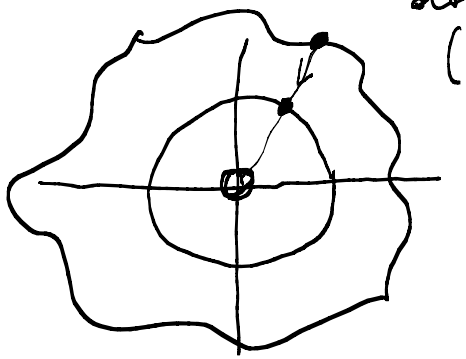


corona circolare

NON È SEMPLIC.

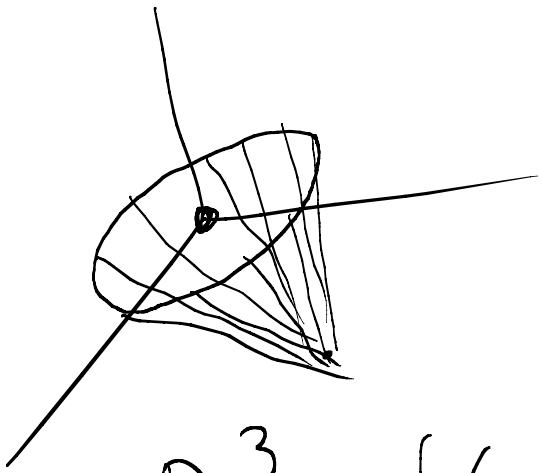
CONNESSA





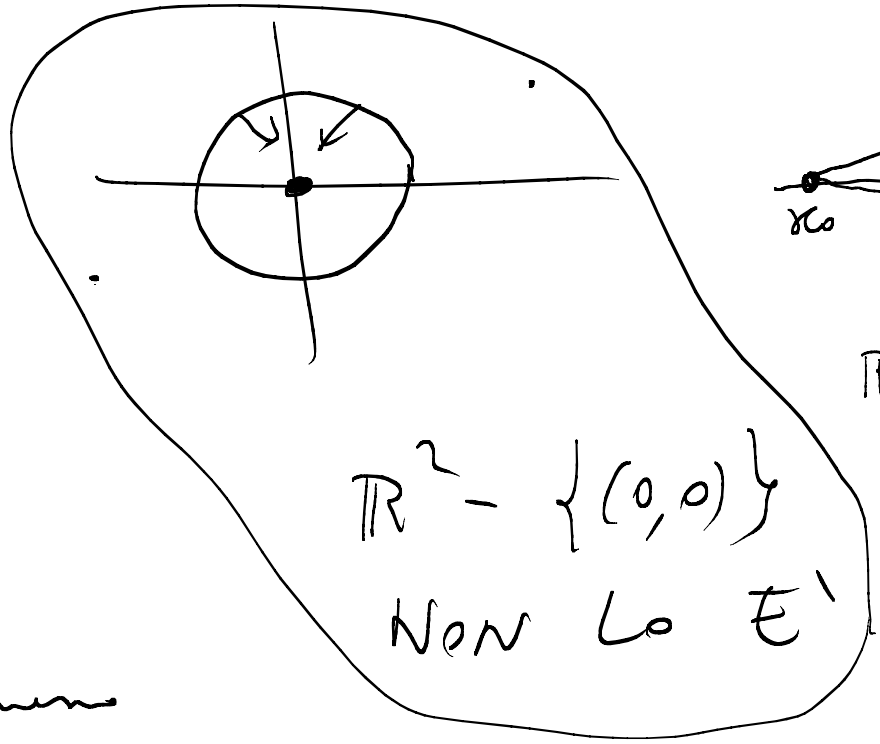
dom $\left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$

$\mathbb{R}^2 \setminus \{0\}$



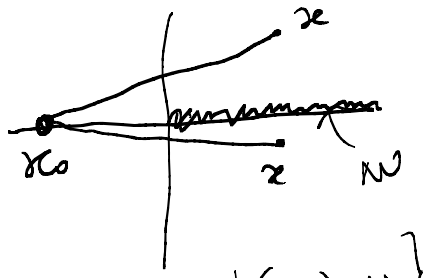
$\mathbb{R}^3 - \{(0,0,0)\}$

è semplic. connesso



$\mathbb{R}^2 - \{(0,0)\}$

NON LO È

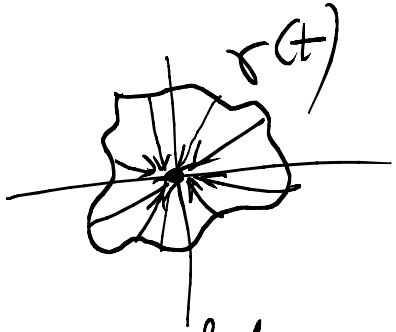


$\mathbb{R}^2 - \{(x,0), x>0\}$

è stellato rispetto a $(\alpha,0)$ con $\alpha<0$.

Th. Tutti gli insiemi stellati sono semplic. connessi

Dim. Supp. $x_0 = 0$



$h(\lambda,t) = (1-\lambda)\gamma(t)$

$h(0,t) = \gamma(t)$

$h(1,t) = 0\,\gamma(t) = 0$

deforma γ in $\sigma(t) \equiv 0$

$$\Rightarrow \int_\gamma A = \int_\sigma A = \int_0^1 A(\sigma(t))\,\dot\sigma(t)\,dt =$$

$$= \int_0^1 \underbrace{A(0)}_{\in \mathbb{R}^n} \cdot 0 \, dt = 0 \qquad \begin{matrix} \sigma(t) \equiv 0 \\ \dot\sigma(t) \equiv 0 \end{matrix}$$

In un insieme stellato rispetto all'origine

$\int_\gamma A$, γ chiusa, vale 0.

Caso generale

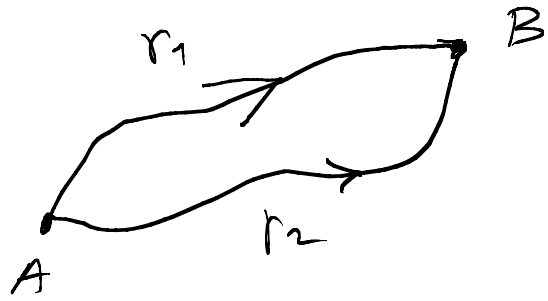
Ω stellato con polo x_0 arbitrario. $\in \Omega$ perché è un punto del segmento di estremi x_0 e $\gamma(t)$

$$h(\lambda, t) = x_0 + (1-\lambda)(\gamma(t) - x_0)$$

continua su $[0,1] \times [0,1]$

$h(0,t) = x_0 + 1(\gamma(t) - x_0) \equiv \gamma(t)$ $\quad h(1,t) = x_0$

$\int_\gamma A = 0$ $\forall \gamma$ chiusa.

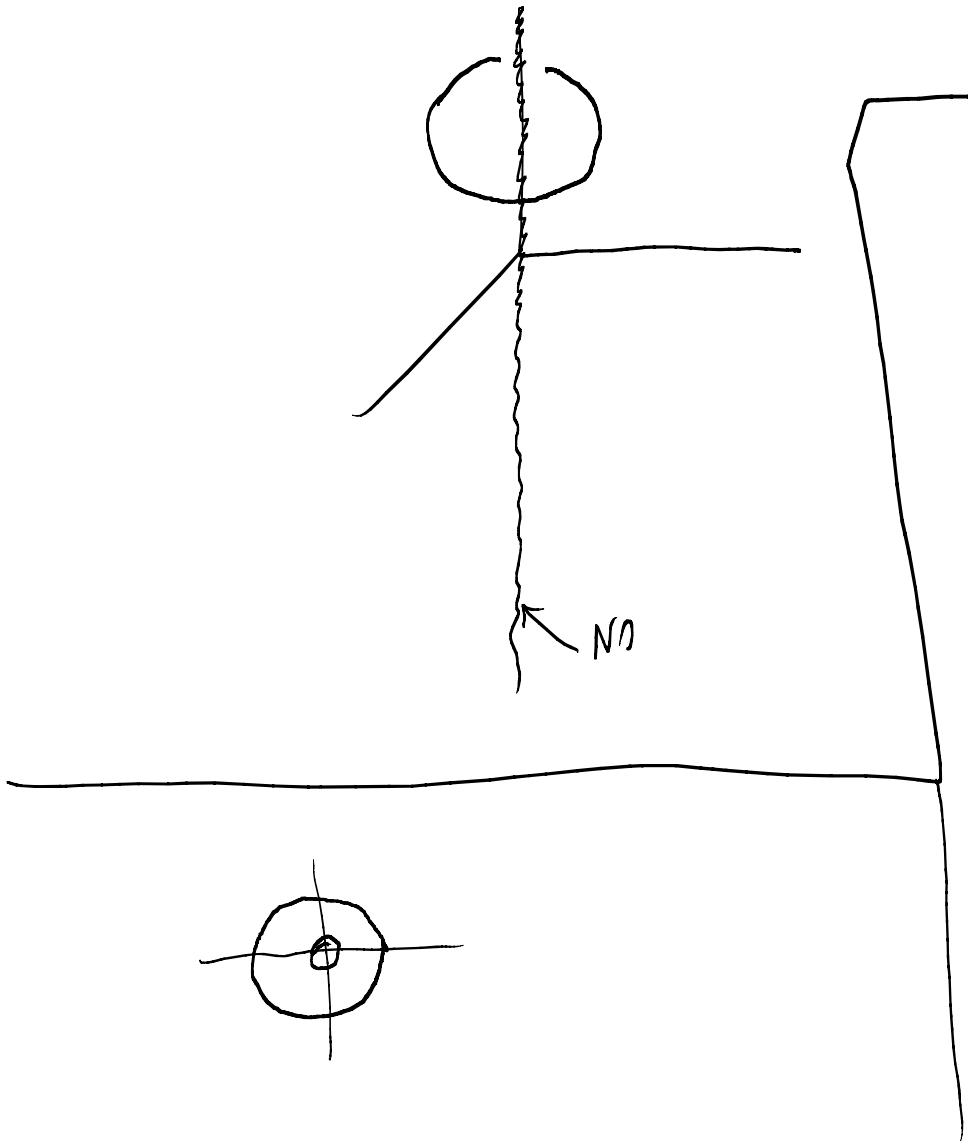


$\int_{\gamma_1} A + \int_{\gamma_2} A = 0$ perché $\gamma_1 \oplus (\gamma_2$ in senso opposto) è chiusa.

$\gamma_1 \oplus (\gamma_2$ in senso opposto) è chiusa.

$\int_{\gamma_1} A - \int_{\gamma_2} A = 0$ $\Longleftrightarrow$ $\int_{\gamma_2} A = \int_{\gamma_1} A$

Th. Un campo irrotazionale (C^1) su un insieme semplicemente connesso è integrabile. (CNS)

$\mathbb{R}^3 \setminus \{(0,0,\alpha) : \alpha \in \mathbb{R}\}$

A sia isotopa nel complemento di tre punti x_1, x_2, x_3