Note Title 4/30/2021

$$\int_\gamma A\,d\gamma = \int_a^b \underbrace{A(\gamma(t))}_{\in \mathbb{R}^n}\ \underbrace{\dot\gamma(t)}_{\in \mathbb{R}^n}\,dt$$

$d\gamma$

$\gamma : [a,b] \to \mathbb{R}^n$

$\dot\gamma \in [a,b] \to \mathbb{R}^n$

$$\alpha(x,w) = \underbrace{A(x)}_{\text{campo associato}}\, w$$

$\int_\gamma \alpha$?

$$\int_\gamma \alpha \equiv \int_\gamma A$$

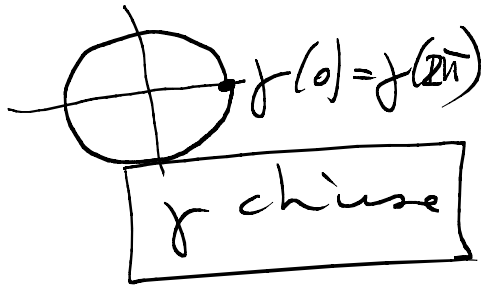
(forma e campo associati hanno la stessa def. integrale sulle curve C^1 a tratti)

$$\alpha(x, w) = \sum_{i=1}^{n} A_i(x)\, w_i = \sum_{i=1}^{n} A_i(x)\, dx_i$$

$$A(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right) \qquad \gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad t \in [0, 2\pi]$$

$$\int_\gamma A =$$

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \Rightarrow$$



γ chiusa

$$= \int_a^b \left[A_1(x,y)\, \dot{x}(t) + A_2(x,y)\, \dot{y}(t) \right] dt \qquad \begin{cases} x(t) = \cos t \\ y(t) = \sin t \end{cases} \quad t \in [0, 2\pi]$$

$$A(x,y) \equiv \begin{pmatrix} A_1(x,y) \\ A_2(x,y) \end{pmatrix} \cdot \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} \equiv \dot{\gamma}(t)$$

$$A_1(x,y) = \frac{-y}{x^2+y^2} \overset{x=\gamma}{=} \frac{-\sin t}{\cos^2 t + \sin^2 t}$$

$$A_2(x,y) = \frac{x}{x^2+y^2} = \frac{\cos t}{\cos^2 t + \sin^2 t}$$

$$\int A = \int_0^{2\pi} \left[\frac{\sin t}{\cos^2 t + \sin^2 t} \cdot \underbrace{(-\sin t)}_{\dot{x}(t)} + \frac{\cos t}{\cos^2 t + \sin^2 t} \cos t \right] dt =$$

$\gamma_{|II}$

$$\int_\gamma \alpha = \int_0^{2\pi} \sin^2 t + \cos^2 t \, dt = 2\pi$$

α

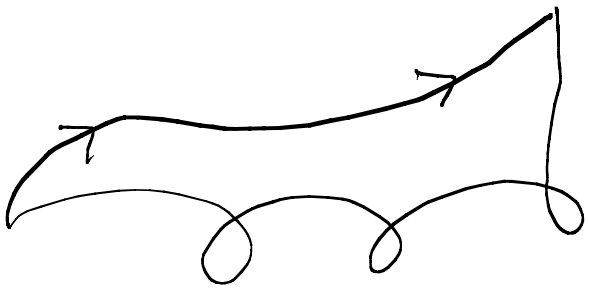
$$\omega(x, dx) = \sum_{i=1}^{n} A_i(x_1, \dots, x_n)\, dx_i$$

$$\gamma(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \qquad t \in [a, b]$$

$$\int_\gamma \omega = \int_a^b \sum_{i=1}^{n} A_i(x_1(t), x_2(t), \dots, x_n(t))\, \dot{x}_i(t)\, dt$$

componenti del campo associato

$$\int_\gamma A$$



$\gamma : [a, b] \to \operatorname{dom} A$

$\rho : [c, d] \to [a, b]$

$\longrightarrow$ strett. crescente, invertibile, C^1

$\rho(c) = a$, $\rho(d) = b$

$$\int_\gamma A = \int_a^b A(\gamma(t))\,\dot\gamma(t)\,dt$$

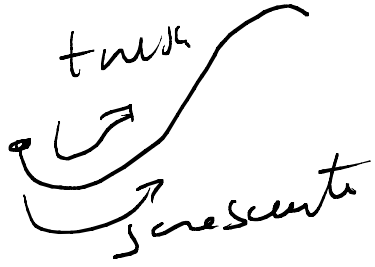
$$\sigma(s) = \gamma(\rho(s))$$

$$\dot\sigma(s) = \dot\gamma(\rho(s))\,\dot\rho(s) \qquad s \in [c,d]$$

$$\int_\sigma A = \int_c^d A(\sigma(s))\,\dot\sigma(s)\,ds = \int_c^d A(\gamma(\rho(s)))\,\dot\gamma(\rho(s))\,\dot\rho(s)\,ds =$$

$t = \rho(s)$

$dt = \dot\rho(s)\,ds$ $\longrightarrow$ $\int_{\rho(c)=a}^{\rho(d)=b} A(\gamma(t))\,\dot\gamma(t)\,dt = \int_\gamma A$



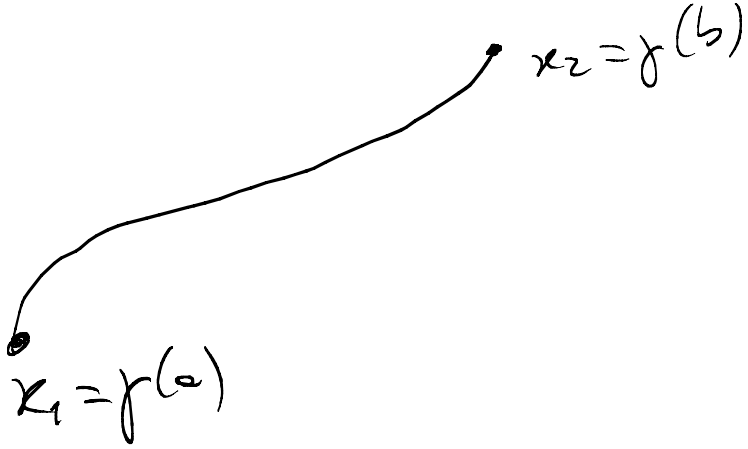
In queste ipotesi γ e σ si dicono equivalenti

equivalenza

Se ρ è strett. decres. $\rho(c)=b$, $\rho(d)=a$ $\int$ cambiano di segno

in questo caso γ e σ saranno <u>equivalenti</u> antiversi, e si avrà

$$\int_{\sigma} A = -\int_{\gamma} A$$

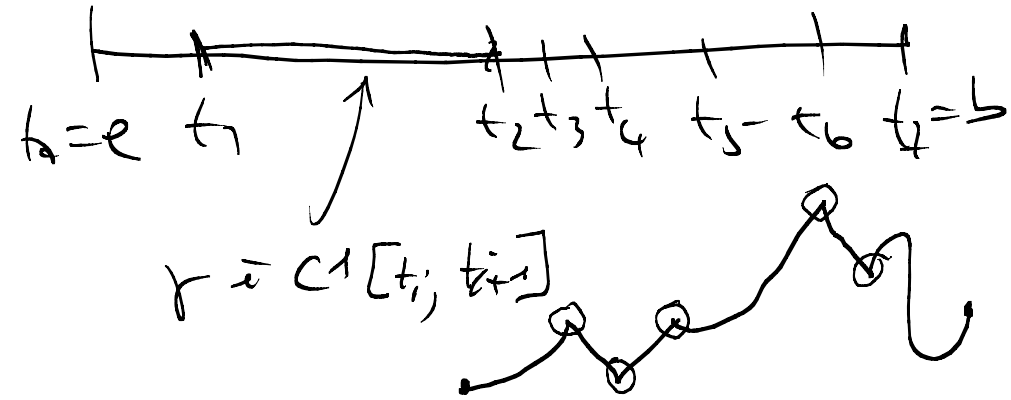


$\gamma : [a,b] \to \mathbb{R}^n$

$\gamma \in C^0[a,b]$

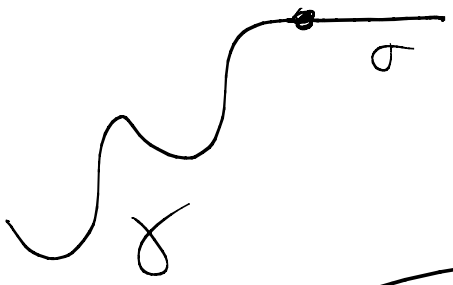
$\gamma \in C^1[t_i, t_{i+1}]$

$a = t_0 < t_1 < t_2 < \dots < t_N = b$



$$\int_\gamma A = \sum_{i=0}^{N-1} \int_{t_i}^{t_{i+1}} A(\gamma(t))\, \dot\gamma(t)\, dt$$

curve C^1 a tratti



Un campo $A : \Omega \to \mathbb{R}^N \quad \Omega \subseteq \mathbb{R}^N$

si dirà <u>integrabile</u> (SINONIMI: potenziale, conservativo, non dissipativo)

se esiste $f : \Omega \to \mathbb{R}$, detta <u>primitiva</u> (oppure potenziale) tale che

$$\nabla f \equiv A \text{ su } \Omega$$

$$\boxed{\nabla f(x) = A(x) \quad \forall x \in \Omega}$$

Una forma $\alpha : \Omega \times \mathbb{R}^N \to \mathbb{R}$

$\Omega \subseteq \mathbb{R}^N$

si dirà integrabile (o esatta)

se esiste $f : \Omega \to \mathbb{R}$ (potenziale, primitiva)

tale che $df \equiv \alpha$ su $\Omega \times \mathbb{R}^N$

$$df(x,w) = \alpha(x,w) \quad \forall x \in \Omega \ \forall w \in \mathbb{R}^N$$

$A(x,y) = \begin{pmatrix} x \\ y \end{pmatrix}$

def. su $\mathbb{R}^2 \to \mathbb{R}^2$

$f(x,y) = \frac{1}{2}(x^2+y^2)$

$f_x = x \qquad f_y = y$

$\nabla f = A$ su $\mathbb{R}^2$