

$$\dim(X+Y) + \dim X \cap Y = \dim X + \dim Y$$

$$\dim X, \dim Y < \infty$$

Def. $\dim X \cap Y = 0 \iff X+Y = X \oplus Y$

$$\dim(X \oplus Y) = \dim X + \dim Y$$

$$\dim(X+Y) =$$

$$\dim X \cap Y = 0$$

$$X \subseteq Y \quad (\text{oppure } Y \subseteq X) \Rightarrow X+Y = Y \quad (X \cap Y = X)$$

$$0 < \dim X \cap Y < \frac{\dim X}{\dim Y}$$

$$\{0\} \subset X \cap Y \subset X \\ \subset Y$$

i) w_1, \dots, w_k base di $X \cap Y \Rightarrow \dim X \cap Y = k$

ii) $w_1, \dots, w_k, x_{k+1}, \dots, x_n$ base di $X \Rightarrow \dim X = n$

$w_1, \dots, w_k, y_{k+1}, \dots, y_m$ base di $Y \Rightarrow \dim Y = m$

Allora si provi che

$w_1, \dots, w_k, x_{k+1}, \dots, x_n, y_{k+1}, \dots, y_m$ sono una base di $X + Y$

e quindi $\dim X + Y = n + m - k$ da cui le tesi.

1) $w_1 - w_k x_{k+1} - x_n y_{k+1} - y_m$ è un sottoinsieme di generatori per $X+Y$

$$\forall z \in X+Y \quad \exists \underline{x} \in X \quad \underline{y} \in Y : \quad x+y = z$$

$$\exists \alpha_1 \dots \alpha_k \beta_{k+1} \dots \beta_n : \quad x = \sum_1^k \alpha_i w_i + \sum_{k+1}^n \beta_j x_j$$

$$\exists \alpha'_1 \dots \alpha'_k \beta'_{k+1} \dots \beta'_m : \quad y = \sum_1^k \alpha'_i w_i + \sum_{k+1}^m \beta'_j y_j$$

$$x+y = \underbrace{\sum_1^k (\alpha_i + \alpha'_i) w_i + \sum_{k+1}^n \beta_j x_j + \sum_{k+1}^m \beta'_j y_j}_{\in \langle w_1 \dots w_k, x_{k+1} \dots x_n, y_{k+1} \dots y_m \rangle}$$

$$\in \langle w_1 \dots w_k, x_{k+1} \dots x_n, y_{k+1} \dots y_m \rangle$$

$w_1 \dots w_k$ $x_{k+1} \dots x_n$ $y_{k+1} \dots y_m$ sono INDEPENDENTI

$$\underbrace{\sum_1^k \alpha_i w_i}_{w} + \underbrace{\sum_{k+1}^n \beta_j x_j}_{x} + \underbrace{\sum_{k+1}^m \gamma_h y_h}_{y} = 0$$

$$w + x + y = 0 \Rightarrow \underbrace{w+x}_{\in X} = \underbrace{-y}_{\in Y} \Rightarrow y \in X \cap Y$$

↓

$$\rightarrow \underbrace{(w+y)}_{\in X \cap Y} + \alpha = 0$$

$\in \langle x_{k+1}, \dots, x_n \rangle$

$\langle w_1, \dots, w_k \rangle$

Per la ripartizione di dimensione

$\langle w_1, \dots, w_k \rangle$ + $\langle x_{k+1}, \dots, x_n \rangle$

nella E' DIRTTA

$w+y=0$ e $\alpha=0$

$$0 = x = \sum_{k+1}^n \beta_j x_j \Rightarrow \beta_j = 0 \quad \forall j = k+1 \dots n$$

not p. much element of one box

$$0 = w + y = \sum_1^k \alpha_i w_i + \sum_{k+1}^m \gamma_h y_h \Rightarrow \alpha_i = 0 \quad \forall i$$

and if

$\gamma_h = 0 \quad \forall h$

$$x_{\langle e_1, \dots, e_n \rangle} = \sum_1^n x_{e_i}$$

$$e_i e_j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

Si provare che $\underbrace{(x - x_{\langle e_1, \dots, e_n \rangle}) e_j = 0}_{\uparrow}, \forall j = 1 \dots n$

$$x_{\langle u_1, \dots, u_n \rangle} = \sum_1^n x_{u_i}$$

$$\left[x - \sum_i^? x_{u_i} \right] u_j = 0 \quad \forall j = 1 \dots n$$

prova

x- maggiore del sostituto
di Gram

X_i sotto spazio di Y , $i = 1 \dots n$

$$W = \bigoplus_1^n X_i \text{ in algebra}$$

1) $W = \sum_1^n X_i$

2) $\forall x_1, \dots, x_n : x_i \in X_i \quad \forall i = 1 \dots n \quad \sum_1^n x_i = 0$
per $x_1 = x_2 = \dots = x_n = 0$

$A: X \rightarrow Y$ $\left\{ \begin{array}{l} 0 < \dim X < \infty \\ 0 < \dim Y < \infty \end{array} \right.$
 e_1, \dots, e_n base of X

e'_1, \dots, e'_m base of Y

$A(e_i) \in Y$ und $\exists A_{ji}$ tal. che

$$A(e_i) = \sum_{j=1}^m A_{ji} e'_j$$

AL-6

$\psi(u) = u'$ \times metre amount
 $\langle e'_1, t, t' \rangle \rightarrow \langle e'_1, -t \rangle = Y$

$$A(e_3) = (t^2)' = 2t = 0 \cdot 2 - 2(-t)$$

A

$\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$
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metri associate ad
 $A(u) = u'$ e alle basi
 $(1, t, t^2)$ del dominio
e $(z_1, -t)$
del codominio

$$A(e_2) = (t)' = 1 = \left(\frac{1}{2}\right) \cdot 2 + 0 \cdot (-t)$$

$$A(e_1) = (1)' = 0 = 0 \cdot 2 + 0 \cdot (-t)$$

$$\langle A_1, A_2, \dots, A_n \rangle = \{ A_2, \dots, A_n \}$$

↑ ↓

can

$$A_1 \in \{ A_2, \dots, A_n \}$$

$$\langle A_1, A_2, \dots, A_n \rangle = \{ A_2, \dots, A_n \} \Rightarrow$$

$$\underline{A_1 \in \{ A_1, A_2, \dots, A_n \}} = \{ A_2, \dots, A_n \}$$

$$\subseteq A_1 \in \{ A_2, \dots, A_n \} \Rightarrow \exists x_i : A_1 = \sum_{i=2}^n x_i A_i$$

(x)

$$\langle A_2, \dots, A_n \rangle \subseteq \langle A_1, A_2, \dots, A_n \rangle$$

$$\sum_2^n \beta_i A_i = \underline{0 \cdot A_1} + \sum_2^n \beta_i A_i \in \text{span}$$

$$B = \sum_1^n \gamma_i A_i = \boxed{\gamma_1 A_1} + \sum_2^n \gamma_i A_i = \text{(*)}$$

$$= \gamma_1 \sum_2^n \alpha_i A_i + \sum_2^n \gamma_i A_i \in \langle A_2, \dots, A_n \rangle$$

$\sum_2^n [\gamma_1 \alpha_i + \gamma_i] A_i$

$\beta \neq 0$

$B \in \langle A_1, \dots, A_n \rangle$

allein so fürs' spannen

B con almen uns die vetter $A_1 - A_n$ denkt man bei
es spannen

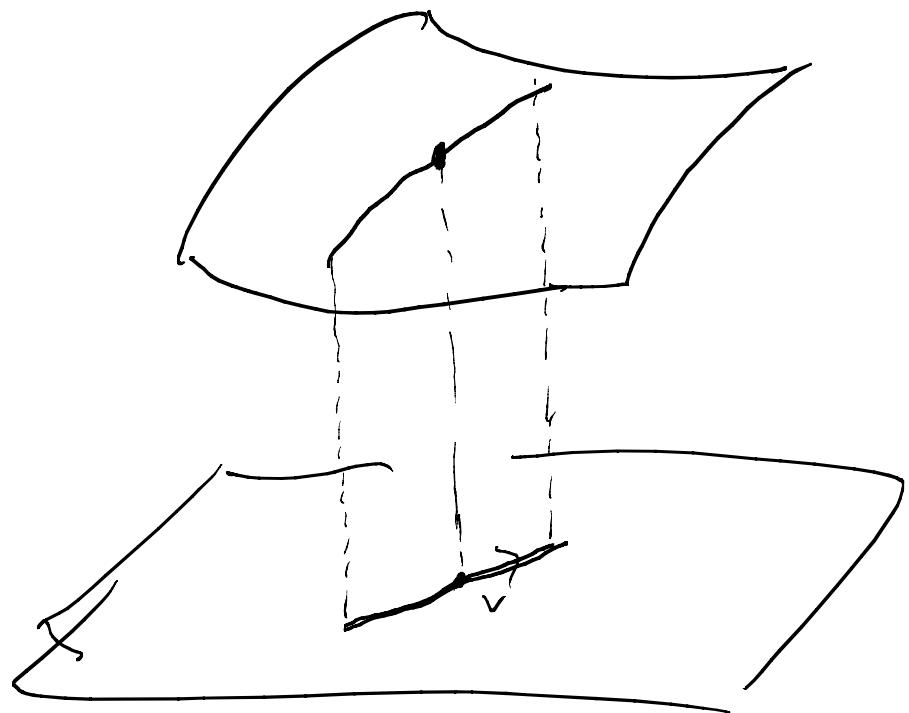
$$\underline{\langle A_1, A_2, \dots, A_n \rangle} = \langle B, A_1, A_2, \dots, A_n \rangle$$

$\exists \alpha_i : B = \sum_{i=1}^n \alpha_i \cdot A_i$ dell'ipot $B \neq 0 \Rightarrow$ almeno uno
 degl' $\alpha_i \neq 0$: per semplicità si $i=1$ (così $\alpha_1 \neq 0$)

$$B = \alpha_1 A_1 + \sum_{i=2}^n \alpha_i A_i \iff \underline{A_1} = \frac{1}{\alpha_1} B - \frac{1}{\alpha_1} \sum_{i=2}^n \alpha_i A_i$$

$$= \frac{1}{\alpha_1} B + \sum_{i=2}^n \left(-\frac{\alpha_i}{\alpha_1} \right) A_i \quad \boxed{\in \langle B, A_2, A_3, \dots, A_n \rangle}$$

Per il Lemma Fondamentale: $\langle B, A_1, A_2, \dots, A_n \rangle = \underline{\langle B, A_2, \dots, A_n \rangle}$



$$f_v(x_0) = \underline{\underline{h'(0)}}$$

$$\underline{\underline{h(t) = f(x_0 + tv)}}$$

$$[x - \sum_i^n \alpha_i u_i] u_j = 0 \quad \forall j=1..n$$

$$\underline{x} u_j = \sum_{i=1}^n \alpha_i u_i u_j$$

↑
negative

$$\begin{pmatrix} u_1 u_1 & u_2 u_1 & \cdots & u_n u_1 \\ u_1 u_2 & u_2 u_2 & & u_n u_2 \\ \vdots & \vdots & & \vdots \\ u_1 u_n & u_2 u_n & & u_n u_n \end{pmatrix}$$

Nature
of
GRAM

$u_1..u_n$ some indp alone $\begin{pmatrix} u_1 u_1 \\ u_1 u_2 \\ \vdots \\ u_1 u_n \end{pmatrix}$ some independent

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \lambda_1 u_1 u_1 \\ \lambda_1 u_1 u_2 \\ \vdots \\ \lambda_1 u_1 u_n \end{pmatrix} + \begin{pmatrix} \lambda_2 u_2 u_1 \\ \lambda_2 u_2 u_2 \\ \vdots \\ \lambda_2 u_2 u_n \end{pmatrix} + \cdots + \begin{pmatrix} \lambda_n u_n u_1 \\ \lambda_n u_n u_2 \\ \vdots \\ \lambda_n u_n u_n \end{pmatrix}$$

$$\left\{ \begin{array}{l} (\sum \lambda_i u_i) u_1 = 0 \\ (\sum \lambda_i u_i) u_2 = 0 \\ \vdots \\ (\sum \lambda_i u_i) u_n = 0 \end{array} \right.$$

Multiplikation mit λ_1 führt zu
der dominante Zeile

$$\sum (\lambda_i u_i) \lambda_1 u_1 + \sum (\lambda_i u_i) \lambda_2 u_2 + \dots + \sum (\lambda_i u_i) \lambda_n u_n = 0$$

$$\downarrow \quad |\sum \lambda_i u_i|^2 = \underbrace{(\sum \lambda_i u_i)}_{\lambda_1} (\sum \lambda_i u_i) = 0 \Rightarrow \underbrace{\sum \lambda_i u_i}_{\lambda_1} = 0$$

$$\left\{ \begin{array}{l} (\sum \lambda_i u_i) \lambda_1 u_1 = 0 \\ (\sum \lambda_i u_i) \lambda_2 u_2 = 0 \\ \vdots \\ (\sum \lambda_i u_i) \lambda_n u_n = 0 \end{array} \right.$$

Jedie u_i
sind linear
unabhängig

$\Rightarrow (\sum$
 $\lambda_i u_i)$

dominante
Zeile
enthält
nur
einheitsvektor $(\sum \lambda_i u_i)$

$$\gamma: [a, b] \rightarrow \mathbb{R}^n$$

$t_0 \in [a, b]$ $\gamma(t_0)$ punto d' tangente

$$\sigma(t) = \underbrace{\gamma(t_0)}_{\text{punto d' tangente}} + t \underbrace{\dot{\gamma}(t_0)}_{\text{direzioni delle tangenti a } f(t_0)}$$

$$\gamma(t) = \begin{pmatrix} e^t \\ e^{2t} \end{pmatrix}$$

tangente nel punto del sostegno $(1, 1)$

$\left\{ \begin{array}{l} e^t = 1 \\ e^{2t} = 1 \end{array} \right. \Rightarrow t = 0 \Leftrightarrow \boxed{t_0}$

$\dot{\gamma}(t) = \begin{pmatrix} e^t \\ 2e^{2t} \end{pmatrix} \Big|_{t=0} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\sigma(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$y = f(x)$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$y = f(x_0) + f'(x_0)(x - x_0)$$

$$(x_0, f(x_0))$$

Surj. fine

$$y(t_0) + \underbrace{\dot{y}(t_0)}_{\text{jacobus}} (t - t_0)$$

vector moment

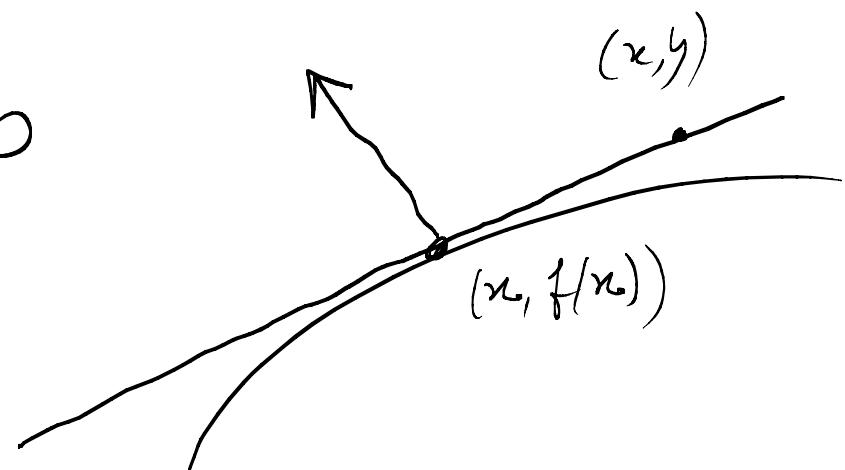
matrix for vector

Vektori normale = vektori perpendicular alle rätte/tangent.

$$y = f(x) + f'(x_0)(x-x_0) \quad f: \mathbb{R} \rightarrow \mathbb{R}$$

$$y - f(x_0) - f'(x_0)(x-x_0) = 0$$

$$\begin{pmatrix} -f'(x_0) \\ 1 \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - f(x_0) \end{pmatrix} = 0$$



↑ vektori normali (orientato verso l'alto)

$$f: \mathbb{R}^N \rightarrow \mathbb{R}$$

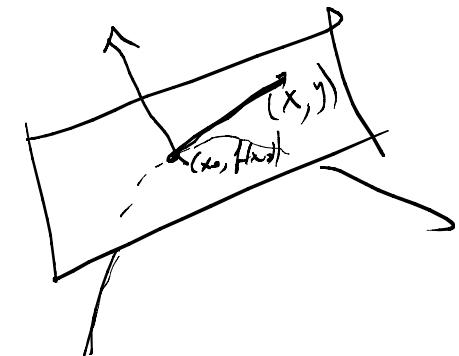
x y

$$y - f(x_0) - \sum f_{x_i}(x_0)(x - x_0)_i = 0$$

$$\begin{pmatrix} (x - x_0)_1 \\ (x - x_0)_2 \\ \vdots \\ (x - x_0)_n \\ y - f(x_0) \end{pmatrix} \begin{pmatrix} -f_{x_1}(x_0) \\ -f_{x_2}(x_0) \\ \vdots \\ -f_{x_n}(x_0) \\ 1 \end{pmatrix} = 0$$

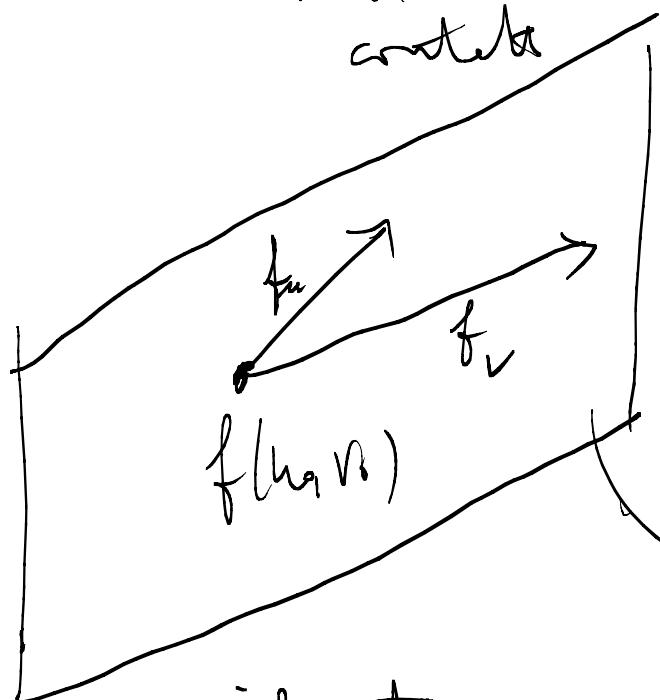
↑
spalten
mit
gleicher
länge.

↑
vektor
mit
orientat
ion
in
AKTO



$$f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$g(u, v) = f(u_0, v_0) + \underbrace{f'(u_0, v_0)}_{\text{punkt}} \begin{pmatrix} u - u_0 \\ v - v_0 \end{pmatrix} \underbrace{\text{increment}}$$



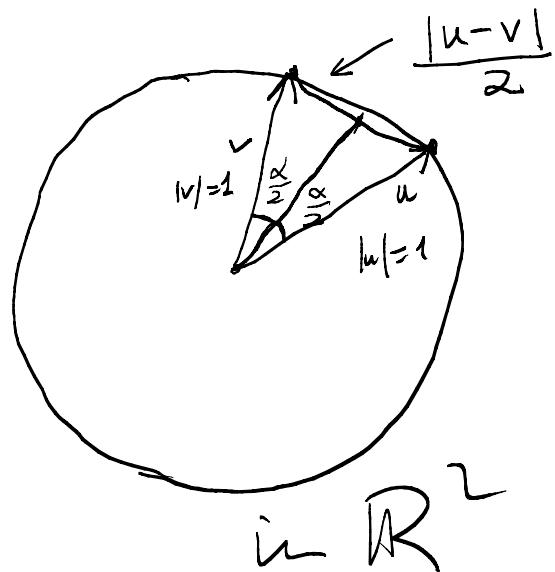
$$(u - u_0) f_u(u_0, v_0) + (v - v_0) f_v(u_0, v_0)$$

combinazione linea di vettori

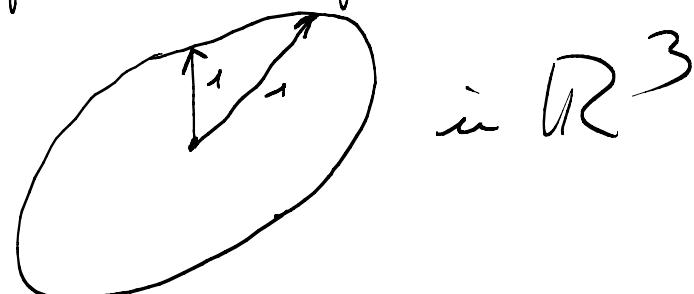
f_u ed f_v in (u_0, v_0)

(le sono indipendenti!!!)

$$\text{il vettore normale } \langle f_u, f_v \rangle = \boxed{f_u \times f_v} \text{ e i suoi multipli}$$



proven fine



$$\begin{aligned}
 \underline{\cos \alpha} &= \cos 2 \frac{\alpha}{2} = 1 - 2 \sin^2 \frac{\alpha}{2} = \sin \frac{\alpha}{2} = \frac{|u-v|}{2} / 1 = \\
 &= 1 - 2 \frac{|u-v|^2}{4} = \\
 &= 1 - \frac{|u|^2 + |v|^2 - 2uv}{2} = \cancel{1} - \cancel{1} + \underline{uv} \Rightarrow \boxed{\cos \alpha = uv} \\
 &\quad |u|=|v|=1
 \end{aligned}$$

Se $|u| \neq 1$ e $|v| \neq 1$ si ha

$$u = |u| \frac{u}{|u|} \quad v = |v| \frac{v}{|v|}$$

$$u \cdot v = |u||v| \left(\frac{u}{|u|} \frac{v}{|v|} \right) = |u||v| \cos \hat{u} \hat{v} \quad \text{FISICA}$$

\nwarrow cos dell'angolo formato da verso

AL-2.1

