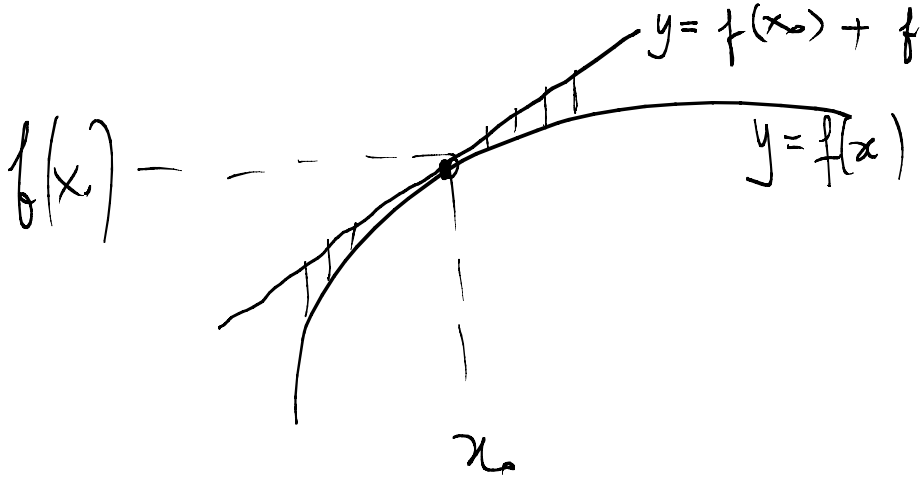
Note Title 4/14/2021

$f : \mathbb{R} \to \mathbb{R}$



$$\{(x,y) \;;\; \underbrace{y = f(x)}_{x \in \text{dom } f}\} \equiv$$

$$\equiv \text{graph } f$$

$y = f(x_0) + f'(x_0)(x - x_0)$ è la retta per $(x_0, f(x_0))$ con coeff. angolare $f'(x_0)$

$f(x)$

$$g(x) = \underbrace{f(x_0)} + \boxed{f'(x_0)(x - x_0)}$$

$w = x - x_0$

$x = x_0 + w$

$$f(x) - g(x) = \underbrace{f(x) - f(x_0) - f'(x_0)(x - x_0)}_{df(x_0,\, x - x_0)}$$

se f è diff. in x_0 allora il II membro è $o(|x - x_0|)$ $|w|$

$f(x) = \sin x$

Tangente al grafico nel punto $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

$\sin\frac{\pi}{4}$

$x_0 = \frac{\pi}{4}$

$f(x_0) = \frac{1}{\sqrt{2}}$

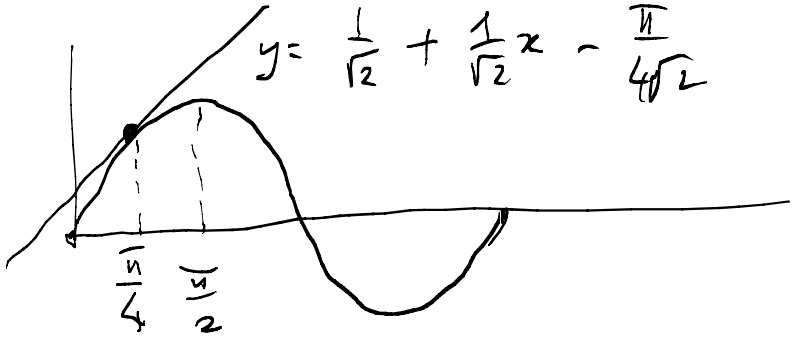
$f'(x) = \cos x$

$f'(x_0) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$y = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$

$f(x_0)$ $\quad \cos\left(\frac{\pi}{4}\right)$

$f'(x_0$



derivata

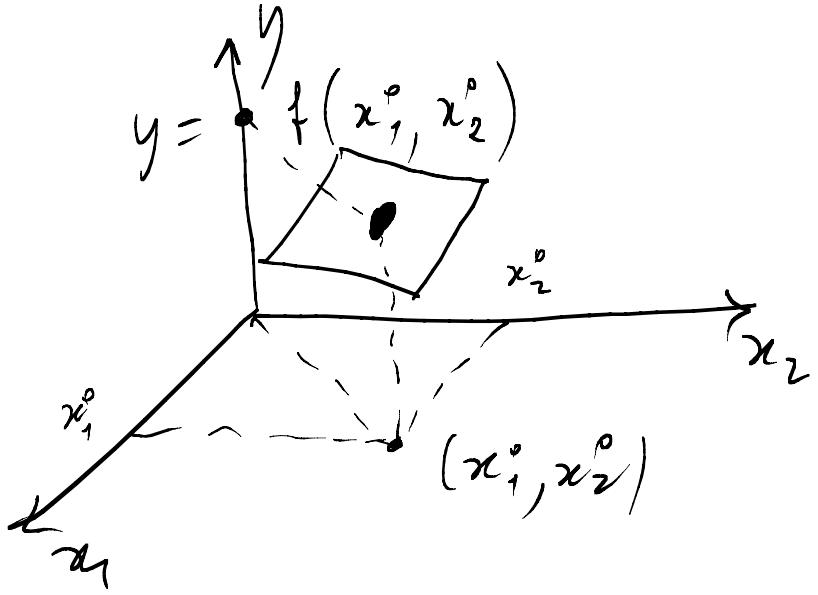
$y = f(x_0) + f'(x_0)(x - x_0)$

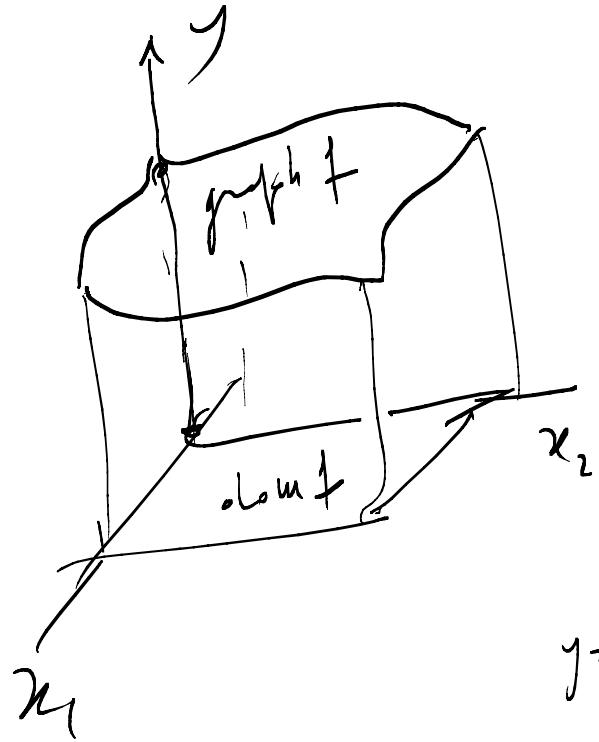
differenziale — incremento

$f(x,y)$
$\in \mathbb{R}^2$

$$f: \mathbb{R}^2 \to \mathbb{R}$$

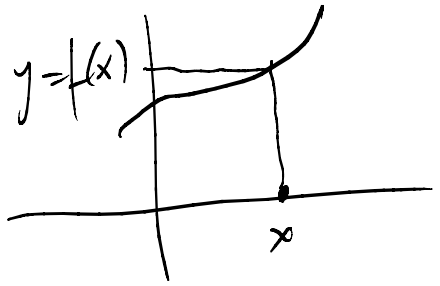
$$\text{graph } f = \{(x,y) \in \mathbb{R}^2 \times \mathbb{R} \,;\, y = f(x)\}$$
$x \in \mathbb{R}^2$





$\mathbb{R}^2$

(x_1, x_2, y)



$f(x_1, x_2)$

$$f(x_1^\circ, x_2^\circ) + df\left((x_1^\circ, x_2^\circ), \begin{pmatrix} x_1 - x_1^\circ \\ x_2 - x_2^\circ \end{pmatrix}\right) =$$
w

$$g(x,y) = f(x_1^\circ, x_2^\circ) + \sum_{i=1}^{2} \frac{\partial f}{\partial x_i}(x_1^\circ, x_2^\circ)(x_i - x_i^\circ)$$

$\nabla f(x^\circ)(x - x_0)$

$$f(x_1, x_2) = x_1^2 x_2$$

$x_1^\circ = 1$
$x_2^\circ = 2$

$y = f(x_1, x_2) = 2$

Piano tangente al grafico di f nel suo punto $(1, 2, 2)$

$x_1^\circ \; x_2^\circ$

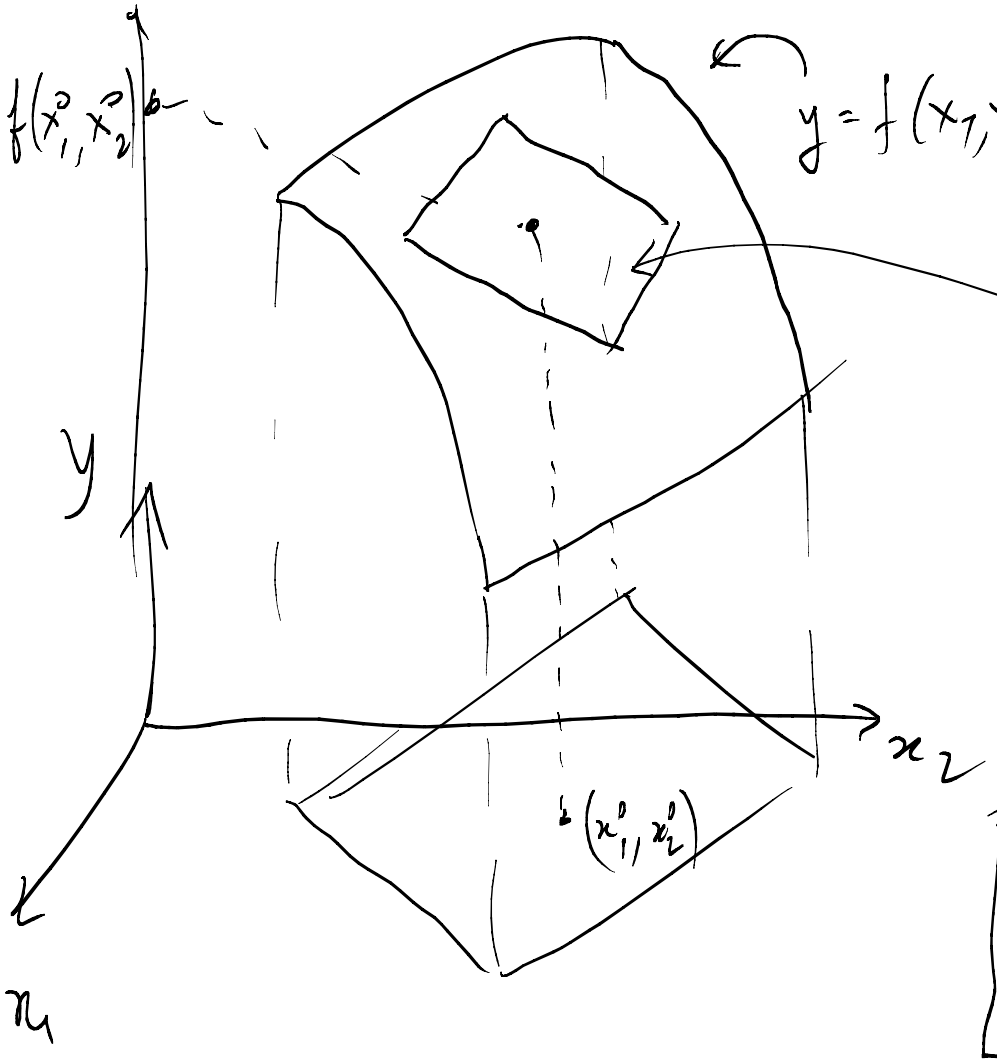
$$f_{x_1}(x_1, x_2)\Big|_{\substack{x_1=1\\x_2=2}} = 2x_1x_2\Big|_{\substack{x_1=1\\x_2=2}} = 4$$

$$f_{x_2}(x_1, x_2)\Big|_{\substack{x_1=1\\x_2=2}} = x_1^2\Big|_{\substack{x_1=1\\x_2=2}} = 1$$

$$y = f(1,2) + 4(x_1 - 1) + 1(x_2 - 2)$$

$f_{x_1}(x_1^\circ, x_2^\circ)$ x_1° $f_{x_2}(\,)$ x_2°

$$y = 2 + 4(x_1 - 1) + (x_2 - 2)$$



$$y = f(x_1^0, x_2^0) + \nabla f(x_1^0, x_2^0)\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix}\right]$$

wolframalpha.com

$f: \mathbb{R} \to \mathbb{R}^N$ (MOTO)

sostegno di f = immagine di f = "curve vere"

(curva parametrica)

$$\gamma(t) = \begin{cases} a + bt = x(t) \\ c + dt = y(t) \end{cases}$$

$\gamma: \mathbb{R} \to \mathbb{R}^2$

$\gamma_0 + tv$

$f(t)$

$$\underbrace{g(t)}_{\in \mathbb{R}^N} = \underbrace{f(t_0)}_{\in \mathbb{R}^N} + \underbrace{f'(t_0)}_{\in \mathbb{R}^N} \underbrace{(t-t_0)}_{\in \mathbb{R}}$$

$g: \mathbb{R} \to \mathbb{R}^N$

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad t \in [0, 2\pi]$$

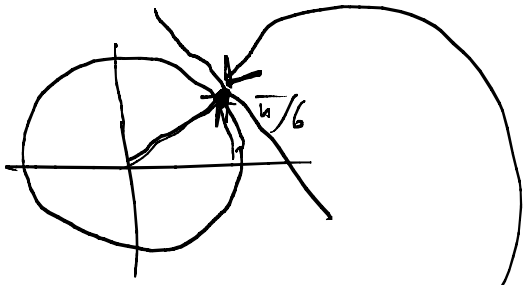
$$\dot{\gamma}(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$$

$$f(t) - g(t) = f(t) - f(t_0) - f'(t_0)(t-t_0) = o(|t-t_0|)$$

se f è differenziabile

retta parametrica tangente a

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad \text{in} \quad \gamma\left(\frac{\pi}{6}\right) = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$$



$$\sigma(t) = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} + \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix} \left(t - \frac{\pi}{6}\right)$$

$$\dot{\gamma}(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}\Big|_{t=\frac{\pi}{6}} = \begin{pmatrix} -\frac{1}{2} \\ \sqrt{3}/2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\sqrt{3}}{2} \\ 1/2 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ \sqrt{3}/2 \end{pmatrix} = 0$$

$s = t - \frac{\pi}{6}$

$$\sigma(s) = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} + s \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$$

$$\simeq \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} + z \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix}$$

$z = \frac{s}{2}$

La velocità $\dot\gamma(t_0)$ è la direzione della tangente a γ in $\gamma(t_0)$

STRATEGIA occorre disporre di t_0 per calcolare la direzione della tg. $\dot\gamma(t_0)$

tangente alla circonf. unitaria in $(1,0) = \gamma(t_0)$ $\in \mathrm{Im}\,\gamma$

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$\gamma(t_0) = \begin{pmatrix} \cos t_0 \\ \sin t_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

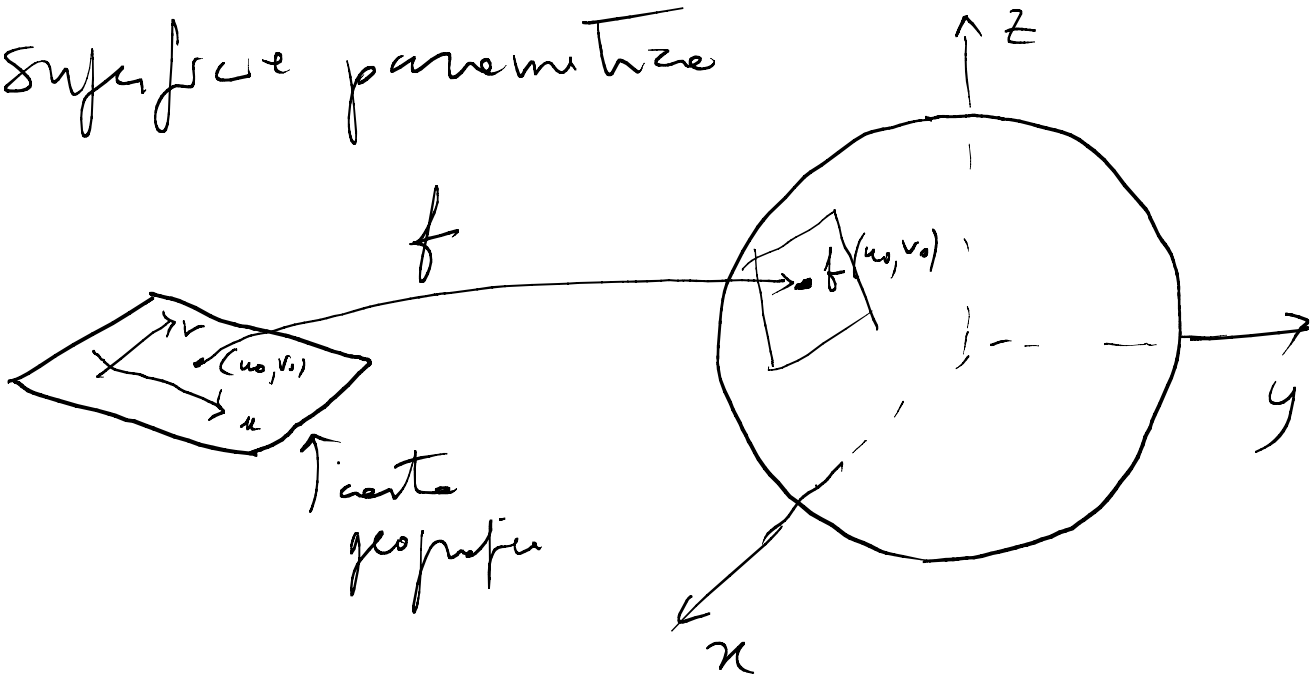
$\cos t_0 = 1$ $\quad t_0 = 0$?

$\sin t_0 = 0$

$$\begin{pmatrix} -\sin 0 \\ \cos 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \dot\gamma(0)$$

$$\sigma(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$f : \mathbb{R}^2 \to \mathbb{R}^3$ superficie parametrizzata



Sostegno (l'immagine) è l'oggetto geometrico più vicino ad una superficie della geometria euclidea.

$$\underbrace{g(\alpha,\beta)}_{\mathbb{R}^3} = \underbrace{f(u_0,v_0)}_{\in \mathbb{R}^3} + df\left((u_0,v_0); \begin{pmatrix} \alpha-u_0 \\ \beta-v_0 \end{pmatrix}\right) = f(u_0,v_0) + \underbrace{f'(u_0,v_0)\begin{pmatrix} \alpha-u_0 \\ \beta-v_0 \end{pmatrix}}_{w} =$$

punto di tangente

$$= f(u_0, v_0) + (\alpha - u_0)\, f_u(u_0, v_0) + (\beta - v_0)\, f_v(u_0, v_0)$$

prima comp. dell'incremento — seconda componente

f_u: prima colonna della jacobiana di f — f_v: seconda colonna della jacobiana

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = f(u_0, v_0) \in \mathbb{R}^3$$

punto di tangenza

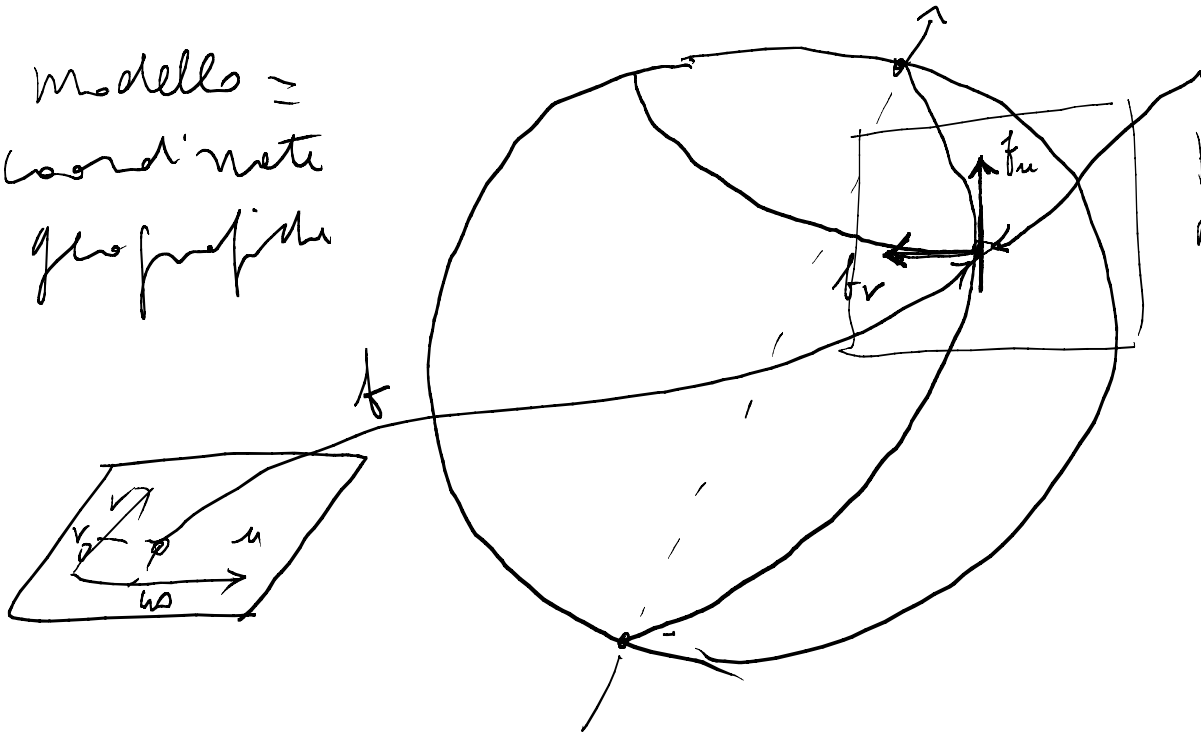
$$A = f_u(u_0, v_0) \in \mathbb{R}^3$$

$$B = f_v(u_0, v_0) \in \mathbb{R}^3$$

PIANO PARAMETRO

$$g(\alpha, \beta) = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + (\alpha - u_0) A + (\beta - v_0) B$$

Modello = coordinate geografiche



$f_u(u_0, v_0)$ richiede di tenere bloccato v_0 e derivare rispetto ad u

Il vettore del piano tangente $f_v(u_0, v_0)$ ho è bloccato e v vere

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \left\langle f_u^{(u_0, v_0)}, f_v^{(u_0, v_0)} \right\rangle$$

$$\| \\ f(u_0, v_0)$$

$$\phi(u,v) = \begin{pmatrix} \cos u \sin v \\ \sin u \sin v \\ \cos v \end{pmatrix} \begin{matrix} = x(u,v) \\ = y(u,v) \\ = z(u,v) \end{matrix}$$

$$\longrightarrow \boxed{(\cos u \sin v)^2 + (\sin u \sin v)^2 + \cos^2 v = 1}$$

piano tangente nel punto $(1,0,0)$