Note Title 4/13/2021

$f \in C^1(B_\delta(x_0)) \Rightarrow f$ è differenziabile in x_0

$$[f(x)]^{g(x)} \equiv e^{g(x)\log f(x)} \qquad f(x) > 0$$

$$f(x,y) = |xy|$$

f non ha una delle derivate parziali in tutti i punti degli assi, salvo $(0,0)$

$\Omega \subseteq \mathbb{R}^N$

$f : \Omega \to \mathbb{R} \qquad x_0 \in \mathring{\Omega}$

$f \in C^1(\Omega) \Rightarrow df(x_0, w) = \underset{\in \mathbb{R}^N}{\nabla f(x_0)} \underset{\in \mathbb{R}^N}{w}$

$f : \mathbb{R} \to \mathbb{R}$ diff.

$$\lim_{w \to 0} \frac{|f(x_0+w) - f(x_0) - aw|}{|w|} = 0$$

$$\left| \frac{f(x_0+w) - f(x_0) - aw}{w} \right|$$

$$\lim_{w \to 0} \frac{f(x_0+w) - f(x_0) - aw}{w} = 0$$

$$\Rightarrow \quad \frac{f(x_0+w) - f(x_0)}{w} - a \quad \Rightarrow \quad a = f'(x_0)$$

$A : \mathbb{R} \to \mathbb{R}$

$A(w) = \underset{\in \mathbb{R}}{a} \cdot \underset{\in \mathbb{R}}{w}$

linear

$$df(x_0, w) = f'(x_0) w$$

$f: \mathbb{R} \to \mathbb{R}^N$ IDENTICA (QUASI)

$\hookrightarrow$

La norma al numeratore è in $\mathbb{R}^N$

$a \in \mathbb{R}^N \quad (w \in \mathbb{R})$

x_0

$$\lim_{w \to 0} \frac{f(x_0+w) - f(x_0) - \underset{\in \mathbb{R}^N}{a} \underset{\in \mathbb{R}}{w}}{w} = 0$$

$\Updownarrow$

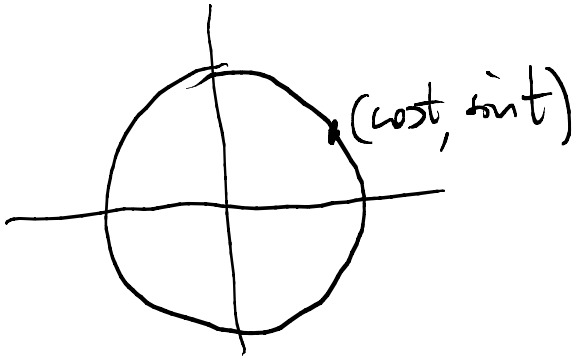
$$\frac{f_i(x_0+w) - f_i(x_0)}{w} - a_i \xrightarrow[w \to 0]{} 0$$

accade se x è una x $\quad f_i$ è derivabile in x_0 e inoltre

$$\alpha_i = \dot{f}_i(x_0)$$

$$df(x_0, w) = \begin{pmatrix} \dot{f}_1(x_0) \\ \vdots \\ \dot{f}_N(x_0) \end{pmatrix} \cdot w \qquad \in \mathbb{R}^N$$

$$\longrightarrow \quad \dot{f}(x_0) = \left(\dot{f}_1(x_0), \dot{f}_2(x_0), \dots, \dot{f}_N(x_0) \right)$$



$t \in [0, 2\pi]$

$$f(t) = (\cos t, \sin t)$$

$$f'(t) = ((\cos t)', (\sin t)') = (-\sin t, \cos t)$$

$f : \mathbb{R}^N \to \mathbb{R}^M$

$$\lim_{w \to 0} \underbrace{\frac{\left| \overset{\in \mathbb{R}^M}{f(x_0+w)} - \overset{\in \mathbb{R}^M}{f(x_0)} - \overset{\in \mathbb{R}^M}{aw} \right|_{\mathbb{R}^M}}{|w|_{\mathbb{R}^N}}}_{\parallel} = 0$$

diff

$\boxed{aw} \in \mathbb{R}^M$

$w \in \mathbb{R}^N$

$a \in \mathbb{R}^{M \times N}$

$$\lim_{w \to 0} \left| \frac{1}{|w|} \left[f(x_0+w) - f(x_0) - aw \right] \right|_{\mathbb{R}^M} = 0$$

$$\Updownarrow$$

$$\boxed{\lim_{w \to 0} \frac{1}{|w|} \left[f_i(x_0+w) - f_i(x_0) - \overset{\in \mathbb{R}^N}{a_i} \overset{\in \mathbb{R}^N}{w} \right] = 0} \quad \forall i = 1 \dots M$$

$$a_i = \underbrace{\left((f_i)_{x_1}, (f_i)_{x_2}, \dots, (f_i)_{x_N} \right)}$$

gradiente (o derivata) della componente i-esima di f

$$f(x) = (f_1(x), \dots, f_M(x))$$

$$a = \begin{pmatrix} (f_1)_{x_1} & (f_1)_{x_2} & \cdots & (f_1)_{x_N} \\ (f_2)_{x_1} & (f_2)_{x_2} & \cdots & (f_2)_{x_N} \\ \vdots & \vdots & & \vdots \\ (f_M)_{x_1} & (f_M)_{x_2} & \cdots & (f_M)_{x_N} \end{pmatrix} (x_0)$$

$$f'(x_0) = a =$$

Matrice Jacobienne (J. JACOBI) $f'(x_0)$

$$f'(x_0) = \left(\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}_{x_1}, \begin{pmatrix} f_1 \\ f_2 \\ \\ f_n \end{pmatrix}_{x_2}, \dots, \begin{pmatrix} f_1 \\ f_2 \\ \\ f_m \end{pmatrix}_{x_N} \right)(x_0)$$

$$\boxed{\left(\underset{\in \mathbb{R}^{M\times N}}{f'(x)}\right)_{ij} = \frac{\partial f_i}{\partial x_j}(x_0) \qquad \begin{array}{l}\forall i = 1..M \\ \forall j = 1..N\end{array}}$$

$$\boxed{f : \mathbb{R}^N \to \mathbb{R}^M \qquad df(x_0, w) = f'(x_0)w}$$

$N = M = 1$ $\quad f'(x_0)$ la derivata dell'unica componente scalare rispetto all'unica variabile

$N=1 \quad M>1$ $f'(x_0)$ è il vettore delle M derivate delle componenti di f rispetto all'unica variabile

$N>1 \quad M=1$ $f'(x_0)$ è il vettore delle N derivate parziali rispetto alle variabili indipendenti

$N>1 \quad M>1$ $f'(x_0)$ è la matrice la cui riga i-esima ($i=1..M$) contiene le N derivate parziali rispetto alle variabili ($j=1..N$)

Th $f: \Omega \to \Sigma$ e $g: \Sigma \to \mathbb{R}^p$ ed esistono

$df(x_0, w)$ e $dg(f(x_0), v)$

Allora, posto $h(x) = g(f(x))$ si ha

$$dh(x_0, w) = dg(f(x_0), df(x_0, v))$$

Th $f: \Omega \to \Sigma$ $g: \Sigma \to \mathbb{R}^p$ differenziabili

f in x_0 e g in $f(x_0)$, posto $h(x) = g(f(x))$ si ha

$$\boxed{h'(x_0) = g'(f(x_0)) \underbrace{f'(x_0)}_{\text{prodotto}}}$$

$\gamma : \mathbb{R} \to \mathbb{R}^N \quad N > 1$ (curva parametrica)

differenziabile $(= \exists \dot{\gamma} = \begin{pmatrix} \dot{\gamma}_1 \\ \vdots \\ \dot{\gamma}_N \end{pmatrix}$

$f : \mathbb{R}^N \to \mathbb{R}$ **differenziabile**

$h(t) = f(\gamma(t)) \qquad h : \mathbb{R} \to \mathbb{R}$

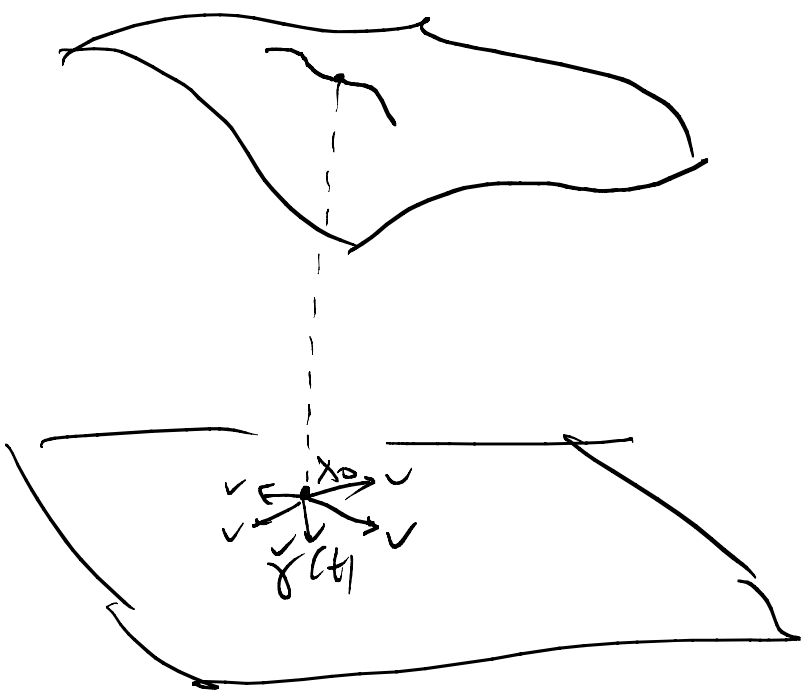
$h'(t) = \underbrace{f'(\gamma(t))}_{\in \mathbb{R}^N} \underbrace{\gamma'(t)}_{\text{prodotto scalare}} \in \mathbb{R}^N = \nabla f(\gamma(t)) \dot{\gamma}(t)$

$h(t) = f(\gamma(t))$

$h'(t) = \nabla f(\gamma(t))\,\dot\gamma(t)$ ←

$\gamma = x_0 + tv$

$\dot\gamma = v$



Se f è diff. allora

$f_v(x_0) = \nabla f(x_0)\,v$

$|\nabla f(x_0)\,v| \le |\nabla f(x_0)|\,|v|$

$|f_v(x_0)|$

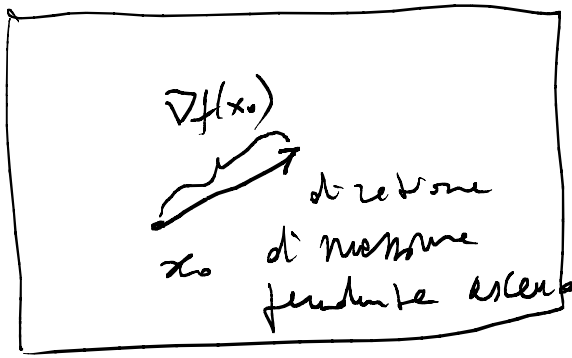
v è parallelo a $\nabla f(x_0)$

$\nabla f(x_0)\,v$ è max

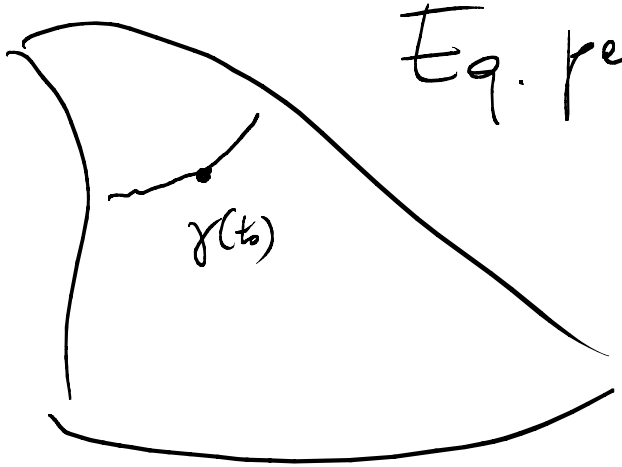
$v = \lambda \nabla f(x_0)$

$\lambda > 0$

dom f



$-\nabla f(x_0)$ è la direzione (a partire da x_0) di massima pendenza discendente.

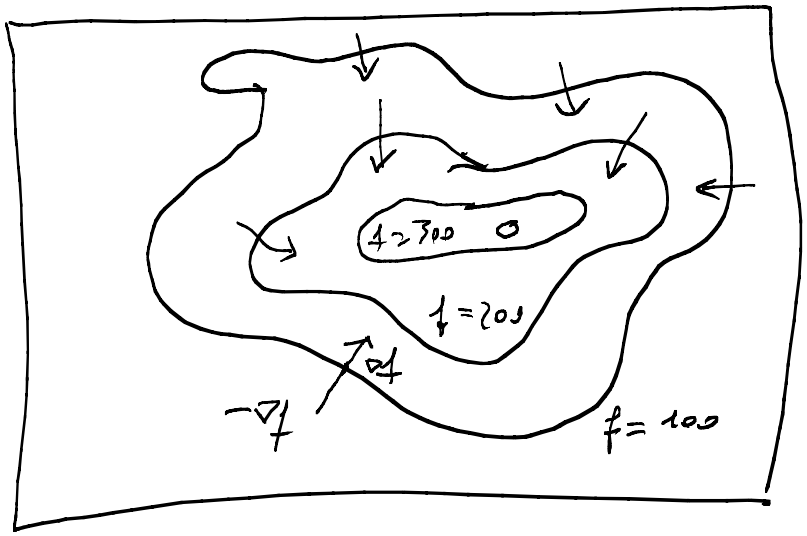


Eq. param. curve di livello

$$f(\gamma(t)) = \text{costante}$$

$$\nabla f(\gamma(t))\,\dot{\gamma}(t) = 0$$

La velocità della curva γ è perpendicolare al gradiente nel punto.



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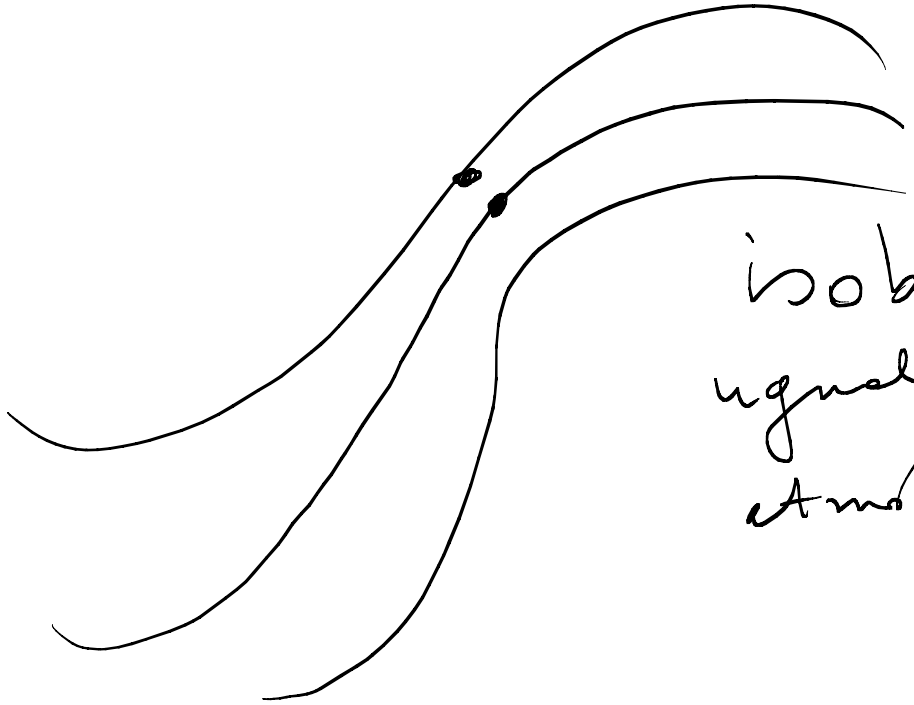
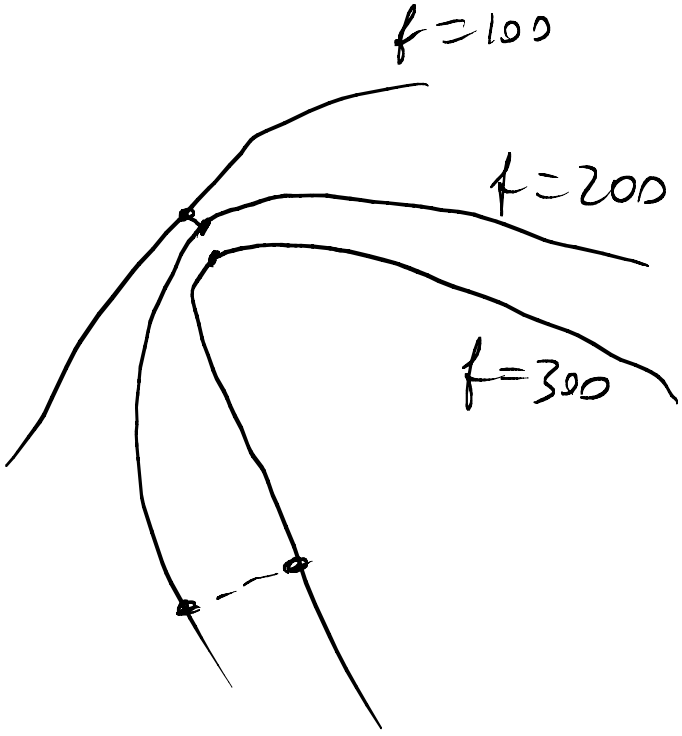
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