Note Title 4/7/2021

Se f è differenziabile in $\underline{\underline{x_0}}$ (il diff. sia A)

allora $f_v(x_0) = A(v)$

facile da calcolare

sconosciuto

Siccome A è lineare $\quad v = \sum v_i e_i \quad e_i$ base canonica

$$A(v) = \sum_1^N v_i A(e_i) = \underbrace{\sum_1^N v_i f_{x_i}(x_0)} = \nabla f(x_0) v$$

$$\boxed{df(x_0, v)}$$

$$\nabla f(x_0) = \begin{pmatrix} f_{x_1}(x_0) \\ f_{x_2}(x_0) \\ \vdots \\ f_{x_N}(x_0) \end{pmatrix}$$

$f(x,y) = |xy|$

in $(0,0)$ è diff cons.?

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = 0$$

$$f_y(0,0) = 0$$

Candidato $\underbrace{df\big(\underset{x_0}{(0,0)}, \underset{v}{(h,v)}\big)}_{A(v)} =$

$$\lim_{(h,k)\to(0,0)} \frac{f(0+h,0+k) - \overset{=0}{f(0,0)} - df\big((0,0),(h,k)\big)}{\sqrt{h^2+k^2}} =$$

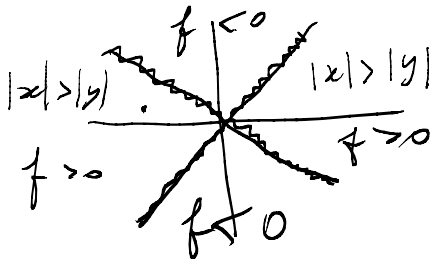
$$\underbrace{f_x(0,0)}_{=0} h + \underbrace{f_y(0,0)}_{=0} k = 0$$

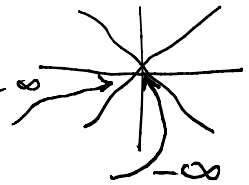
$$= \lim_{(0,0)} \frac{|hk|}{\sqrt{h^2+k^2}} = 0$$

1-omog.

definita dappertutto su $|(h,k)| = 1$

$$\lim_{(0,0)} \frac{1}{x^2-y^2} = \infty$$





$$\left| \frac{1}{x^2-y^2} \right| > \varepsilon \quad \text{per ogni} \quad |(x,y)| < \delta \quad (x,y) \neq (0,0)$$

$$|x^2-y^2| < \frac{1}{\varepsilon}$$

x^2-y^2 è continua in $(0,0)$ e quindi $\lim_{(0,0)} x^2-y^2 = (x^2-y^2)|_{(0,0)} = 0$

$$\lim_{0} f(x) = 0 \implies \lim_{0} \frac{1}{f(x)} = \infty \qquad x \in \mathbb{R}^N$$

$$\frac{1}{|f(x)|} > \varepsilon \iff |f(x)| < \frac{1}{\varepsilon} \impliedby |x| < \delta \quad x \neq 0$$

$$\lim_{(0,0)} \frac{1}{x^2+2y^2-xy} = +\infty$$

x^2+2y^2-xy

def. >0

$$\begin{pmatrix} 1 & -1/2 \\ -1/2 & 2 \end{pmatrix}$$

$$\lim_{(0,0)} (x^2+2y^2-xy) = 0$$

funzione continua

$$\begin{vmatrix} 1-\lambda & -\frac{1}{2} \\ -\frac{1}{2} & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - \frac{1}{4}$$

$$\boxed{\lambda^2 - 3\lambda + 2 - \frac{1}{4}}$$

$$\lambda = \frac{3 \pm \sqrt{9 - \frac{28}{4}}}{2} = \begin{cases} \frac{3-\sqrt{2}}{2} \\ \frac{3+\sqrt{2}}{2} \end{cases}$$

$f > 0$

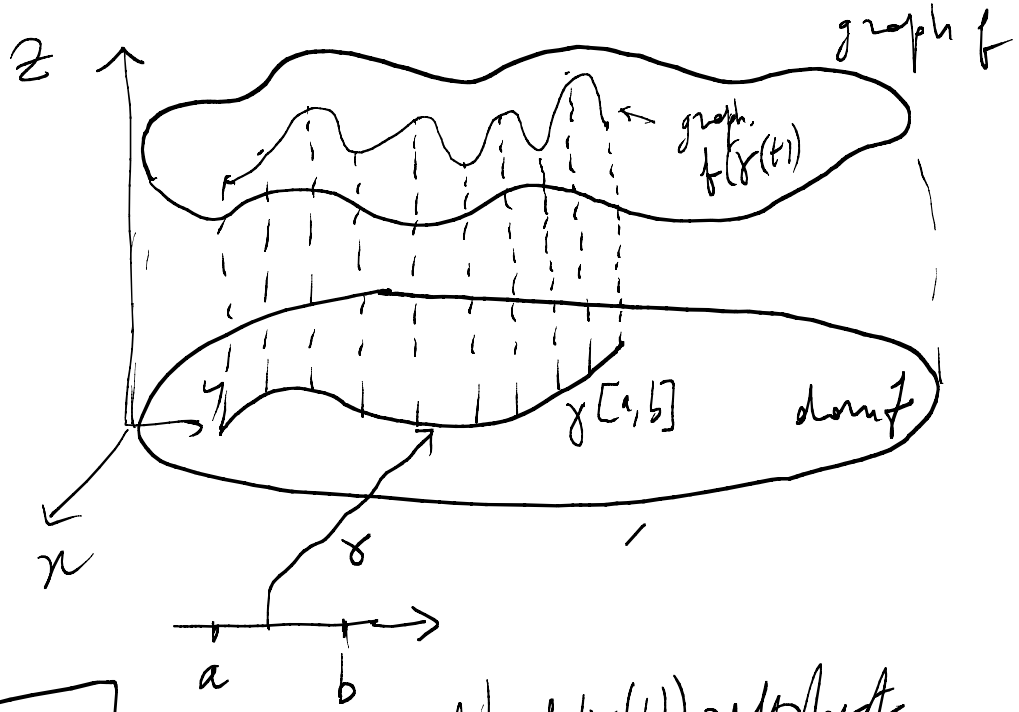
ore definita

$\Rightarrow \lim$

$|f(x)| > \varepsilon$

$f > 0$

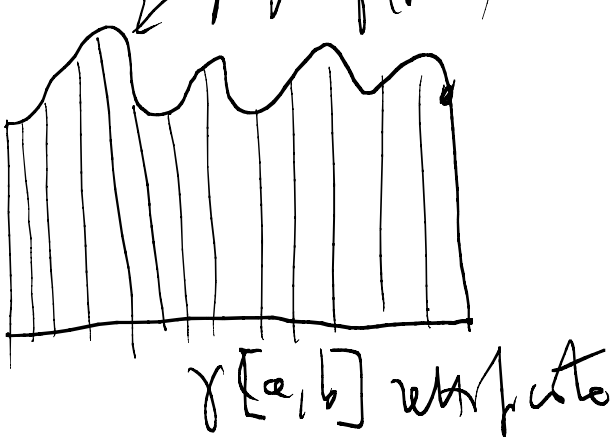
$\Leftrightarrow \underline{\underline{f(x) > \varepsilon}}$



$\gamma : [a,b] \to \Omega$

$f : \Omega \to \mathbb{R}$

$f > 0$



$$\int_\gamma f \, d\sigma = \int_a^b f(\gamma(t)) \, |\dot\gamma(t)| \, dt$$

$d\sigma$: lunghezza tratto nel tempo dt

$$\frac{\partial f}{\partial x_i}(x_0) \equiv f_{e_i}(x_0) = \lim_{h \to 0} \frac{f(x_0 + h e_i) - f(x_0)}{h}$$

tutte le componenti di x_0 restano costanti salvo l'i-esima che è incrementata di h

$$\frac{\partial (x^2 - y^2)}{\partial x}(1,2) = \lim_{h \to 0} \frac{f(1+h,2) - f(1,2)}{h} =$$

$$(1,2) + h(1,0) = (1+h, 2)$$

$$\lim_{h \to 0} \frac{(1+h)^2 - 2^2 - (1^2 - 2^2)}{h} =$$

$f(1+h,2)$ $\qquad$ $f(1,2)$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h} =$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h} = 2$$

$(1, 2, 3) + h(0, 1, 0)$ $\qquad \frac{\partial}{\partial y}$

x_0 $\qquad e_2$

$f(x_0 + t e_2) \equiv f(1, 2+h, 3)$

$f: \mathbb{R} \to \mathbb{R}$ $\qquad df(x_0, h) = f'(x_0)h$

$f: \mathbb{R}^N \to \mathbb{R}$ $\qquad f'(x_0) = \nabla f(x_0) = \begin{pmatrix} f_{x_1}(x_0) \\ \vdots \\ f_{x_N}(x_0) \end{pmatrix}$

$$f': \Omega \to \mathbb{R}^N$$

$$df(x_0, v) = \nabla f(x_0) v$$

N variabili $\qquad$ N variabili

$$df: \Omega \times \mathbb{R}^N \to \mathbb{R}$$

f α-homog. + f kontin. na $\mathrm{dom} f \cap \{|x|=1\}$

$\Rightarrow \lim_0 f(x) = 0$

$$f(x) = f\left(|x| \frac{x}{|x|}\right) \overset{\text{homog.}}{=} |x|^\alpha f\left(\frac{x}{|x|}\right)$$

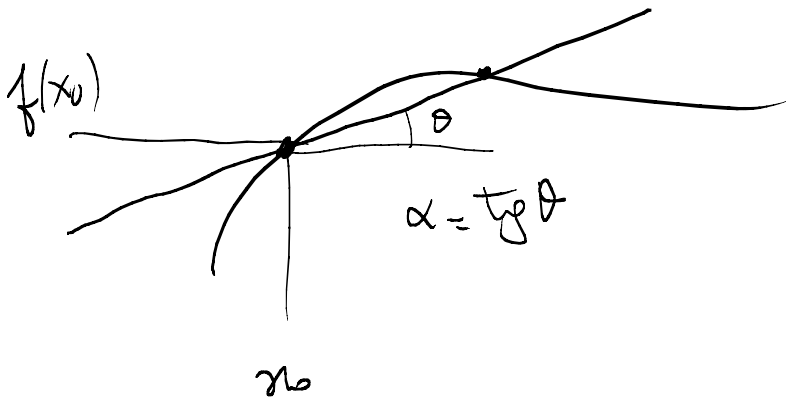
$\hookrightarrow \in \{|x|=1\}$

$$0 \le |f(x)| = |x|^\alpha \cdot \underbrace{\left|f\left(\frac{x}{|x|}\right)\right|}_{\le K} \le K|x|^\alpha \xrightarrow[x\to 0]{} 0$$

$\downarrow x \to 0$

0

$\le K$ jelikož f je kontin. na $\{|x|=1\}$



$(h = x - x_0)$

$y = f(x_0) + \alpha(x - x_0) = g(x)$

$g(x)$

$$\frac{f(x) - \overbrace{[f(x_0) + \alpha(x - x_0)]}}{(x - x_0)} = \frac{f(x) - f(x_0)}{x - x_0} - \alpha$$

funzione derivabile

$\downarrow$

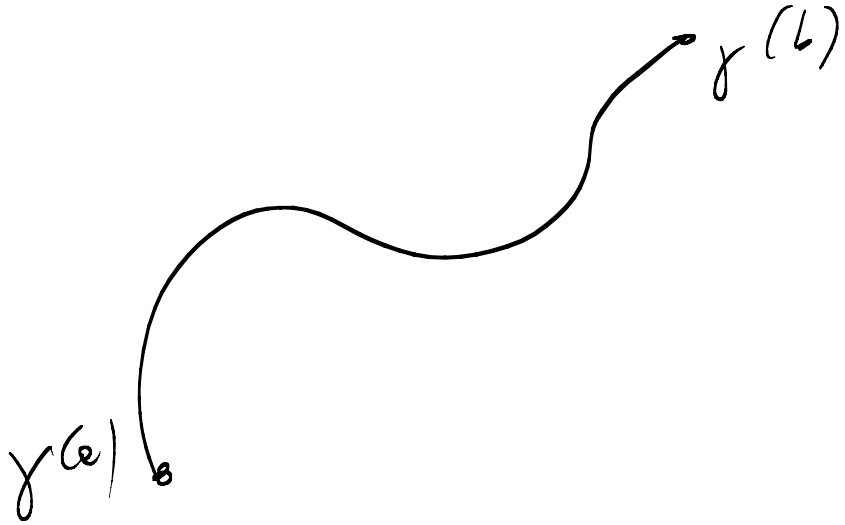
$f'(x_0) - \alpha$

$f(x) - g(x)$ è di ordine 1 rispetto a $(x - x_0)$

se $\alpha \neq f'(x_0)$

è di ordine superiore ad 1 se $\alpha = f'(x_0)$

$\gamma : [a,b] \to \mathbb{R}^N$



$\rho : [0,1] \to [a,b]$

$\rho(0) = b$

$$\rho(t) = b + t[(a-b)]$$

$\rho(0) = b \quad \rho(1) = a \quad \dot{\rho} < 0$

$$\left| \int_a^b \dot{\gamma}(t)\,dt \right| \qquad \int_a^b |\dot{\gamma}(t)|\,dt$$

$$\approx \qquad \approx$$

$$\left| \sum_1^N \dot{\gamma}(\xi_i)(t_{i+1}-t_i) \right| \leq \sum_1^N |\dot{\gamma}(\xi_i)|(t_{i+1}-t_i)$$

$t_{i+1}-t_i > 0$

Dreiecksungleichung

$$\frac{p(t)}{q(t)}$$ funzioni razionali = rapporti di polinomi

$$\int \frac{p(t)}{q(t)}\,dt$$ si deve sapere cosa si può fare e come farlo

$\boxed{f(\cos x, \sin x)}$ con f razionale si trasforma in una razionale ponendo $\boxed{t = \operatorname{tg}\frac{x}{2}}$

$$\boxed{\cos t = \frac{1-t}{1+t^2} \qquad \sin t = \frac{2t}{1+t^2}}$$

$$f_v(x_0) = \lim_{t \to 0} \frac{f(x_0 + tv) - f(x_0)}{t}$$

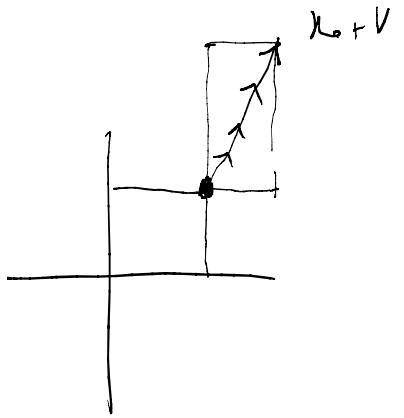
de derivata functie

$$f_{e_1}(x_0) \equiv \frac{\partial f}{\partial x}(x_0)$$

$f(x,y) = x^2 y$

$$\frac{\partial f}{\partial (1,2)}(1,1) =$$

x_0, v



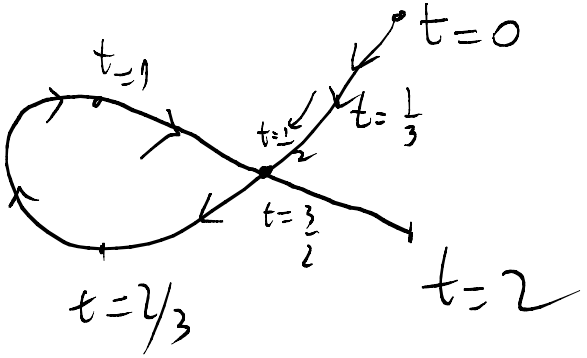
$$(1,1) + t(1,2) = (1+t, 1+2t)$$

$x_0 + t$ ✓

$$\nabla f(x,y) = \begin{pmatrix} 2xy \\ x^2 \end{pmatrix}\Big|_{(1,1) = x_0} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\frac{\partial f}{\partial (1,2)}(1,1) = \lim_{0} \frac{1}{t}\left[\underbrace{(1+t)^2}_{x^2}\underbrace{(1+2t)}_{y} - 1^2 \cdot 1\right]$$

$$f_v(1,1) = (2,1)(v_1, v_2) = 2v_1 + v_2$$

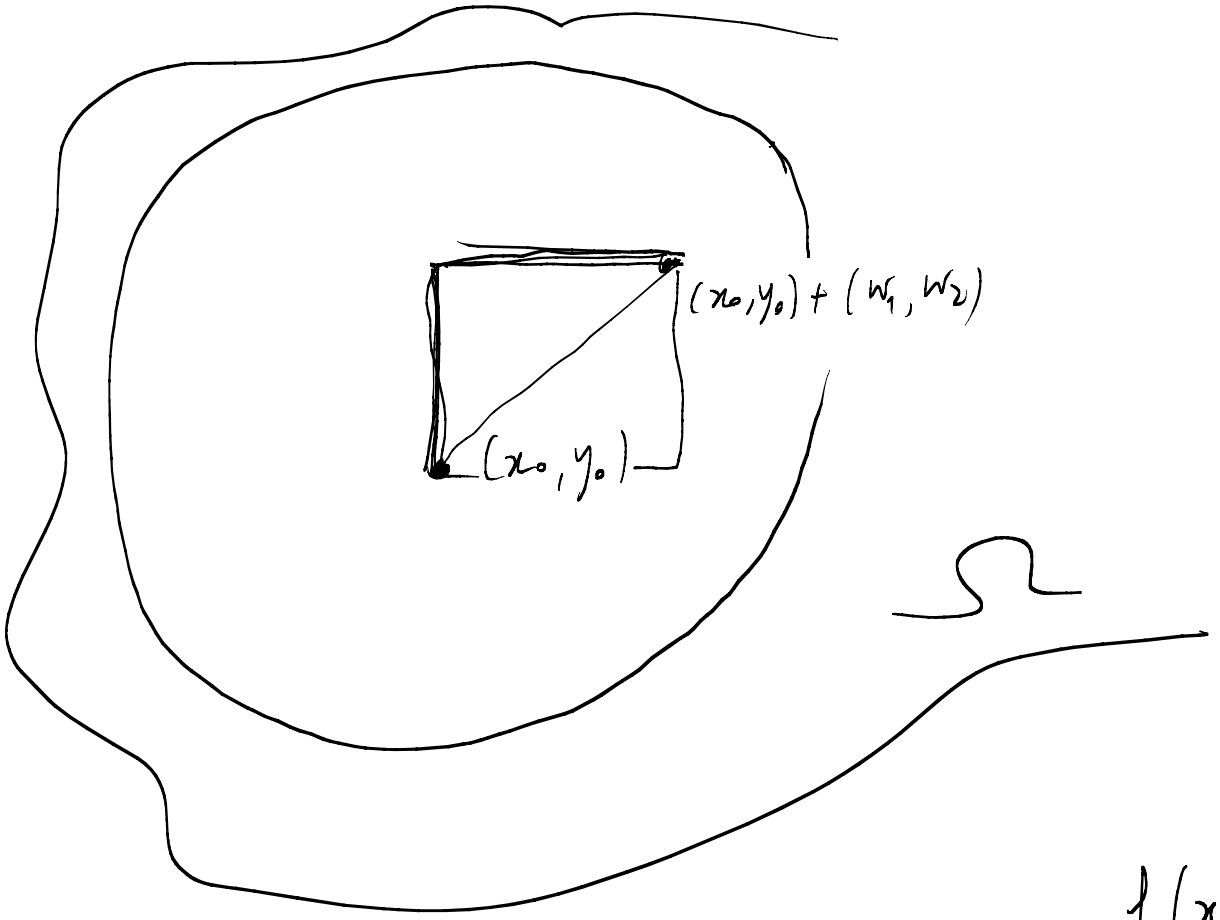


$\gamma : [0,2] \to \mathbb{R}^2$

$\gamma\left(\frac{1}{2}\right) = \gamma\left(\frac{3}{2}\right)$

NON INIETTIVA

cioè NON semplice



|w| piccolo

$$df(x_0, w) = \sum_1^N \frac{\partial f}{\partial x_i}(x_0) w_i = \sum_1^N \frac{\partial f}{\partial x_i} dx_i$$

$f(x)$ è lineare

allora $df(x_0, v) = f(v)$

$\Pi_x(x, y) = x \qquad \Pi_y(x, y) = y \qquad d\Pi_x((x_0, y_0), (h, k)) = \Pi_x(h, k) = h \Rightarrow d\Pi_x \equiv dx$

$$\lim_{(0,0)} \frac{xy - \sin xy}{3x^2 + 2y^2} \qquad (x,y) \neq (0,0)$$

$$\frac{xy - \sin xy}{(xy)^3} \cdot \frac{(xy)^3}{3x^2+2y^2} \longrightarrow 0$$

$xy \neq 0$

$(x,y) \to (0,0)$

$\frac{1}{6}$

4-omogenea, continua sempre definita in $|(x,y)| = 0$

$\downarrow$

0

$(x,y) \to (0,0)$

$xy \neq 0$

$\underline{xy = 0}$ il numeratore è 0 sempre $\lim_{\substack{\boxed{xy=0} \\ (x,y)\to(0,0)}} \left[\frac{xy - \sin xy}{3x^2+2y^2}\right] = 0$

$\Downarrow$

0

$$\lim_{(0,0)} \frac{\sin(2x^2-3y^2)}{x^2+y^2}$$

$t = x^2-3y^2$

$$\frac{\sin(x^2-3y^2)}{x^2-3y^2} \cdot \frac{x^2-3y^2}{x^2+y^2}$$

$\to 1$

sui raggi ho compat. diversi

$x^2-3y^2 \neq 0$

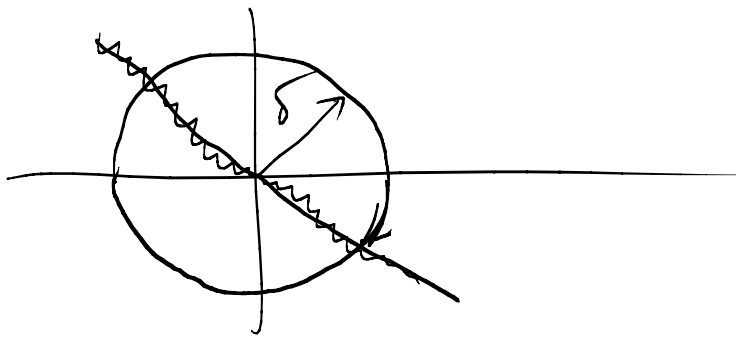
$$\frac{1-\cos\sqrt{x^2+y^2}}{x^2+2y^2} = \frac{1-\cos\sqrt{x^2+y^2}}{x^2+y^2} \cdot \frac{x^2+y^2}{x^2+2y^2}$$

$\to \frac{1}{2}$

Dominante non costante

$$\frac{\sin x^2 + y^2}{\sqrt{x^2+2y^2}} =$$

$y = -x$



$$\sin x^2 = x^2 - (x^2 - \sin x^2)$$

$$x = (\cos\theta)\,\delta$$

$$y = (\sin\theta)\,\delta$$

$\theta \to -\frac{\pi}{4}$

$$= \frac{\sin x^2 + y^2}{x^2+y^2} \cdot \frac{x^2+y^2}{\sqrt{x^2+2y^2}} \to 0$$

$x \neq 0$

$$\frac{x^2\left(\frac{\sin x^2}{x^2} + \frac{y^2}{x^2}\right)}{x^2\left(1 + \frac{y^2}{x^2}\right)}$$

1-omogenea

continua in $|(x,y)| = 1$

$$\frac{\sin x^2 + y^2}{\sqrt{x^2 + 2y^2}} = \frac{(\sin x^2 - x^2) + x^2 + y^2}{\sqrt{\quad}} =$$

$$= \frac{\sin x^2 - x^2}{x^6} \cdot \frac{x^6}{\sqrt{x^2 + 2y^2}} + \frac{x^2 + y^2}{\sqrt{x^2 + 2y^2}}$$

$-\frac{1}{6}$ $\quad t = x^2$ $\qquad$ 0 5-omogenea continua in $B_1(0)$ $\qquad$ 0 1-omog. continua in $B_1(0)$

$\sin t = t - \frac{t^3}{6} + o(t^3) \qquad \frac{\sin t - t}{t^3}$

$$\lim_{(0,0)} \frac{x \sin y}{(x^2+y^2)^{2/3}}$$

$$\frac{x \sin y}{xy} \cdot \frac{xy}{(x^2+y^2)^{2/3}}$$

2-omog.

il rapporto è $\frac{2}{3}$-omog. continua su $x^2+y^2=1$

$\frac{4}{3}$ omog.

$\to 1$

$\to 0$