Note Title 3/31/2021

Lemma fondamentale

CNS finché $\langle A_1, A_2, \dots, A_n \rangle = \langle A_2, \dots, A_n \rangle$

è che $A_1 \in \langle A_2, \dots, A_n \rangle$

C.N. $\langle A_1, A_2 \dots A_n \rangle = \langle A_2 \dots A_n \rangle \Longrightarrow A_1 \in \langle A_2 \dots A_n \rangle$

$\Downarrow$

$A_1 \in \langle A_1 \dots A_n \rangle = \langle A_2, \dots, A_n \rangle \Rightarrow A_1 \in \langle A_2 \dots A_n \rangle$

$A_1 = 1 \cdot A_1 + 0 A_2 + \dots + 0 A_n$

C.S. $A_1 \in \langle A_2, \dots, A_n \rangle \Rightarrow \langle A_1, \dots, A_n \rangle = \langle A_2 \dots A_n \rangle$

$\Downarrow$

$\exists\, \alpha_2 \dots \alpha_n : A_1 = \sum_2^n \alpha_i \cdot A_i$ $\quad B \in \langle A_1 \dots A_n \rangle$ generico $\exists\, \beta_1 \dots \beta_n : B = \sum_1^n \beta_j A_j$

Sottraendo ad $A_1$ $\sum_2^n \alpha_i A_i$ si ottiene

$$B = \beta_1 \sum_2^n \alpha_i A_i + \sum_2^n \beta_j A_j \in \langle A_2, \dots, A_n \rangle$$

AL_3.1, medio

$$\lim_{(0,0)} \frac{\ln\left(1+\sqrt{x^2+y^2}\right)-\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$$

$t = \sqrt{x^2+y^2}$

$$g(t) = \frac{\ln(1+t)-t}{t} = \text{NON È DEF. IN } 0$$

$t \to 0$

$$\ln(1+t) = t - \frac{t^2}{2} + o(t^2)$$

$$f' = \frac{1}{1+t}\Big|_0 = 1$$

$$f'' = -\frac{1}{(1+t)^2}\Big|_0 = -1$$

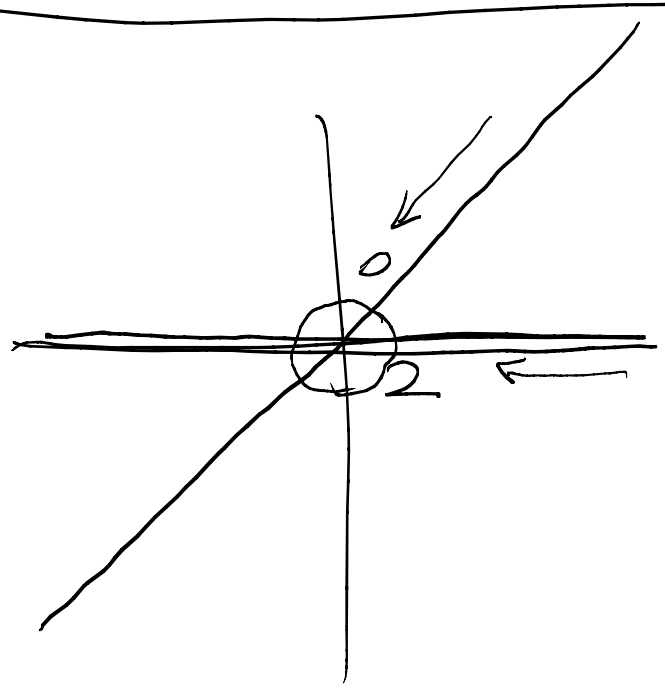
$$\frac{t - \frac{t^2}{2} + o(t^2) - t}{t} =$$

$$= -\frac{t}{2} + \frac{o(t^2)}{t} \to 0$$

$$\lim_{(0,0)} \frac{x^2 - y^2}{1 - \cos\sqrt{x^2+y^2}}$$

# NON ESISTE

$$\frac{x^2-y^2}{x^2+y^2} \cdot \frac{x^2+y^2}{1-\cos\sqrt{x^2+y^2}} \to 2$$




0 su $y = x$

1 su $y = 0$

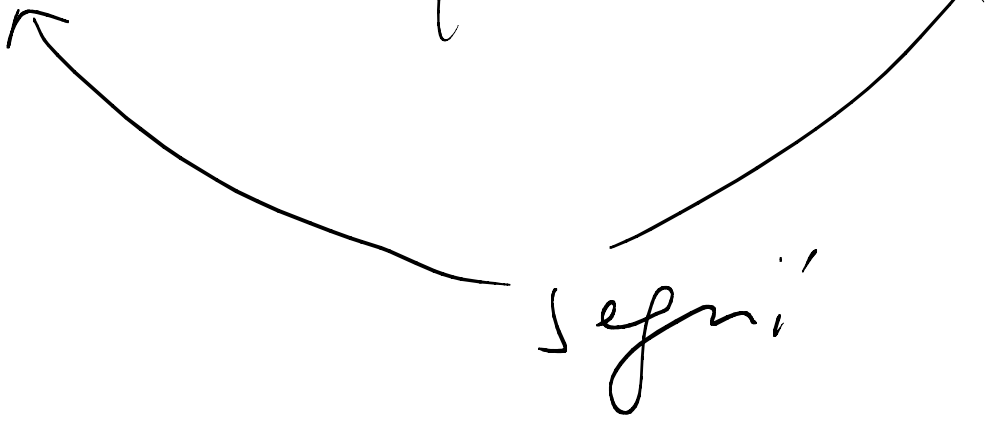
$$\lim_{\infty} (x^2 + 2xy - 3y^2)$$

NON ESISTE

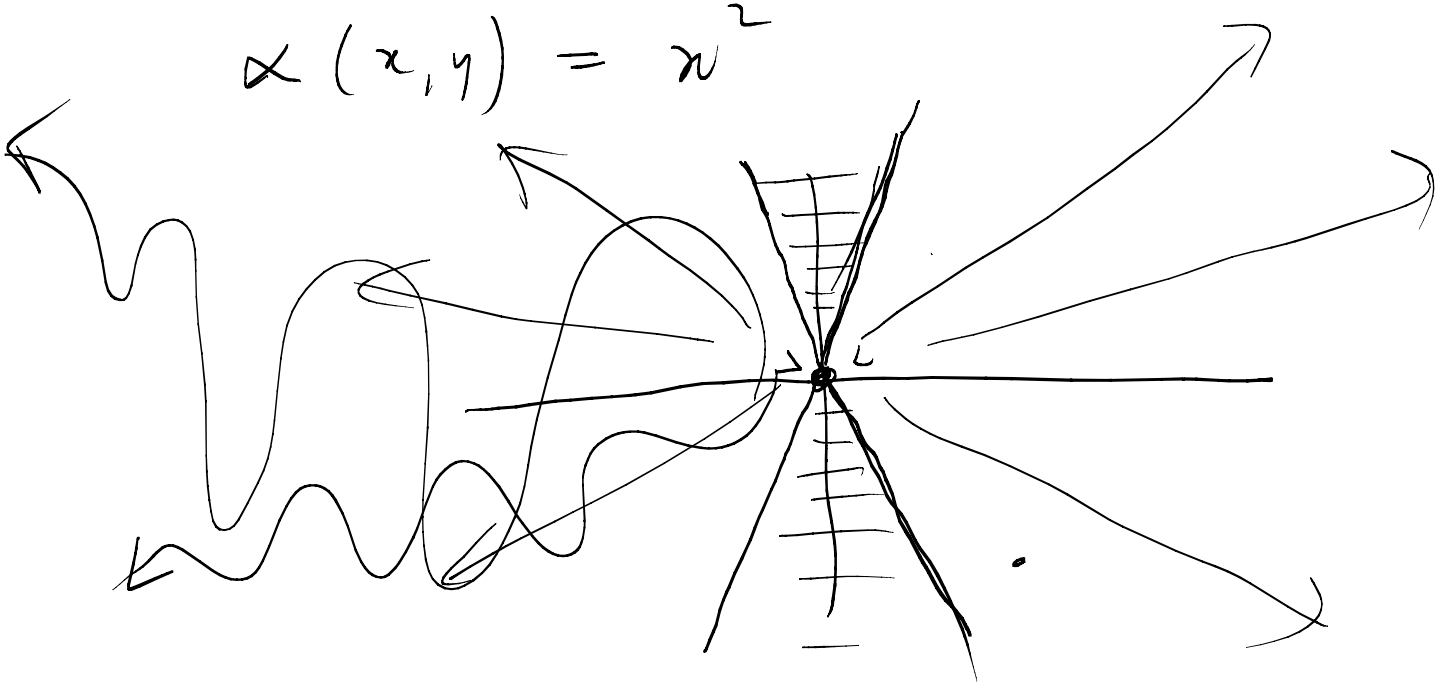
perché la forma è indefinita!

$$\lambda |x|^2 \leq q(x_1 \dots x_n) \leq \Lambda |x|^2$$

quadratica



$\alpha(x,y) = x^2$



$|y| \le \alpha |x|$

$\alpha > 0$

$\lim_{\infty} \quad x^2$

$|y| \le \alpha |x|$

$x^2 > \varepsilon$

$\Updownarrow$

$|x| > \varepsilon^{1/2}$

$\sqrt{x^2+y^2} \ge |x| \sqrt{1+\alpha^2}$

$|(x,y)| > \delta$

$|x| \sqrt{1+\alpha^2} > \delta$

$\Downarrow$

$|x| > \frac{\delta}{\sqrt{1+\alpha^2}}$

$\frac{\delta^2}{1+\alpha^2} > \varepsilon$

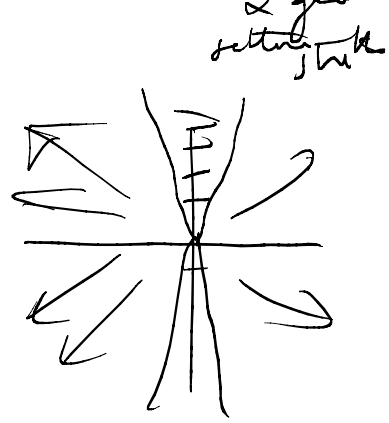
$|x| > \frac{\varepsilon^{1/2}}{\sqrt{1+\alpha^2}}$

$$\lim_{\infty} x^2 = +\infty$$

$$|y| < \alpha |x|$$

$\exists \delta : \quad |(x,y)| > \delta \Rightarrow x^2 > \varepsilon$

$$|x| > \varepsilon^{1/2}$$




$$|(x,y)| > \delta \Rightarrow \delta < |x|\sqrt{1+\alpha^2}$$

$$\sqrt{x^2+y^2} \qquad \frac{\delta}{\sqrt{1+\alpha^2}} < |x|$$

$$\wedge$$

$$|x|\sqrt{1+\alpha^2}$$

$$\boxed{\varepsilon^{1/2} < \frac{\delta}{\sqrt{1+\alpha^2}}} \qquad \delta > \varepsilon^{1/2}\sqrt{1+\alpha^2}$$

$$\frac{x^2+y^2}{x-y}$$

$$x = \rho\cos\theta$$
$$y = \rho\sin\theta$$

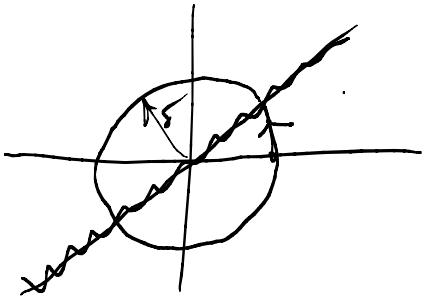
$$\frac{\rho^2\cos^2\theta + \rho^2\sin^2\theta}{\rho\cos\theta - \rho\sin\theta} = \frac{\rho^2}{\rho(\cos\theta-\sin\theta)} = \rho\,\frac{1}{\cos\theta-\sin\theta}$$

$0\cdot\infty$

$\rho \to 0$ quando $(x,y)\to(0,0)$

$$\frac{\delta}{\cos\theta-\sin\theta} > \varepsilon$$

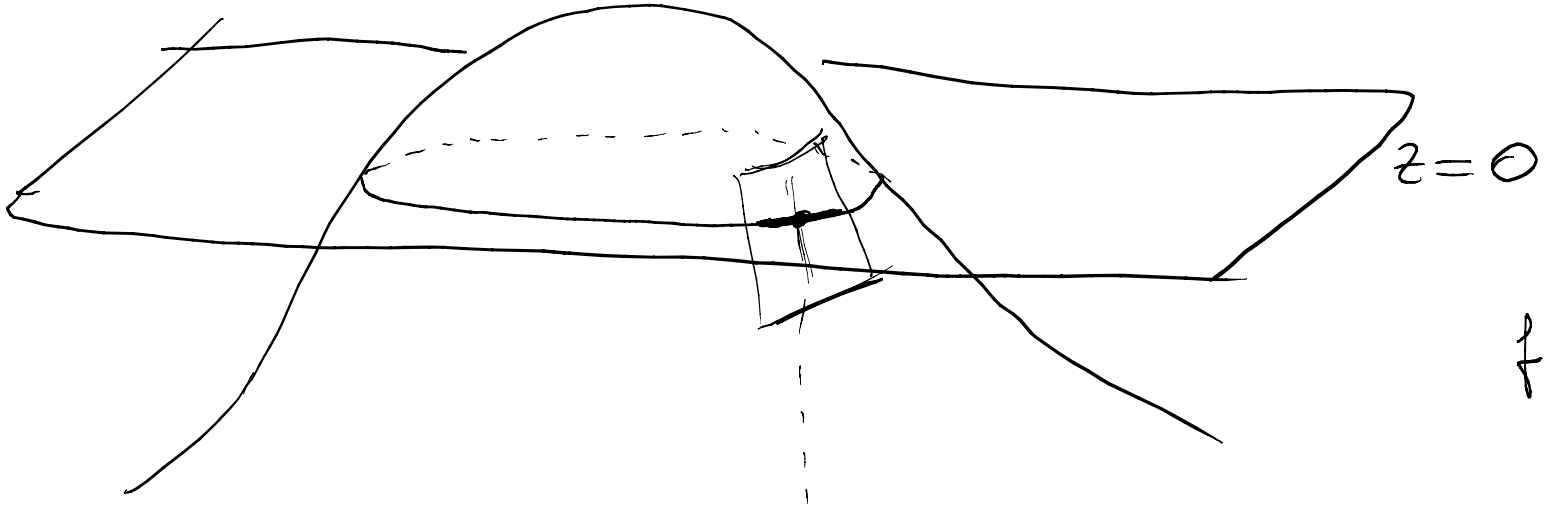
$\theta \to \frac{\pi}{4}^-$

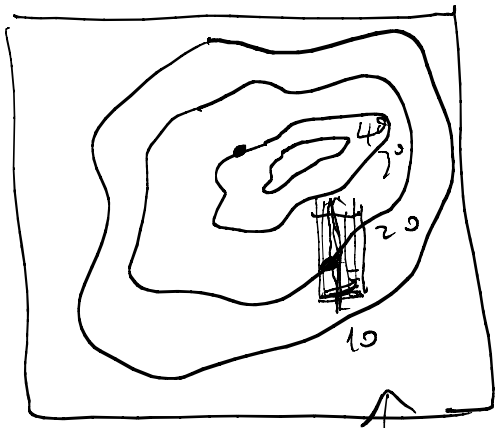



$y = \alpha x$

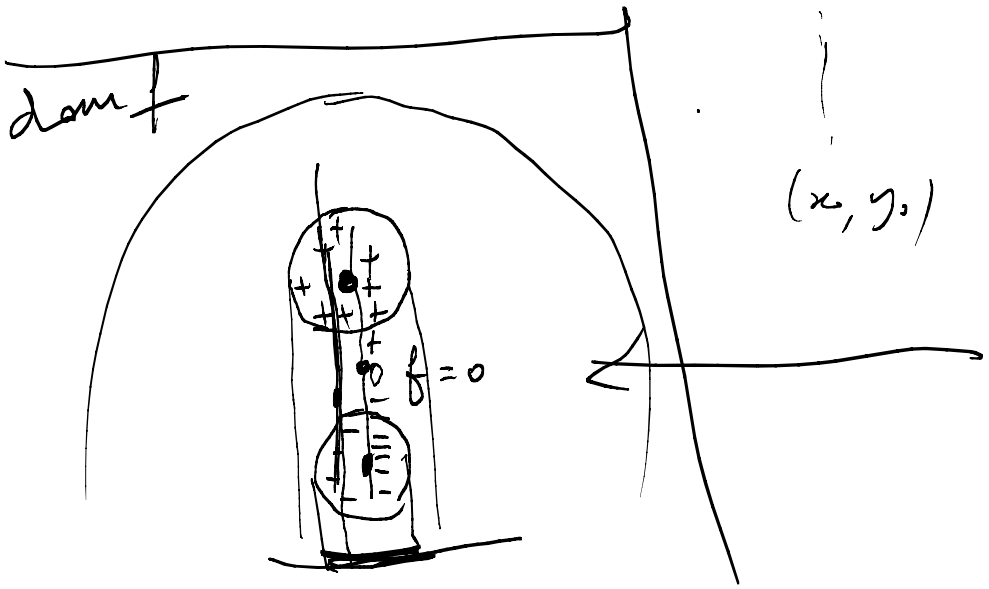
$$\frac{x^2+\alpha^2x^2}{x-\alpha x} = \frac{x^2}{x}\,\frac{1+\alpha^2}{1-\alpha}$$

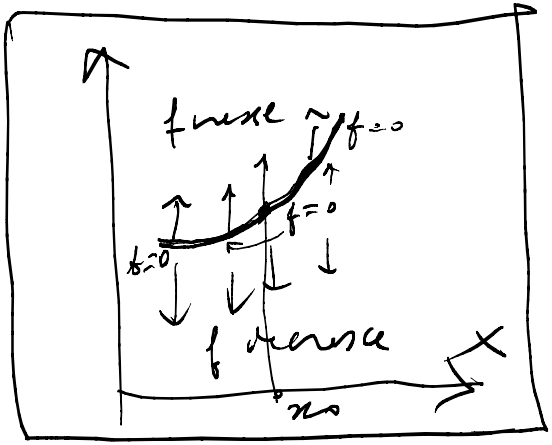
$x \to 0$




$f(x_0, y_0) = 0$ ←




$f(x, \varphi(x)) = 0$

$[x_0 - \delta, x_0 + \delta]$

$f(tx) = t^\alpha f(x)$ $\quad$ $f$ è $\alpha$-omog.

$g(ty) = t^\beta g(y)$ $\quad$ $g$ è $\beta$-omog.

$$g(f(tx)) = g\underbrace{(t^\alpha f(x))}_{s} = \underbrace{(t^\alpha)^\beta}_{s^\beta} g(f(x))$$

$\alpha\beta$-omog.

polinomio non costante $p(z_0) \neq 0$

$$\exists\, z^* \in \mathbb{C} : \quad |p(z^*)| < |p(z_0)|$$

$$\exists\, \bar{w} : |q(\bar{w})| < 1$$

$$|q(w)| \leq |1 - w^k \alpha_k| + |w|^{k+1} |\tilde{q}(w)| \quad (*)$$

Scegliamo $w$ : $-w^k \alpha_k \in$ reale negativo $\left(\arg(w^k \alpha_k) = \pi\right)$

$$\arg \alpha_k + k \arg w = \pi$$

$$\arg w = \frac{\pi - \arg \alpha_k}{k}$$

$$|1 - \alpha_k w^k| = 1 - |\alpha_k w^k|$$

differenza di moduli

$1$ reale positivo

$\alpha_k w^k$ reale negativo di modulo più piccolo

$$= |\alpha_k| |w|^k$$

$$|\alpha_k w^k| < 1$$

$$\Rightarrow |w| < \frac{1}{|\alpha_k|^{1/k}}$$

Da (*) segue

$$|q(w)| \le 1 - |\alpha_k||w|^k + |w|^{k+1}|\tilde{q}(w)| =$$

$$= 1 - |w|^k \underbrace{\left[ |\alpha_k| - |w||\tilde{q}(w)| \right]}_{>0}$$

$|w|^k \to 0$ se $w \neq 0$

$w \to 0$

$|\alpha_k| > 0$

per perm. del segno $[\ ] > 0$ se $|w| < \delta$

$\arg w = \dfrac{\pi - \arg \alpha_k}{k}$ $\quad |w| < \min\left\{\delta, \dfrac{1}{|\alpha_k|^{1/k}}\right\} \Rightarrow$ il II termine è $>0$

e quindi $|q(w)| < 1$