Note Title 3/26/2021

funzioni α-omogenee $\quad \alpha < 0$

$$f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$$

$$f(tx,ty) = \frac{1}{\sqrt{t^2(x^2+y^2)}} = |t|^{-1} f(x,y) \underset{t>0}{=} t^{-1} f(x,y)$$

$$\lim_{(x,y)\to(0,0)} \frac{1}{\sqrt{x^2+y^2}} = +\infty$$

$\forall \varepsilon > 0 \quad \exists \delta > 0 : \quad \forall (x,y) \in \operatorname{dom} f, \ |(x,y)| < \delta, \ (x,y) \neq 0$

$(x,y) \neq (0,0)$

$$\frac{1}{\sqrt{x^2+y^2}} > \varepsilon$$

estraendo i membri > 0

$$\sqrt{x^2+y^2} < \frac{1}{\varepsilon}$$

$t \to \frac{1}{t}$ è strett. decresc.

$\delta = \frac{1}{\varepsilon}$

$f : X \to \mathbb{R}$ $\quad X$ cono $\quad \alpha < 0$

$\boxed{x \in X \subseteq \mathbb{R}^N}$ $\quad f$ sia α-omogenea.

$$|f(x)| = \left| f\left(|x| \frac{x}{|x|}\right) \right| \overset{|x| \neq 0}{=}$$

$$= \underbrace{|x|^\alpha}_{+\infty} \underbrace{\left| f\left(\frac{x}{|x|}\right) \right|}_{?}$$

f diverge?

$\bar{x}$ fissato in dom f

$$f(t\bar{x}) = t^\alpha f(\bar{x})$$

$$t \to 0^+ \Rightarrow t^\alpha \to +\infty$$

$\boxed{\text{Supponiamo che } \exists k > 0 : \left| f\left(\frac{x}{|x|}\right) \right| \geq k \quad \forall x \in X \text{ dom } f}$

$$|x|^\alpha \left| f\left(\frac{x}{|x|}\right) \right| \geq k |x|^\alpha \xrightarrow[x \to 0]{} +\infty$$

perché $\alpha < 0$

se fosse $\inf \left| f\left(\frac{x}{|x|}\right) \right| = 0$

$\forall \sigma = \frac{1}{n} \quad \exists x_n : \quad \left| f\left(\frac{x_n}{|x_n|}\right) \right| < \frac{1}{n}$

$\boxed{y_n = \frac{x_n}{|x_n|}}$

$\Downarrow$

$\lim_{0} f(x) \neq \infty$ perché se lo fosse

f diverge ∞ se

$\forall \varepsilon > 0 \quad \exists \delta > 0 \; ; \; x \in \operatorname{dom} f \quad |x| < \frac{\delta}{\bar{\delta}} \quad x \neq 0$

$\varepsilon = 1 \qquad \bar{\delta}$

$\boxed{|f(x)| > \varepsilon \quad (1)}$

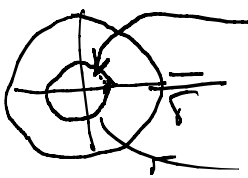
$B_{\bar{\delta}}(0)$

fermo

$|f(x)| = |x|^{\alpha} \left| f\left(\frac{x}{|x|}\right) \right|$

$\left| \frac{\bar{\delta}}{2} \frac{x_n}{|x_n|} \right| = \frac{\bar{\delta}}{2}$

$\left| f\left(\frac{\bar{\delta}}{2} \frac{x_n}{|x_n|} \right) \right| = \left(\frac{\bar{\delta}}{2} \right)^{\alpha} \left| f\left(\frac{x_n}{|x_n|}\right) \right| \xrightarrow{n \to \infty} 0$



raggio $\frac{\bar{\delta}}{2}$

Lemma Se f è α-omog., $\alpha<0$, su un cono X

allora

$\lim_{0} f = \infty$ se e solo se $\exists k>0$:

$\boxed{\mathcal{F}B_1(0) = \{ |x,y| = 1 \}}$ $|f(x)| > k$ su $X \cap \mathcal{F}B_1(0)$

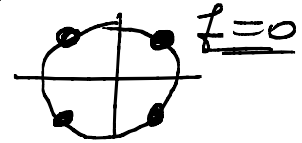
$$\lim_{(0,0)} \frac{x^2-y^2}{x^4+y^4} = \infty \; ? \; \text{NO}$$

$\frac{1}{x^2-y^2}$ è (-2)-omogenea sul cono $(x,y) \neq (0,0)$

$\frac{x^2-y^2}{x^4+y^2}$ su $\underbrace{\mathcal{F}B_1(0)}$?

Rapporto di funzioni continue su $\boxed{x^2+y^2=1}$ che si annulla

$\boxed{\text{NO}}$

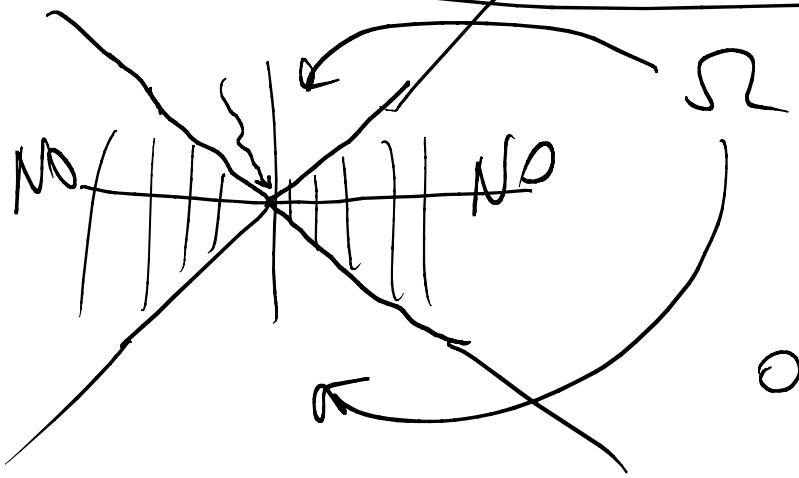


$$\lim_{\substack{(x,y)\to(0,0)\\(x,y)\in\Omega}} \underbrace{x \lg y}_{f(x,y)}$$

$\operatorname{dom} f = \{y > 0\}$

$(0,0)$ è di accum. per $\operatorname{dom} f$

1) $\Omega = \{(x,y) : |y| > |x|\}$



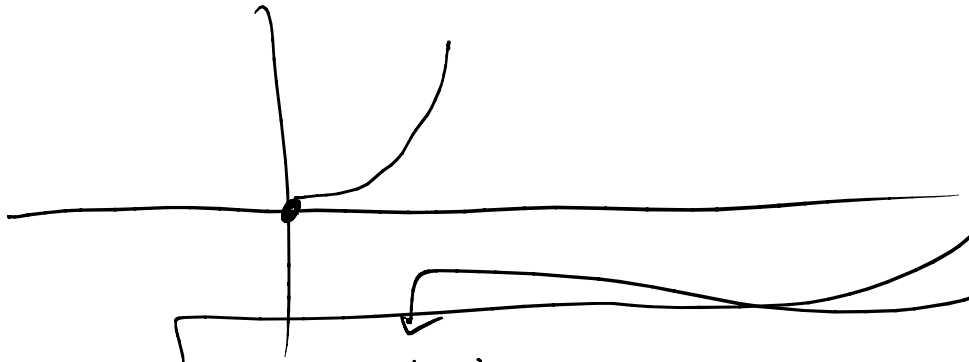
$(0,0)$ è di accum. anche per Ω

$$0 \le |x \lg y| = |x|\,|\lg y| \underset{\text{su }\Omega}{\le} |y|\,|\lg y| = |y \lg y| \to 0$$

per l'assol. 1°

Th. confronto lim = 0

2) $\Omega = \{y > 0\} = \text{dom } f$ massimale.



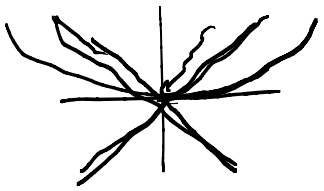
$\boxed{x \to 0}$

$\boxed{y = e^{-\frac{1}{x^2}}}$

$$\lim_{x \to 0} e^{-\frac{1}{x^2}} = 0$$

$$f\left(x, e^{-\frac{1}{x^2}}\right) =$$

$$= x \lg\left(e^{-\frac{1}{x^2}}\right) = x\left(-\frac{1}{x^2}\right) \not\to 0$$



$$\lim_{(0,0)} \frac{1}{x^2+2y^2-xy}$$

$\lim_{\infty}$

il modulo diverge a $+\infty$

$-\frac{1}{x}$ oscilla se $x \to 0$