Note Title 3/23/2021

$a_n \to L$ in $\mathbb{R}^N \iff (a_n)_i \to L_i \quad \forall i = 1..N$

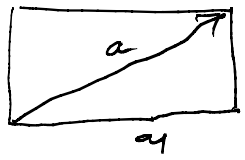
$\left(\frac{n+1}{n}, \left(1+\frac{1}{n}\right)^n, \frac{n-1}{n} \right) \to \left(1, e, 1\right)$

$\forall\, a \in \mathbb{R}^N$

$$\to \quad |a_i| \le |a| \le \sqrt{N} \max_{1..N} |a_i|$$

costante dipendente solo dalla dimensione

$|a_i|$: modulo in $\mathbb{R}$; $|a|$: norma in $\mathbb{R}^n$



$\max\{|a_1|, |a_2|\} \le |a| \le \sqrt{2} \max\{|a_1|, |a_2|\}$

$$a_n \to L \iff \boxed{|a_n - L| \to 0}$$

$\Longrightarrow$ $\quad |a_n - L| \to 0 \iff \forall \varepsilon > 0 \ \exists \nu : \forall n > \nu$

$$|a_n - L| < \varepsilon$$

Poiché $|a| \geq |a_i| \ \forall i = 1..N$ segue, per gli stessi ε e ν

$$|(a_n - L)_i| < |a_n - L| < \varepsilon$$

$$\lim_n |(a_n)_i - L_i)| = 0 \iff \lim_n (a_n)_i = L_i \ \forall i = 1..N$$

$\Longleftarrow$ $\quad$ se $(a_n)_i \to L_i \quad \forall i = 1..N$

$\varepsilon > 0 \quad \exists \nu_i : \forall n > \nu_i \quad \boxed{|(a_n)_i - L_i| < \varepsilon}$

$$\nu = \max_{i=1..N} \nu_i$$

$$\Rightarrow \max_{1..N} |(a_n)_i - L_i| < \varepsilon \Rightarrow |a_n - L| \leq \varepsilon\sqrt{N}$$

$$|a_n - L| \to 0$$

$$\forall$$

$$|(a_n - L)_i|$$

$$\|$$

$$|(a_n)_i - L_i| \geq 0$$

$$\Rightarrow |(a_n)_i - L_i| \to 0$$

de $|(a_n)_i - L_i| \to 0 \quad \forall i \Rightarrow \max_{1..N} |(a_n)_i - L_i| \to 0$

$$\| \qquad \to 0$$

$$0 \leq |a_n - L| \leq \sqrt{N}$$

$$f : \Omega \to \mathbb{R}^N$$

è continua in x_0 se lo sono le sue componenti scalari

$$f_i : \Omega \to \mathbb{R} \qquad f(x) \equiv \begin{pmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_N(x) \end{pmatrix}$$

è convergente ad L in x_0 o all'∞ se e solo se

$$\boxed{f_i(x) \underset{x \to x_0}{\longrightarrow} L_i} \qquad f(x) \to L \Leftrightarrow f_i(x) \to L_i \quad \forall i = 1..N$$

□

$$\left(\frac{1}{n}, \frac{n+1}{2n}\right) \xrightarrow[\mathbb{R}^2]{} \left(0, \frac{1}{2}\right)$$

$|a| \geq |a_i| \quad \forall i = 1..N$

$\forall i = 1..N \quad |a|^2 = a_1^2 + a_2^2 + \dots + a_N^2 \geq a_i^2$

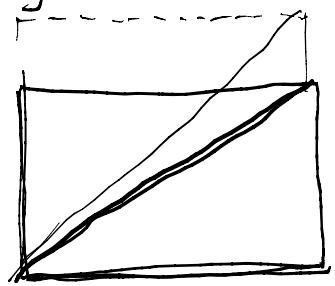
da $|a|^2 \geq a_i^2$, passando alla radice si ha

$$|a| \geq |a_i|$$

disuguaglianza del bordo.

$$\max_{1..N} |a_i| = |a_j| \geq |a_i| \quad \forall i = 1..N$$

$$\Rightarrow a_j^2 \geq a_i^2 \quad \forall i = 1..N$$



$$|a| \leq \sqrt{N} \max_{1..N} |a_i|$$

$$\rightarrow |a|^2 = a_1^2 + \dots + a_N^2 \leq N a_j^2$$

passando alla radice $|a| \leq \sqrt{N}\,|a_j|$

$$\|(1,-2)\| \geq \begin{matrix} |1| \\ |-2| \end{matrix}$$

$$\|\tilde{a}\| \geq \begin{matrix} |a_1| \\ |a_2| \end{matrix}$$

$$\|(1,-2)\| \geq \begin{matrix} (a_1\ a_2) \\ |a| \end{matrix}$$

$$\sqrt{1+4} = \sqrt{5} \geq \begin{matrix} \to 1 \\ \to 2 \end{matrix} \qquad N=2$$

dell'altro $\|(1,-2)\| \leq \sqrt{2} \max\{|1|, |-2|\} =$

$$= \sqrt{2}(2) = 2\sqrt{2}$$

$$\boxed{\sqrt{5} \leq 2\sqrt{2}} \iff 5 \leq 4 \cdot 2 = 8$$

vera

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \qquad \gamma : \mathbb{R} \to \mathbb{R}^2$$

è continua perché le componenti sono continue.

$$f_i(x) \underset{x \to x_0}{\not\longrightarrow} f_i(x_0)$$

$$\boxed{|f_i(x) - f_i(x_0)| \overset{\text{No}}{<} \varepsilon}$$

$\forall x : |x - x_0| < \delta$

$x \in \operatorname{dom} f_i$

$$|f_i(x) - f_i(x_0)| \leq |f(x) - f(x_0)|$$

$$\left(n, \frac{1}{n}\right) \longrightarrow ?$$

$n \to +\infty \quad \frac{1}{n} \to 0$

$$\lim_n a_n = \infty$$

$$\Updownarrow$$

$$\lim_n |a_n| = +\infty$$

$$\Uparrow$$

$$\left|\left(n, \frac{1}{n}\right)\right| = \sqrt{n^2 + \frac{1}{n^2}} \longrightarrow +\infty$$

$n^2 \to +\infty \quad \frac{1}{n^2} \to 0$

$$\sqrt{\sum_{i}^{N} (a_n)_i^2}$$

$$\longrightarrow \sqrt{(a_n)_1^2 + (a_n)_2^2 + \dots + (a_n)_N^2} \longleftarrow$$

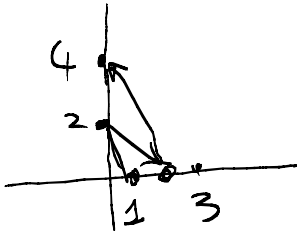
$$(0,1), (2,0), (0,3), (4,0) \dots, (n,0), (0,n+1) \dots$$

$a_n \qquad a_{n+1}$

$$|a_n| = n \Rightarrow \lim |a_n| = +\infty$$

$$\Downarrow$$

$$a_n \to \infty$$



$(a_n)_1 = 0, 2, 0, 4, 0, 6 \dots$

$(a_n)_2 = 1, 0, 3, 0, 5, 0 \dots$

$$\lim_{(0,0)} \frac{x-y}{x+y} \qquad f(x,y) = \frac{x-y}{x+y}$$

f continua

$$\lim_{(x,y)\to(1,2)} f(x,y) = f(1,2)$$

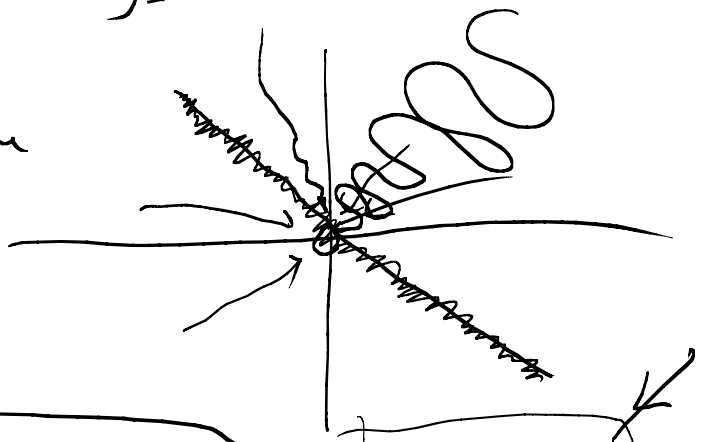
$$\boxed{\frac{\alpha x}{\beta x} \equiv \frac{\alpha}{\beta} \quad x \neq 0}$$

$$\frac{\frac{1}{2}}{\frac{3}{2}}$$

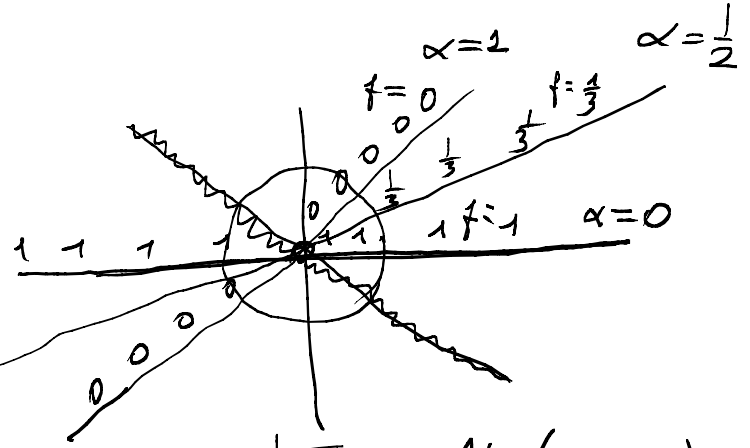
$$\text{dom}\,\frac{x-y}{x+y} = \{y \neq -x\}$$

Studiamo la restrizione di f alla retta

$$\boxed{y = \alpha x} \quad x \neq 0$$



dom f



$x \neq 0$

$$f(x, \alpha x) = \frac{x - \alpha x}{x + \alpha x} = \boxed{\frac{1-\alpha}{1+\alpha}}$$

$\lim_{(0,0)} f(x,y)$ NON ESISTE perché in ogni intorno di $(0,0)$.

$$f(x,y) = \frac{x^2 y}{x^4+y^2}$$ è definita fuori di $(0,0)$

$$\underset{\geq 0}{x^2} + \underset{\geq 0}{y^4} = 0 \iff \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$\boxed{f \text{ ristretta alla retta } \boxed{y=\alpha x}}$ $\quad \underline{x \neq 0}$

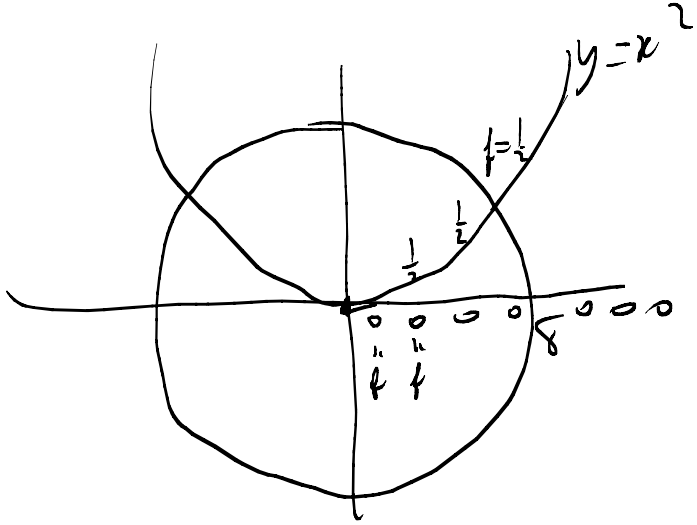
$$f(x,\alpha x) = \frac{x^2(\alpha x)}{x^4+\alpha^2 x^2} = \frac{\alpha x^3}{x^2(x^2+\alpha^2)} \overset{x\neq 0}{=} \frac{\alpha x}{x^2+\alpha^2}$$

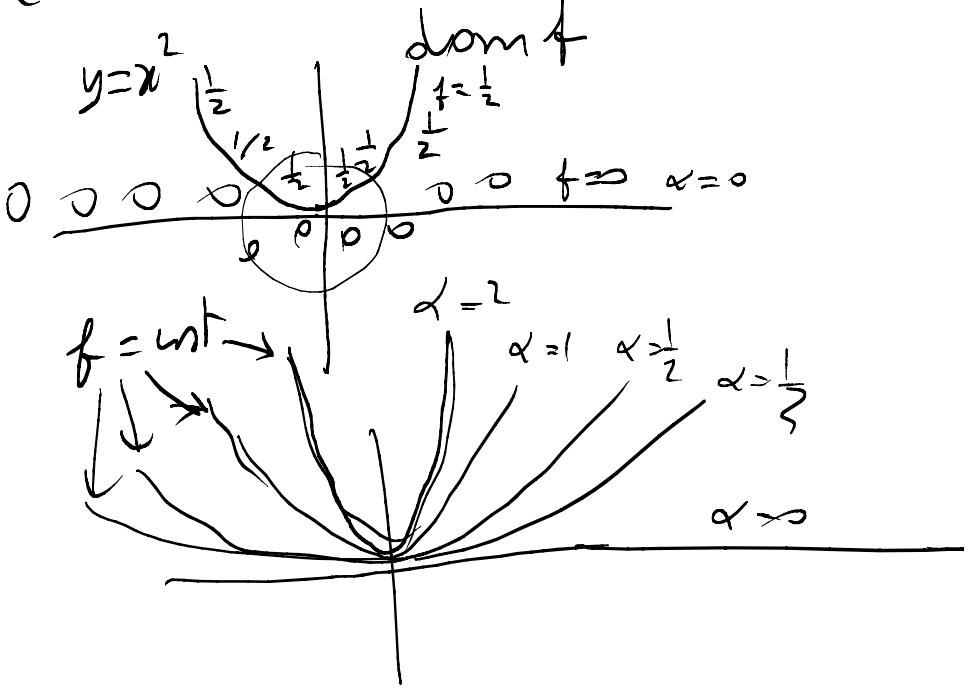
$$\lim_{x\to 0} \frac{\alpha x}{x^2+\alpha^2} = \begin{cases} 0 & \alpha \neq 0 \\ 0 & \alpha = 0 \end{cases} = \boxed{0}$$

È vero che $\displaystyle\lim_{(0,0)} \frac{x^2 y}{x^4+y^2} = 0$? $\boxed{\text{NO}}$

Consideriamo, invece, la restrizione ad una parabola $y = \alpha x^2$

$$f(x, \alpha x^2) = \frac{x^2(\alpha x^2)}{x^4 + \alpha^2 x^4} \underset{x \neq 0}{=} \frac{\alpha}{1 + \alpha^2}$$





$$\frac{(x^2+y^2) - \sin(x^2+y^2)}{(x^2+y^2)^3} = \frac{t - \sin t}{t^3}$$

$t = x^2+y^2$

$(x,y) \to 0 \quad t \to 0 = L$

Taylor

$$\sin t = t - \frac{t^3}{6} + o(t^3)$$

$$\frac{t^3/6 + o(t^3)}{t^3} \longrightarrow \frac{1}{6}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2 - \sin(x^2+y^2)}{(x^2+y^2)^3} \longrightarrow \frac{1}{6}$$

funzione più esterna

fermé la

$\frac{t - \sin t}{t^3}$ NON è definita in $t = 0 = L$