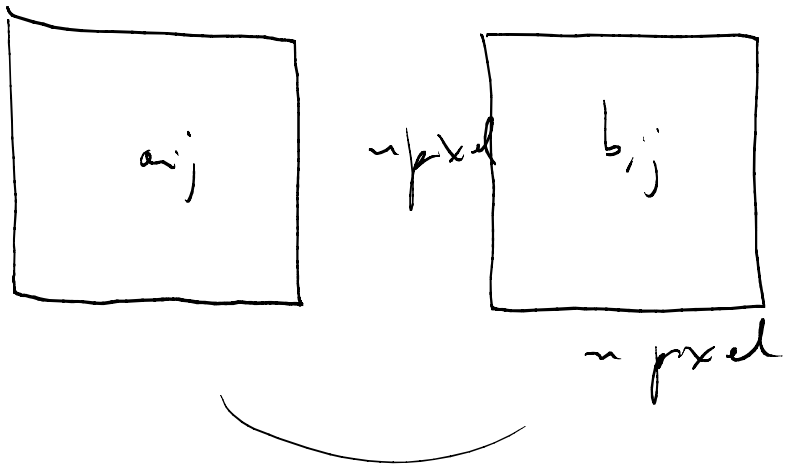
Note Title 3/17/2021

$\lim_{\infty} xy$ NON ESISTE $\lim_{\infty} x$ NON ESISTE $(x,y) \to x$

$\lim_{\infty} x^2+y^2 = \infty$ polinomi da $\mathbb{R}^N \to \mathbb{R}$

$p : \mathbb{C} \to \mathbb{C}$ $\quad p(z) = a_0 + a_1 z + \cdots + a_n z^n$

$\lim_{\infty} |p(z)| = \infty$ se $\deg p > 0$



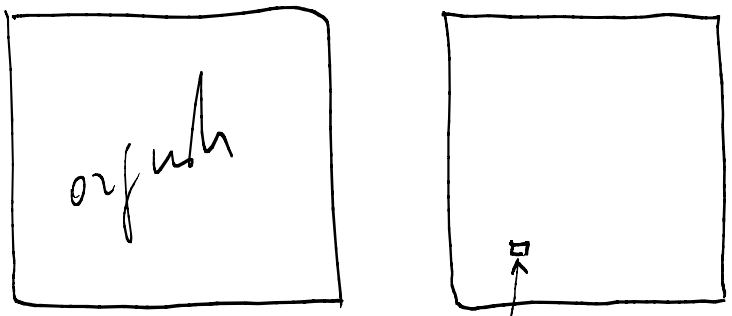
$$\sum_{i,j} (a_{ij} - b_{ij})^2$$

$$b_{ij} = a_{ij} + 1$$

in quest caso

$= n^2$

$n = 1024$



$\Longrightarrow = 1$

$b_{ij} = a_{ij}$

per $i \neq \bar{i}$, $j \neq \bar{j}$

$= 256$

$i = \bar{i}$, $j = \bar{j}$

bianco

in una zona scura

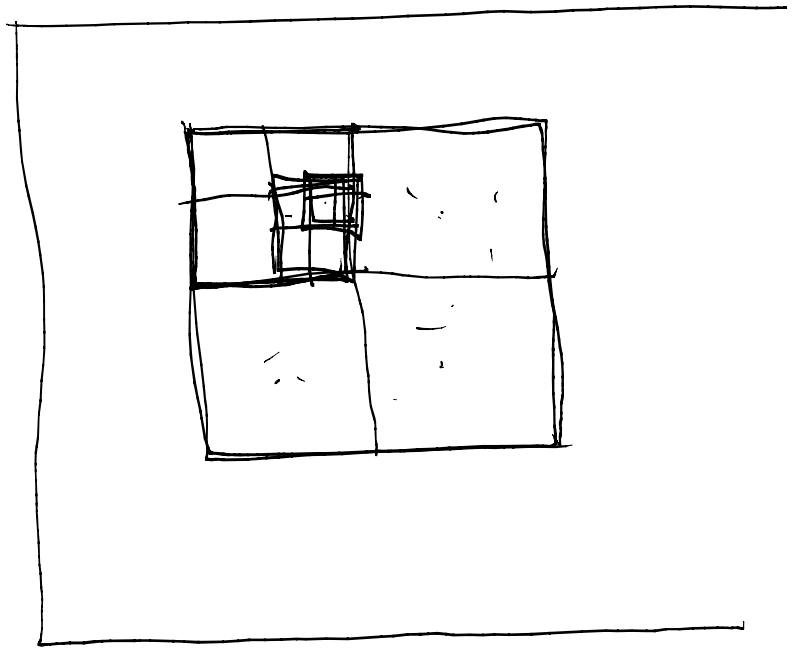
dist. giusta

è il max $|a_{ij} - b_{ij}|$

Se Ω è chiuso e limitato (compatto)

allora ogni successione in Ω contiene una sottosuccess.

convergente ad un punto di Ω



$$\sup \{ f(x) : x \in \Omega \}$$

$$x_n \in \Omega \; ; \; \lim_n f(x_n) = \sup f$$

Se $\sup f(x) = +\infty$ allora fissato $K = n$ $\exists\, x_n$:

$f(x_n) \geq n$ $\qquad$ $f(x_n) > n$ e quindi diverge x il th. confronto

$$\sup f(x) = L < +\infty$$

fissato $\varepsilon = \frac{1}{n}$ $\exists\, x_n$:

$$L - \frac{1}{n} < f(x_n) \leq L$$

L

$$\lim_n f(x_n) = L$$

Per la compattezza $\exists\, n_k$: $x_{n_k} \to x^* \in \Omega$

$$\lim_k f(x_{n_k}) = f(x^*) \leftarrow$$

continuità di f

$$f(x^*) = \lim f(n_k) = \sup_\Omega f$$

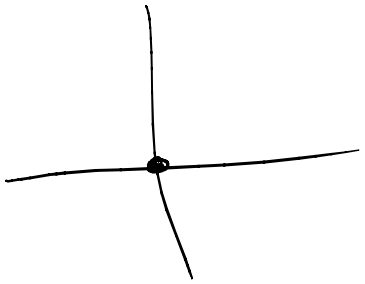
$a + bi + cj + dk$

3 unità immaginarie

$$\begin{cases} i^2 = j^2 = k^2 = -1 \\ ijk \ ? \end{cases}$$

Hamilton

$\rho e^{i\theta}$



modello delle rotazioni in $\mathbb{R}^3$

1) $\lim_{x \to x_0} f(x) = L$

2) $\lim_{y \to L} g(y) = M$

$\Longrightarrow \boxed{\lim_{x \to x_0} g(f(x)) = M}$

3) ?

$\lim_{x \to x_0} g(f(x)) \underset{y = f(x)}{=\!=} \lim_{y \to L} g(y)$

$x \to x_0 \Rightarrow f(x) \to L$

$$|g(\underbrace{f(x)}_{y}) - M| < \varepsilon$$ è vera, per un'opportuno valore di $\eta > 0$ per l'ipotesi 2)

se

1) $f(x) \in \operatorname{dom} g$

2) $|f(x) - L| < \eta \Leftarrow \exists \delta : \begin{array}{l} |x - x_0| < \delta \\ x \neq x_0 \\ x \in \operatorname{dom} f \circ g \end{array}$

$\rightarrow$ 3) $\boxed{f(x) \neq L}$

E' GARANTITA dall'ipotesi 3)

Le tre possibilità per l'ipotesi 3) sono

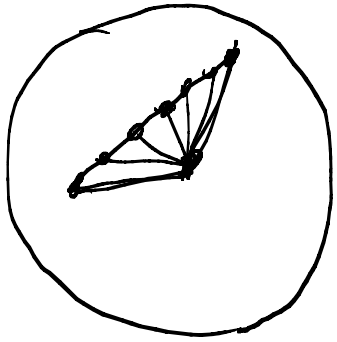
I) richiedere che g sia continua, perché, in questo caso non occorre $f(x) \neq L$

OPPURE

II) verifichiamo che g non è diffusa in L

III) $f(x) \neq L$ su $B_\delta(x_0) \setminus \{x_0\}$.

$$\forall x, y \in \Omega \Rightarrow (1-\lambda)x + \lambda y \in \Omega \quad \forall \lambda \in [0,1]$$



$$|(1-\lambda)x + \lambda y| \leq |(1-\lambda)x| + |\lambda y| =$$

($1-\lambda \geq 0$, $\lambda \geq 0$)

$$= (1-\lambda)|x| + \lambda|y| \leq$$

(B)

$$\leq (1-\lambda)M + \lambda M = M \cdot [1-\lambda+\lambda] = M$$

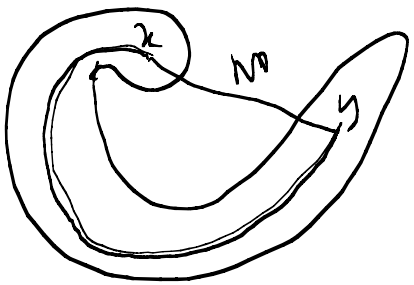
$B = \{\lambda : |\lambda| \leq M\}$

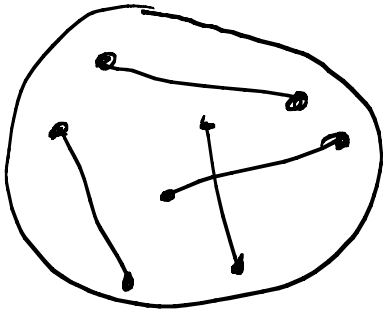
$\exists \delta > 0$ $\exists \varphi : [x_0 - \delta, x_0 + \delta]$ tale che

1) $\varphi(x_0) = y_0$

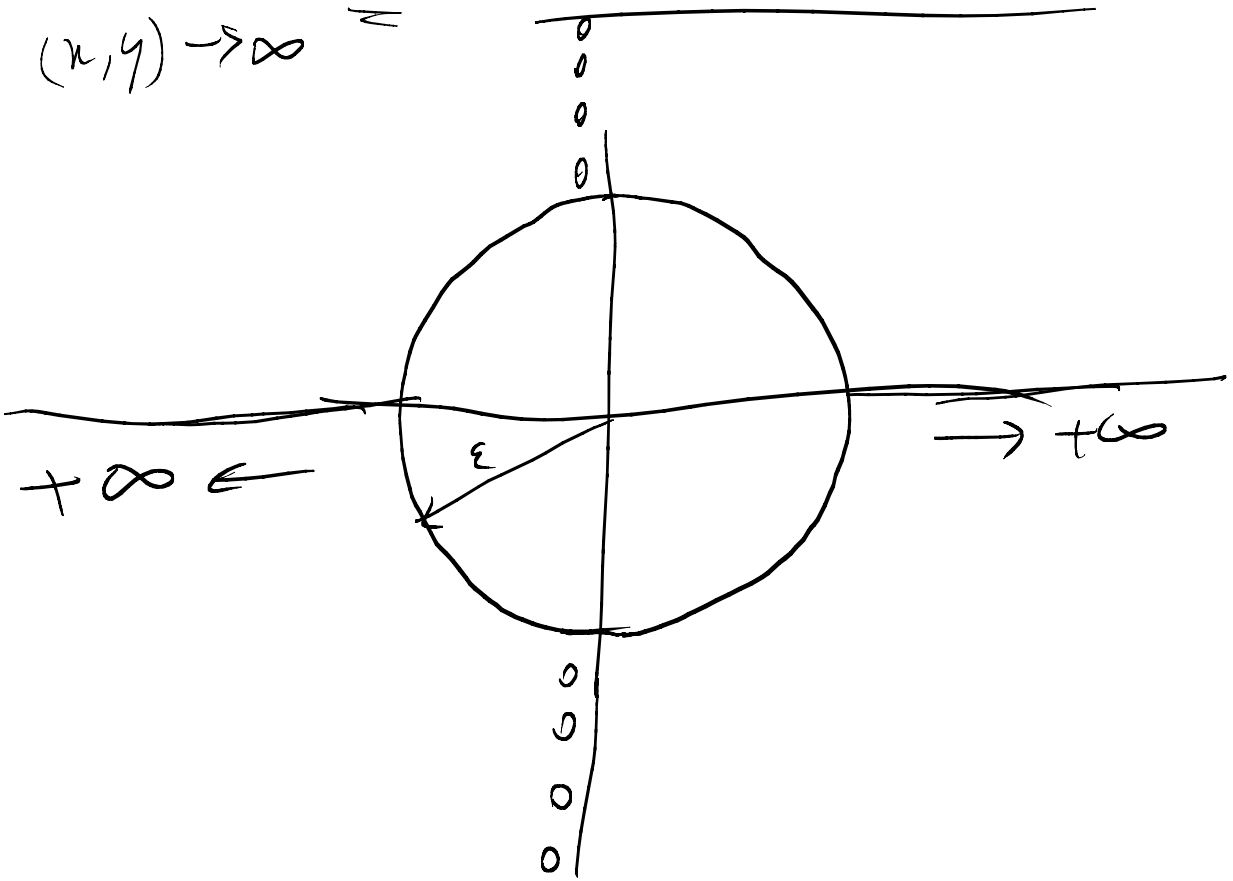
2) $f(x, \varphi(x)) \equiv 0$ su $[x_0 - \delta, x_0 + \delta]$

3) φ è continua su $[x_0 - \delta, x_0 + \delta]$



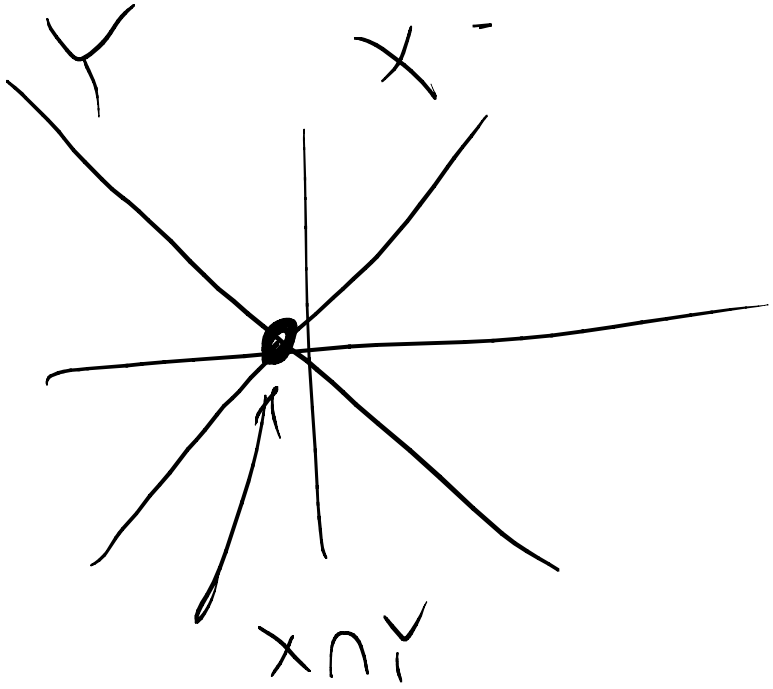


$$\lim_{(x,y)\to\infty} x^2 \text{ NON ESISTE}$$



$$X + Y = X \oplus Y \iff X \cap Y = \{\emptyset\}$$

$$X \cap Y = \emptyset$$



La somma di X e Y è diretta

$$\Longrightarrow \quad X \cap Y = \{0\}$$

Dim

$w \in X \cap Y$

$$w = w + 0 = 0 + w \Rightarrow w = 0$$

$w \in X$, $0 \in Y$, $0 \in X$, $w \in Y$

$X \cap Y = \{0\} \Rightarrow X + Y$ è diretta?

$x + y = 0 \overset{?}{\Rightarrow} x = y = 0$ di perché

$x+y=0 \Rightarrow x=-y \Rightarrow (x,-y) \in X \cap Y$

$x \in X \quad -y \in Y$

$\Downarrow$

$x,(-y)=0$

$\Downarrow$

$x,y=0$

AL_3.2 (e 3.3)

$$p(z) = z^n \left[\alpha_n + \frac{\alpha_{n-1}}{z} + \cdots + \frac{\alpha_1}{z^{n-1}} + \frac{\alpha_0}{z^n} \right]$$

$$|p(z)| = |z|^n \left| \alpha_n + \cdots + \frac{\alpha_0}{z^n} \right|$$

$f(z)$ $\qquad$ $z \to \infty$

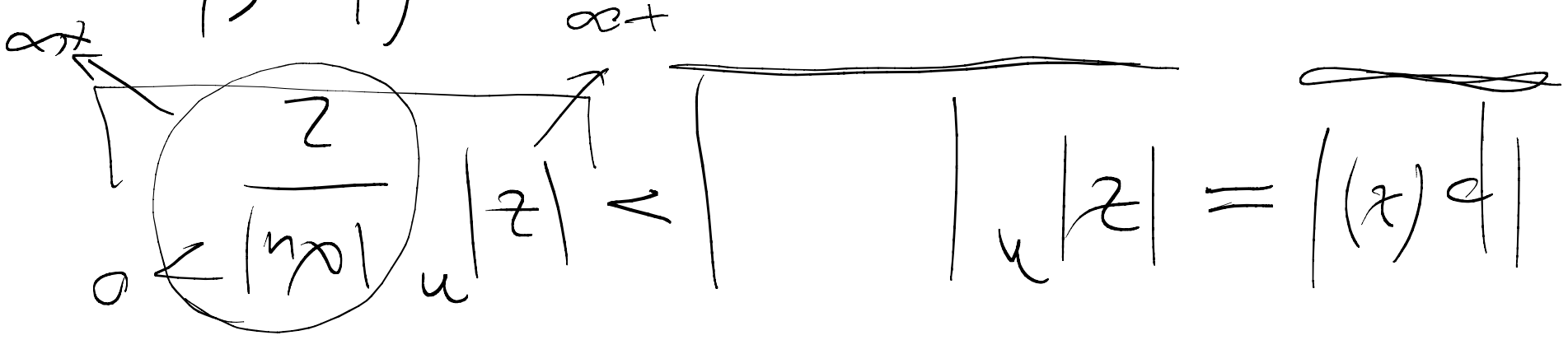
$$L = |\alpha_n| > 0$$

$$\lim_{\infty} f(z) = L > 0 \iff L - \varepsilon < f(z) < L + \varepsilon \quad \forall z : |z| > \delta$$

$$\varepsilon = \frac{|\alpha_n|}{2} \left(= \frac{L}{2}\right)$$

$$|p(z)| = |z|^n \left| \qquad \right| > |z|^n \frac{|\alpha_n|}{2}$$

$|z|^n \to +\infty$, $\frac{|\alpha_n|}{2} \to 0$, $|z|^n \to +\infty$

$(L - \varepsilon)$

$\varepsilon = |\alpha_n|/2$