

$$\lim_{x \rightarrow x_0} f(x) = L$$

(Note: In the original image, both the limit expression and the result 'L' are enclosed in boxes, with arrows pointing to them from the definition below.)

$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$
f convergent in x_0

$\forall \epsilon > 0 \exists \delta > 0 : \forall x \in \text{dom } f$

$\lim_{x \rightarrow x_0} f(x) = L \in \mathbb{R}^m$

$$|x - x_0| < \delta \quad x \neq x_0 \Rightarrow$$

$$|f(x) - L| < \epsilon$$

f divergent in x_0

$\lim_{x \rightarrow x_0} f(x) = \infty$

$\forall \epsilon > 0 \exists \delta > 0 \forall x \in \text{dom } f$

$$|x - x_0| < \delta \quad x \neq x_0$$

$$|f(x)| > \epsilon$$

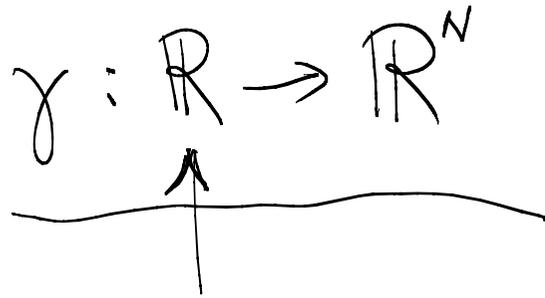
$$\lim_{\infty} f(x) = \underline{L}$$

$$\forall \varepsilon > 0 \exists \delta > 0 \forall x \in \text{dom} f$$

$$|x| > \delta \Rightarrow |f(x) - L| < \varepsilon$$

$$\lim_{\infty} f(x) = \underline{\infty} \quad \forall \varepsilon > 0 \exists \delta > 0 \forall x \in \text{dom} f$$

$$|x| > \delta \Rightarrow |f(x)| > \varepsilon$$



$$\lim_{t \rightarrow t_0^-}$$

$$\forall \varepsilon > 0 \exists \delta > 0 \forall t \in \text{dom} f$$

$$|t - t_0| < \delta, t \neq t_0, t < t_0$$

$$t_0 - \delta < t < t_0$$

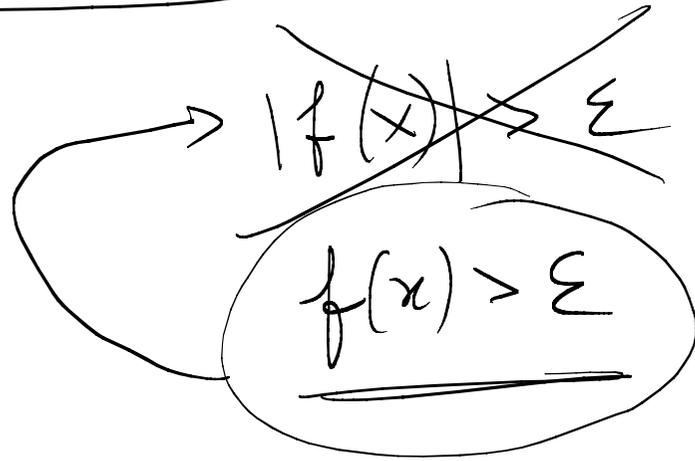
$$\lim_{t \rightarrow +\infty}$$

~~$|t| > \delta$~~

$$\underline{t > \delta}$$

$f: \mathbb{R}^N \rightarrow \underline{\mathbb{R}}$ ha senso dire che

$\lim_{?} f(x) = +\infty$



$f: \mathbb{R} \rightarrow \mathbb{R}^N$

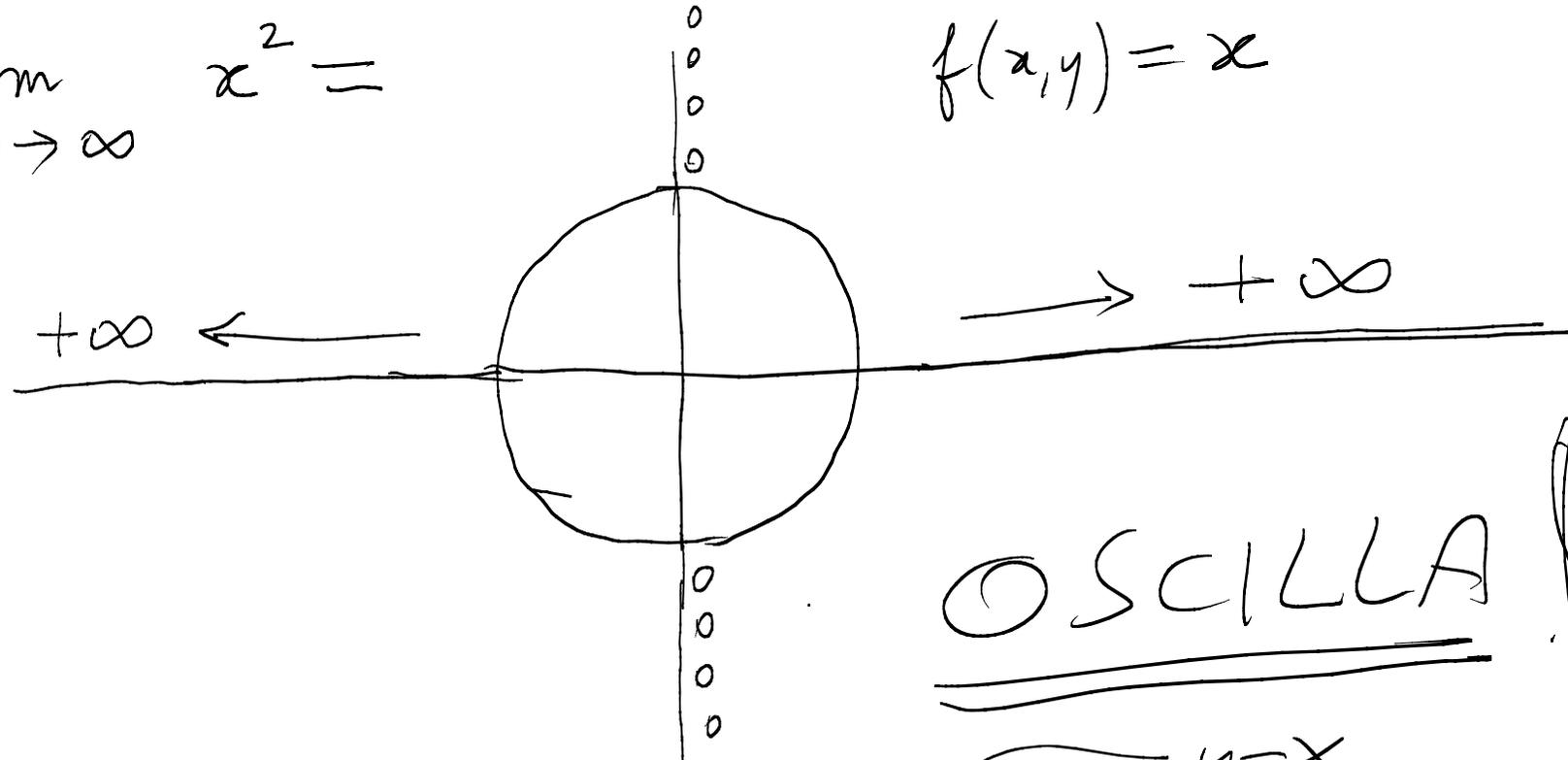
$\lim_{t \rightarrow t_0^-} f(x) = L$

$\forall \epsilon > 0 \exists \delta > 0$ tale che
 $|t - t_0| < \delta, t \neq t_0, t < t_0 \implies |f(x) - L| < \epsilon$

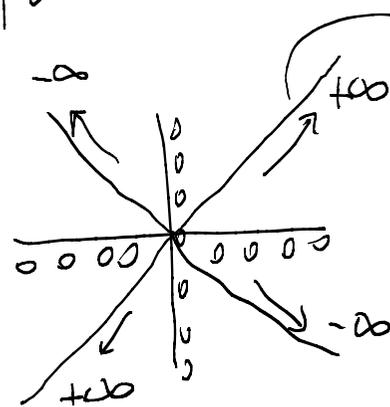
In \mathbb{R} $\lim_{t \rightarrow +\infty} p(t)$ dipende dal termine di grado max.

$$\lim_{(x,y) \rightarrow \infty} x^2 =$$

$$f(x,y) = x$$



$$\lim_{\infty} xy = ?$$



$$y=x$$

$$f(x,x)$$

ustra. di f
alle bisettrici $y=x$

$$\lim_{(x,y) \rightarrow \infty} x^2 + y^2 = +\infty$$

$$f(x,y) \quad f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\forall \varepsilon > 0 \exists \delta > 0 \forall (x,y) \in \mathbb{R}^2 \quad |f(x,y) - \infty| > \varepsilon \iff |x^2 + y^2| > \delta$$

dove

$$|x^2 + y^2| > \varepsilon \iff x^2 + y^2 > \varepsilon \iff \sqrt{x^2 + y^2} > \varepsilon^{1/2}$$

$$\delta = \sqrt{\varepsilon}$$

$p(z)$ polinomio a coeff. $\in \mathbb{C}$ NON COSTANTE

$$p: \mathbb{C} \rightarrow \mathbb{C} \quad \deg p > 0 \implies \lim_{z \rightarrow \infty} p(z) = \infty$$

$$p(z) = \alpha_n z^n + \alpha_{n-1} z^{n-1} + \dots + \alpha_1 z + \alpha_0$$

$$\alpha_n \neq 0$$

$n > 0$

$$|p(z)| = \left| z^n \left(\alpha_n + \frac{\alpha_{n-1}}{z} + \frac{\alpha_{n-2}}{z^2} + \dots + \frac{\alpha_0}{z^n} \right) \right| =$$

$$= |z|^n \left| \alpha_n + \frac{\alpha_{n-1}}{z} + \dots + \frac{\alpha_0}{z^n} \right|$$

problem is that $\lim_{\infty} \frac{\alpha}{z^n} = 0$

$\left| \frac{\alpha}{z^n} \right| < \varepsilon$

$\iff \frac{|\alpha|}{|z|^n} < \varepsilon$

$|z|^n > \frac{|\alpha|}{\varepsilon}$

$|z| > \left[\frac{|\alpha|}{\varepsilon} \right]^{1/n} = \delta$

$\left| \alpha_n + \frac{\alpha_{n-1}}{z} + \dots + \frac{\alpha_0}{z^n} \right| \uparrow \underbrace{|\alpha_n|}_{> 0}$

$$\varepsilon = \frac{|\alpha_n|}{2} \stackrel{L/2}{=} \delta; \quad \underline{|z| > \delta}$$

$$L + \varepsilon > \left| \alpha_n + \dots + \frac{\alpha_n}{z^n} \right| > \underbrace{|\alpha_n|}_L - \underbrace{\varepsilon}_{\frac{|\alpha_n|}{2}} = \frac{|\alpha_n|}{2}$$

da wir

$$\underline{|p(z)|} > \frac{|\alpha_n|}{2} |z|^n \rightarrow +\infty$$

th. component

weil $|z|^n \rightarrow +\infty$ in $|z|^n > \varepsilon$

$$|z| > \varepsilon^{\frac{1}{n}} = \delta$$

$$|p(z)| \rightarrow +\infty$$

$$\frac{a_n}{b_n} \rightarrow L \neq 0$$

$$\underline{a_n, b_n > 0}$$

$b_n \text{ div.} \Rightarrow a_n \text{ div.}$

$$L - \varepsilon < \frac{a_n}{b_n} < L + \varepsilon$$

$$\varepsilon < L$$

$$\underline{a_n} > (L - \varepsilon) \underline{b_n}$$

$$\sum \frac{1}{n+1}$$

ε divergent

$$\frac{\frac{1}{n+1}}{\frac{1}{n}} \rightarrow 1$$

$$\frac{1}{n+1} > (1 - \varepsilon) \frac{1}{n} \quad \forall n > N$$

Th $f: \mathbb{R}^N \rightarrow \underline{\mathbb{R}^M}$ $\lim_{\infty} f(x) = \infty$ f continua

Allora $|f(x)|$ ha minimo.

x_0 è di minimo per $|f(x)|$
e $|f(x)| \geq |f(x_0)| \quad \forall x \in \text{dom} f$

DIM. Scelto $x_0 \in \text{dom} f$ ad
arbitrio, si pone $\varepsilon = |f(x_0)|$

Se $|f(x_0)| = 0$ allora x_0 è il minimo richiesto, e la

prova è conclusa.

Se $\varepsilon > 0$ per l'ipotesi $\lim_{\infty} f(x) = \infty \quad \exists \delta: \underline{|x| > \delta}$

$|f(x)| > \varepsilon = |f(x_0)|$

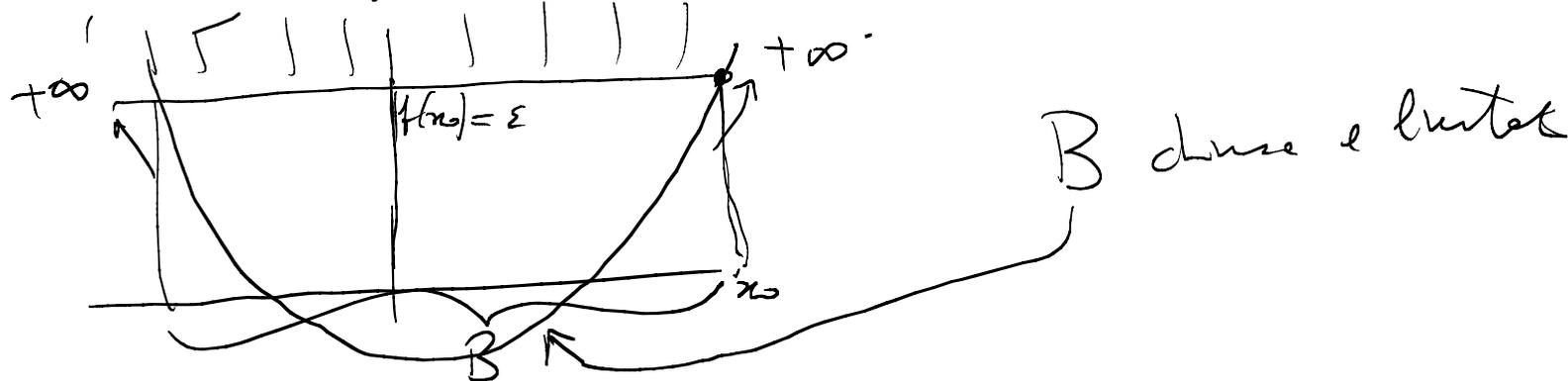
Definiamo $B = \overline{B_\delta(0)} = \{x \in \mathbb{R}^N: |x| \leq \delta\}$

1°) $x_0 \in B$ perché se fosse $x_0 \notin B$ si avrebbe $|x_0| > \delta$
 e quindi $|f(x_0)| > \varepsilon = |f(x_0)|$, assurdo!

Per il th. di Weierstrass, essendo f continua su B ,
 chiusa e limitata, avrà minimo x^* . Da ciò segue

$$\left. \begin{array}{l} \text{dentro } B \\ x_0 \in B \end{array} \right\} |f(x^*)| \leq |f(x)| \quad \forall x \in B \text{ (cioè } |x| \leq \delta) \left. \vphantom{\begin{array}{l} \text{dentro } B \\ x_0 \in B \end{array}} \right\} |f(x^*)| \leq \inf_{x \in \mathbb{R}^n} |f(x)|$$

$$\text{fuori } B \quad |f(x^*)| \leq |f(x_0)| < |f(x)| \quad \forall x : |x| > \delta$$



Lemme (di Hermite) p polinomio complesso non costante, $p(z_0) \neq 0$.

Allora $\exists \bar{z} : |p(\bar{z})| < |p(z_0)|$

DIM. $q(w) = \frac{1}{p(z_0)} p(z_0 + w) \leftarrow$

$w = z - z_0$

$q(0) = \frac{p(z_0)}{p(z_0)} = 1$

le tesi diventa $\exists w : |q(w)| < 1$

$\deg q = \deg p \Rightarrow q$ non costante

$q(w) = 1 + \alpha_k w^k + w^{k+1} \tilde{q}(w)$

k è il minimo intero > 0 tale che $\alpha_k \neq 0$

$|q(w)| \leq \underbrace{|1 + \alpha_k w^k|}_{\text{+residuo}} + \underbrace{|w|^{k+1} |\tilde{q}(w)|}_{\text{+residuo}}$

$\left| \frac{p(z_0 + w)}{p(z_0)} \right| = \frac{|p(z_0 + w)|}{|p(z_0)|} < 1$

\int_S

w tali che

1) $\alpha_k w^k$ se reale e negativo

$$q(w) = 1 + \alpha_1 w + \alpha_2 w^2 + \dots + \alpha_n w^n$$

$$q(0) = 1$$

↑

non tutti nulli.

α_k il coeff. del termine di grado minimo
effettivamente presente.

Teoremi fondamentali dell'Algebra