Note Title 3/2/2021

RICEVIM. Mercoledì 15:30 → 18:30

ENRICO GIUSTI ANALISI MATEMATICA II

Distanza in $\mathbb{R}^N$

$$d(x,y) \equiv |x-y|$$

In $\mathbb{R}^N$

1) $d(x,y) \geq 0$ ok

$$d(x,y) = |x-y| \geq 0$$ per l'assioma delle norme ($|z| \geq 0 \ \forall z$)

2) $d(x,y) = 0 \iff x = y$

$$|x-y| = 0 \overset{\text{assioma norme}}{\iff} x-y = 0 \overset{\text{spazio vett.}}{\iff} x = y$$

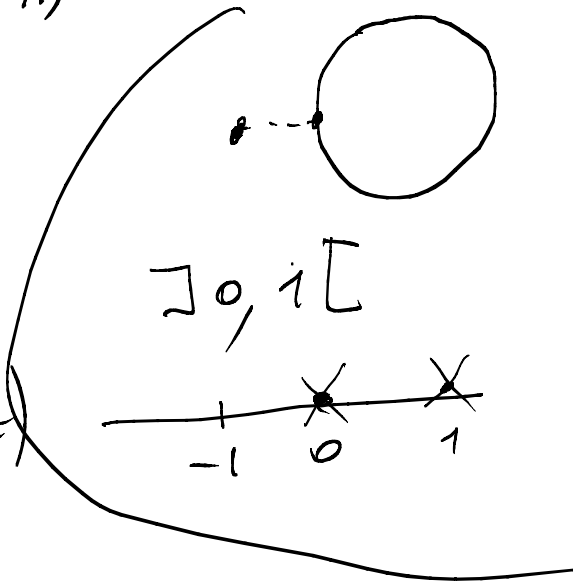
1) $d(x,y) \geq 0 \quad \forall x,y \in \Omega$
2) $d(x,y) = 0 \iff x = y$
3) $d(x,y) = d(y,x) \quad \forall x,y \in \Omega$
4) $d(x,y) \leq d(x,z) + d(z,y) \quad \forall x,y,z$

d su Ω

$d : \Omega \times \Omega \to \mathbb{R}$

metrica o distanza

Ω spazio metrico



3) $d(x,y) = d(y,x)$? sì

$$\| \\ |x-y| = |-(y-x)| = |-1|\,|y-x| = d(y,x)$$

$\uparrow$ omogeneità della norma

4) $d(x,y) \le d(x,z) + d(z,y)$

$$\| \\ |x-y| = |x-z+z-y| \le |x-z| + |z-y| =$$

dis. triang. della $|\cdot|$

LO STESSO IN $\mathbb{C}^n$

Sfera di centro $x_0 \in \mathbb{R}^N$ e raggio $\rho \geq 0$

$$S(x_0, \rho) = \{ x \in \mathbb{R}^N : d(x, x_0) = \rho \}$$

sfera

$$B(x_0, \rho) = \{ x \in \mathbb{R}^N : d(x, x_0) < \rho \}$$

palla aperta

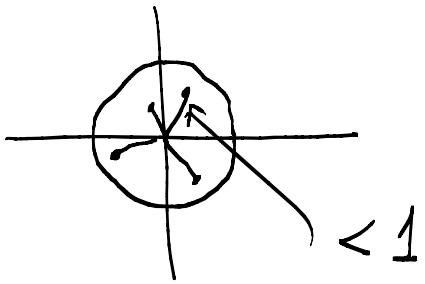
$$\overline{B}(x_0, \rho) = \{ x \in \mathbb{R}^n : d(x, x_0) \leq \rho$$

palla chiusa

$B(x_0, \rho)$ è l'insieme dei punti di $\mathbb{R}^N$ che distano da x_0 "meno" di ρ.

$$B(0,1) \subseteq \mathbb{R}^2$$

$$d(x,y) = |x-y|$$



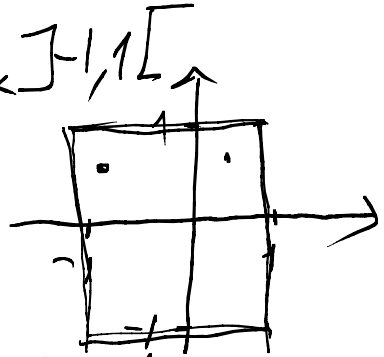
cerchio di centro l'origine e raggio 1, detto circonferenza (cioè) $S(0,1) = \{(x,y) \in \mathbb{R}^2 : |(x,y) - (0,0)| = 1\}$

$|(x,y)| = \sqrt{x^2+y^2}$

$= \mathbb{R}^2$ $\quad \|(x,y)\| = \max\{|x|, |y|\}$ $\quad$ scelgo

$\in \mathbb{R}^2$ $\quad$ $\in \mathbb{R}^2$

$B(0,1) =]-1,1[\times]-1,1[$



$= $ punto del piano con valore assoluto delle coordinate < 1

$\|(x,y) - (0,0)\| =$

$= \|(x,y)\| = \max\{|x|, |y|\}$

3°) $\|(x,y)\| = |x| + |y|$

Successione in $\mathbb{R}^N$ $\quad a : \mathbb{N} \to \mathbb{R}^N$ $\quad$ dominio $\mathbb{N}$ codominio $\mathbb{R}^N$

$a(n) = a_n$

Esempi

$a_n = \underbrace{\left(n, \frac{n+1}{n}\right)}_{\in \mathbb{R}^2}$ succ. in $\mathbb{R}^2$ $\quad \mathbb{N} = \{1, 2, \dots\}$

$a_n = \underbrace{\left(\frac{n+1}{n-1} \;/\; \frac{1}{n} \;/\; \frac{1}{n^2+1}\right)}_{\in \mathbb{R}^3}$ $\quad$ $1 \quad 0 \quad 0$

$a_n = \underbrace{\left((-1)^n, (-1)^{n+1}\right)}_{\in \mathbb{R}^2}$

$a : \mathbb{N} \Rightarrow \mathbb{R}^2$ $\quad b_n, c_n \in \mathbb{R}$

$a_n = (b_n, c_n)$

$$n \longrightarrow \begin{cases} (a_n)_1 \\ (a_n)_2 \\ \vdots \\ (a_n)_N \end{cases} \qquad \begin{pmatrix} (a_n)_1 \\ \vdots \\ (a_n)_N \end{pmatrix} \in \mathbb{R}^N$$

$a : \mathbb{N} \to \mathbb{R}^N$

al variare di n è una successione in $\mathbb{R}^N$

$(a_n)_{n \in \mathbb{N}}$ successione in $\mathbb{R}^N$ sarà CONVERGENTE ad

$\to L \in \mathbb{R}^N$ se e solo se $\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ |a_n - L|_{\mathbb{R}^N} < \varepsilon$

AN 1

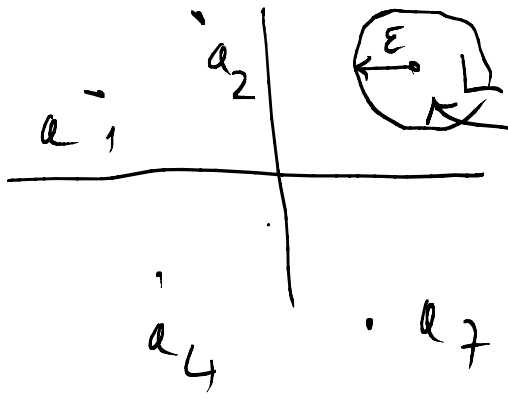
$\lim a_n = L \quad \forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ L - \varepsilon < a_n < L + \varepsilon \Rightarrow \forall \varepsilon > 0 \ \exists \nu : \forall n > \nu$

FA SCHIFO in $\mathbb{R}^N$ $\to$ $|a_n - L| < \varepsilon$

$\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu$ $\quad a_n \in B(L, \varepsilon)$ $\quad$ geom.

$|a_n - L| < \varepsilon$

$d(a_n, L)$



$]L-\varepsilon, L+\varepsilon[$

$B(L, \varepsilon)$ in $\mathbb{R} \subseteq \mathbb{R}^1$

$a_n \to L$ $\quad \lim a_n = L$ $\quad \lim_{n \to \infty} a_n = L$ $\quad \lim_n a_n = L$

a_n converge ad L

$a_n \to L$
$b_n \to M$ $\Longrightarrow$ $a_n + b_n \to L + M$

limite della somma = somma dei limiti

$$|a_n+b_n-(L+M)| =$$

$$= |a_n-L+b_n-M| \leq$$

dis. triang. 1)

$$\leq |a_n-L| + |b_n-M| <$$

$\sigma = \frac{\varepsilon}{2}$ $< \frac{\varepsilon}{2}$ $< \frac{\varepsilon}{2} = \eta$ 2ª ip

$\forall n > \nu_1$ ip. 1 $\quad \forall n > \nu_2$

ν_1 $\quad \mu$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

Il ν della tesi è il più grande fra ν_1 e ν_2

Is. $\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu$

$|a_n+b_n-(L+M)| < \varepsilon$

cosa da provare

Hp $\forall \sigma > 0 \ \exists \nu : \forall n > \nu$ cose che so

1) $|a_n - L| < \sigma$

$\forall \eta > 0 \ \exists \mu : \forall n > \mu$

2) $|b_n - M| < \eta$

per $n > \nu = \max(\nu_1, \nu_2)$