Note Title 2/4/2021

1) $\det(e_1, \dots, e_n) = 1$

2) Alternante

3) Multilineare

} "Assiomi"

$\Downarrow$

$\det A^T = \det A$

$\det AB = \det A \det B$ BINET

$$(Ax)\,y = x\,(A^* y)$$

→ aggiunta

$$A(u)\,u = u\,A(u)$$

$$\| \qquad\qquad \|$$

$$(\lambda u)\,u \qquad\qquad u\,(\lambda u)$$

$$\| \qquad\qquad \|$$

$$\lambda |u|^2 \qquad\qquad \overline{\lambda}\,|u|^2$$

$$\left(A^T\right)_{ij} = A_{ji}$$

$$\left(A^*\right)_{ij} = \overline{A_{ji}}$$

$$\begin{cases} Ax = b \\ \det A \neq 0 \end{cases} \qquad x_i = \frac{\det(A_1 \dots A_{i-1}\, b\, A_{i+1} \dots A_n)}{\det A}$$

REGOLA DI CRAMER
(FORMULA)

Th. Cramer $A : \mathbb{R}^n \to \mathbb{R}^n$ allora

è iniettiva se e solo se è suriettiva

SOLO per $n = 3$

Sviluppo di Sarrus

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{matrix}$$

$$X = \underbrace{\langle v_1 \dots v_k \; u_{k+1} \dots u_n \rangle}$$

$v_{k+1} \in X =$ e quindi

$$\boxed{v_{k+1} = \sum_{1}^{k} \alpha_i v_i + \sum_{k+1}^{n} \beta_j u_j}$$

se $v_{k+1} = 0$ il sistema dei v_i è già dipendente

se $v_{k+1} \neq 0$ allora si può scambiare con uno

degli elementi che ha coeff. non nullo

se qualche β_j è $\neq 0$ si scambierà v_{k+1} con quell'elem. $u_i \in \{u_i\}$

se tutti i β_j fossero $= 0$ allora avrei $v_{k+1} = \sum_{1}^{k} \alpha_i v_i \Rightarrow v_i$ dip.

$$\begin{cases} B \in \langle A_1, \dots, A_n \rangle \\ B \neq 0 \end{cases}$$

$$\updownarrow$$

$$\exists\, A_i \;:\; \langle A_1, \dots, A_n \rangle = \langle A_1 \dots A_{i-1}\, B\, A_{i+1} \dots A_n \rangle$$

DIM. $B \in \langle A_1 \dots A_n \rangle$ $\quad B = \sum_1^n \alpha_i A_i$

se tutti gli α_i fossero nulli, mi segue $B=0$ contro l'ipotesi.

Sia $\alpha_i \neq 0$. Sia $B = \alpha_i A_i + \sum_{j \neq i} \alpha_j A_j$

Risolvendo rispetto ad A_i segue $A_i = \frac{1}{\alpha_i} B - \sum_{j \neq i} \frac{\alpha_j}{\alpha_i} A_i$

equiv $A_i \in \langle B, A_j \; j \neq i \rangle$

AL_3.1

$\langle A_1 \dots A_n \rangle = \langle A_1, \dots, A_n, B \rangle \equiv \langle A_1 \dots A_{i-1} B, A_{i+1} \dots A_n \rangle$

↑ Lemma fundam.

$$\cos \widehat{uv} = \frac{uv}{|u||v|}$$

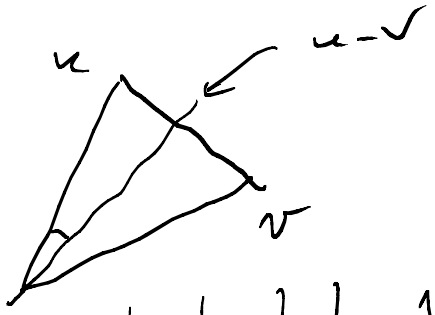
$$|u-v|^2 = |u|^2 + |v|^2 - 2uv$$

$$uv = |u||v| \cos \widehat{uv}$$

In $\mathbb{R}^3$ (anche $\mathbb{R}^2, \dots \mathbb{R}^n$)

$|u| = |v| = 1$

AL_2.1



$$\frac{|u-v|}{2} = |u| \sin\frac{\vartheta}{2}|$$

$$\Updownarrow$$

$$|u|^2 + |v|^2 - 2uv = 4|u|^2 \sin^2\frac{\vartheta}{2}$$

$|u| = |v| = 1$

$$\Leftrightarrow \quad 2 - 2uv = 4\sin^2\frac{\vartheta}{2}$$

$$1 - uv = 2\sin^2\frac{\vartheta}{2} \Rightarrow uv = \cos\vartheta$$

$$\underbrace{\sum_{1}^{k} \alpha_i w_i + \sum_{k+1}^{n} \beta_j v_j}_{\in X} = \underbrace{-\sum_{k+1}^{m} \gamma_h y_h}_{\in Y}$$

$$\Rightarrow \quad -\sum_{1}^{m} \gamma_h y_h \in X \cap Y$$

$$-\sum_{k+1}^{m} \gamma_h y_h = \sum_{1}^{k} \alpha_i' w_i$$

$$0 = \sum_{1}^{k} \alpha_i' w_i + \sum_{k+1}^{m} \gamma_h y_h$$

w_i, y_h are linearly independent $\rightarrow$ $\alpha_i' = 0 \; \forall i$, $\gamma_h = 0 \; \forall h$

$$\sum \alpha_i w_i + \sum \beta_j x_j = \sum \alpha'_i w'$$

$\in X \cap Y$

$$\sum (\alpha_i - \alpha'_i) w_i + \sum \beta_j x_j = 0$$

$\Rightarrow \beta_j = 0 \ \forall j$