Note Title 1/11/2021

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\2\\1\end{pmatrix}\right\} \longrightarrow \left\{\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}1\\3\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$$

e_1'

$e_1\ e_2\ e_3$

	1	1	1	1	1	1
	0	1	2	1	3	1
	0	0	1	2	1	1
II	0	1	0	-3	1	-1
I	1	0	0	2	-1	1

II − 2III

I − II − III

1	0	0	2 −1 1
0	1	0	−3 1 −1
0	0	1	2 1 1

← matrice di cambio base

VERIFICA

Idem

$$2\begin{pmatrix}1\\0\\0\end{pmatrix} - 3\begin{pmatrix}1\\1\\0\end{pmatrix} + 2\begin{pmatrix}1\\2\\1\end{pmatrix} \overset{?}{=} \begin{pmatrix}1\\1\\2\end{pmatrix}$$

e_1 e_2 e_3 e_1'

$$-1\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}1\\1\\0\end{pmatrix} + 1\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\3\\1\end{pmatrix} = e_2'$$

AL_6.3

AL_6.4

$$\begin{vmatrix} 1 & 3 & 1 & -1 \\ -3 & -1 & -1 & 1 \\ 1 & 1 & 3 & -1 \\ 1 & 1 & 1 & -3 \end{vmatrix}$$

$$\begin{matrix} 0 & 8 & 2 & -2 \\ \rightarrow\ 0 & -2 & 2 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -2 & 0 & -2 \\ 0 & 8 & 2 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & -10 \\ 0 & 0 & 0 & 12 \end{matrix} \quad \begin{matrix} II + 3I \\ III - I \\ IV - I \\ II \\ IV \\ III - II \\ IV + 4II \\ IV - III \end{matrix}$$

1 scambio

$$\begin{vmatrix} 1 & 3 & 1 & -1 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 12 \end{vmatrix} = -(-48) = 48$$

AL_4.6 / 4.9 / 4.10

$$\dim \underbrace{\left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix} \right\rangle + \left\langle \begin{pmatrix}0\\1\\2\end{pmatrix}, \begin{pmatrix}0\\3\\2\end{pmatrix} \right\rangle}$$

$$\left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}, \begin{pmatrix}0\\3\\2\end{pmatrix} \right\rangle$$

Gauss, segurando com bem le colocam

PIVOT

$$\begin{array}{cccc} 1 & -1 & 0 & 0 \\ 2 & 1 & 1 & 3 \\ 1 & 1 & 2 & 2 \end{array}$$

$$\vdots$$

$$(x_1,x_2,x_3,x_4)\left\langle \begin{pmatrix}1\\-1\\2\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle^{\perp} = X^{\perp} \iff \begin{cases} 1x_1 - 1x_2 + 2x_3 + 0x_4 = 0 \\ 0x_1 + 1x_2 + 1x_3 + 1x_4 = 0 \\ 1x_1 + 1x_2 + 1\cdot x_3 + 1\cdot x_4 = 0 \end{cases}$$

i coeff. sono le componenti dei generatori.

Teoria

Pratica

x_1	x_2	x_3	x_4
1	−1	2	0
0	1	1	1
1	1	1	1
0	2	−1	1
0	0	−3	−1
0	0	3	1

III − I

II − 2II

III / (−1)

Scambiando x_3 con x_4 e sopprimendo x_1 x_2 x_4 come prima

x_4	x_3
1 −1 0	−2
0 1 1	−1
0 0 1	−3
0 1 0	2
1 0 0	0

II − III

I + II

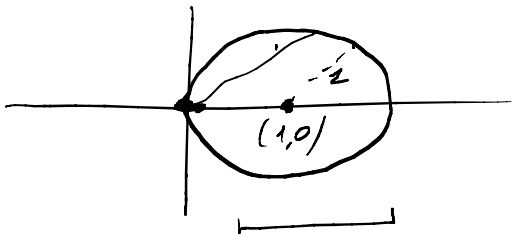
$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} = \begin{pmatrix}0\\2x_3\\x_3\\-3x_3\end{pmatrix} = x_3\begin{pmatrix}0\\2\\1\\-3\end{pmatrix}$$

$$X^{\perp} = \langle (0,2,1,-3) \rangle$$

AL_2.6

LONI 14/I/2021 ore 9

$$\int \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy =$$



$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{2\cos\theta} \rho\, d\rho$$

$$(x-1)^2 + y^2 \le 1$$

$$x^2+y^2-2x+1 \le 1$$

$$x^2+y^2-2x \le 0$$

$$\rho^2\cos^2\theta + \rho^2\sin^2\theta - 2\rho\cos\theta \le 0$$

$$\rho^2 \le 2\rho\cos\theta \qquad \rho \ge 0$$

$$\rho \le 2\cos\theta \Rightarrow \cos\theta \ge 0$$

$$\theta \in \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right]$$

50 % del tempo
va dedicato al dominio

$0, u_1, u_2, \dots, u_n$ SONO <u>DIPENDENTI</u> perché

$3 \cdot 0 + 0u_1 + 0u_2 + \dots + 0u_n = 0$

$1 \neq 0$ $= 0$

i coeff. sono NON TUTTI NULLI

$$A = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 4 & -1 \\ 4 & -6 & 5 \end{pmatrix}$$

$A(u) = Au \qquad u \in \mathbb{C}^3 \quad (\mathbb{R}^3)$

$\mathbb{C}^3$ ed $\mathbb{R}^3$ sono invarianti per A

- A NON è SIMMETRICA (NON È AUTOAGGIUNTA)

$$0 = \begin{vmatrix} 5-\lambda & 3 & 1 \\ 2 & 4-\lambda & -1 \\ 4 & -6 & 5-\lambda \end{vmatrix} = (\lambda^2 - 10\lambda + 25)(4-\lambda) - 12 - 12 - [4(4-\lambda) + 6(5-\lambda) + 6(5-\lambda)] =$$

$$= -\lambda^3 + 14\lambda^2 - 65\lambda + 100 - 24 - 16 + 4\lambda - 30 + 6\lambda - 30 + 6\lambda =$$

$$= -\lambda^3 + 14\lambda^2 - 49\lambda = -\lambda(\lambda^2 - 14\lambda + 49)$$

$-65+16=49$

$\lambda = 0$ semplice $\qquad \lambda = 7$ doppia

Il sistema degli autovettori, per $\lambda = 7$ è

u_1	u_2	u_3
$5-7$	3	1
2	$4-7$	-1
4	-6	$5-7$

u_1	u_2	u_3
-2	3	1
$\cancel{2}$	$\cancel{-3}$	$\cancel{-1}$
$\cancel{4}$	$\cancel{-6}$	$\cancel{-2}$

$\simeq$

multiplo della I

u_1	u_2	u_3
-2	3	1

NON PIVOT

PIVOT

l'autospazio ha dim 2

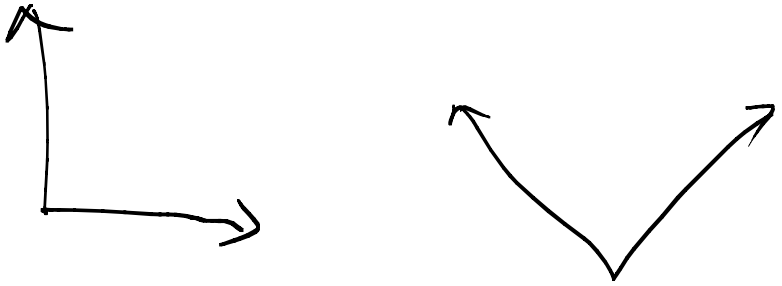
$$u_3 = 2u_1 - 3u_2$$

due parametri

$$\longrightarrow \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ 2u_1 - 3u_2 \end{pmatrix} =$$

$$= u_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + u_2 \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$$

base dell'autospazio



$$\boxed{\langle u_1 \dots u_n \rangle = \langle v_1 \dots v_m \rangle}$$

$u_1 \dots u_n \mid v_1 \dots v_m$ ha sempre soluzione $+$ $v_1 \dots v_m \mid u_1 \dots u_n$ ha sempre soluzione

DIMENSIONE Dell'autospazio di λ_0 =

= numero di NON-PIVOT nella riduzione a scala (GAUSS) del sistema $(A - \lambda_0 I)u = 0$

$$\longrightarrow \begin{array}{cccc|c} u_1 & u_2 & & u_n & \\ \hline a_{11}-\lambda & a_{12} & \cdots & a_{1n} & 0 \\ \vdots & & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda & 0 \end{array}$$

$$\boxed{AL\_7.8/7.9}$$

$$X = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle \quad (u)$$

$$Y = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle \quad (v_1, v_2)$$

$u \in \langle v_1, v_2 \rangle$	$X \cap Y = \emptyset$ *	$X \cap Y \neq \emptyset$ *
oui	parallèle	$X \subseteq Y$
non	gauches	incidentes

$X \cap Y$

$\exists \alpha, \beta, \gamma \in \mathbb{R}$:

$$\alpha \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow$$

α	β	γ	
1	−1	0	1
0	0	−1	1
1	0	−1	1
1	−1	0	1

Si on prend $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$

$\bar{\alpha} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ est un élément de $X \cap Y = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \bar{\beta} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \bar{\gamma} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$u \in \langle v_1, v_2 \rangle$ se $\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right)$ ha soluzioni

$\left(\begin{array}{c|cc} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}\right)$ ha soluzioni? NO!

$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right)$ incompatibile

G_1.4 / 1.6

rette spazi affini

0 0 1 III - II

INSOLUBILE

$$(1,1,1)_{\langle \underbrace{(1,i,0)}_{e_1},\ \underbrace{(1,-i,3)}_{e_2} \rangle} \overset{\text{Eulero-Fourier}}{=}$$

$$(1,1,1)_{(1,i,0)} + (1,1,1)_{(1,-i,3)}$$

$$\boxed{\begin{aligned} &e_1 e_2 = 0 \ ? \\ &(1,i,0)(1,-i,3) = \\ &= 1\cdot\bar{1} + i\,\overline{(-i)} + 0\cdot 3 = \\ &= 1 - 1 + 0 = 0 \ !!! \end{aligned}}$$

MIRACOLO!!!

$$\left\{ \left[\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \alpha \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} - \beta \begin{pmatrix} 1 \\ -i \\ 3 \end{pmatrix} \right] \begin{matrix} (1,i,0) = 0 \\ (1,-i,3) = 0 \end{matrix} \right.$$

$$x - x_{\langle e_1, e_2 \rangle}$$

ha sempre soluzione $(\bar{\alpha}, \bar{\beta})$ e $\bar{\alpha}(1,i,0) + \bar{\beta}(1,-i,3)$ è la proiezione richiesta.

$\int \frac{1}{x} dx$ su $\{x \neq 0\}$ sono $\log|x| + \begin{cases} C_1 & x<0 \\ C_2 & x>0 \end{cases}$

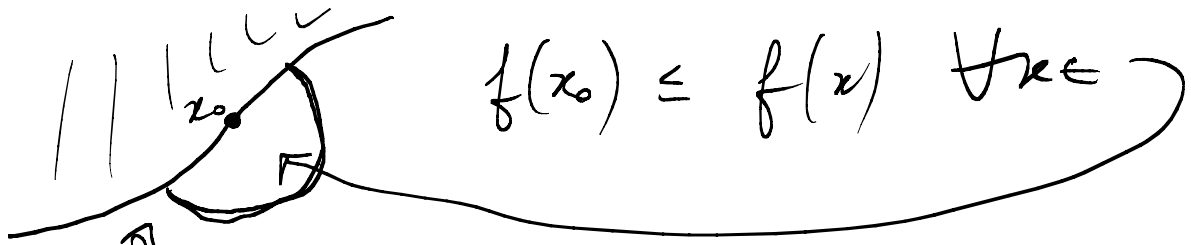
$$\left[\log|x| + \frac{x}{|x|}\right]' = \frac{1}{x}$$

$$\rightsquigarrow \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

IL Th. di LAGRANGE vale SOLO

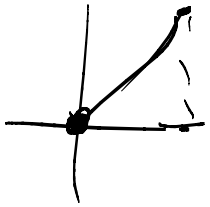
sugli INTERVALLI

Th. Fermat

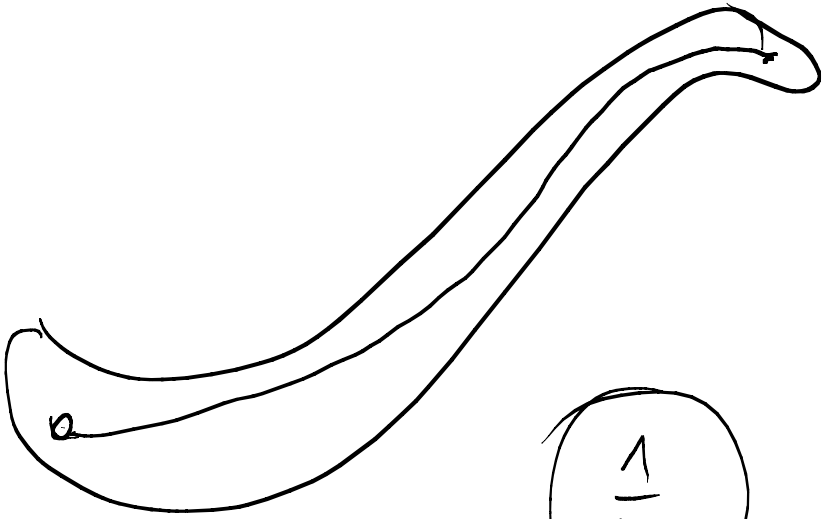


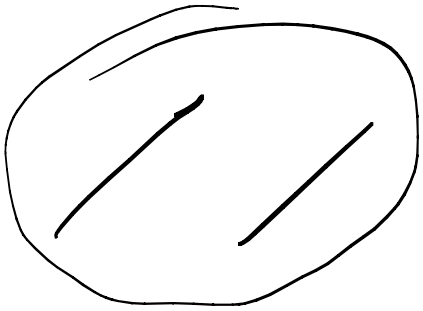
x_0

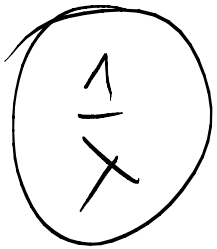
- $\Omega = [0,1]$



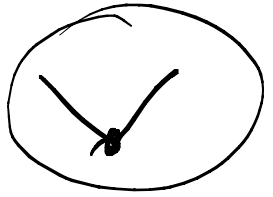
- $f(x) = x$
- 0 è minimo locale
- NON È INTERNO
- $f'(0) = 1 \neq 0$







$f(x) = |x|$



NO

$A(x)$ operatore

$A : X \to X$ $\quad x \in X$ $\quad X$ spazio dim. finita ($\neq 0$)

(Th. esistenza autovettori)

$v_1 - v_n$ base in X $\quad x$

$$A(x) = \sum_{i=1}^{n} \left[\sum_{j=1}^{n} A_{ij} x_j \right] v_i$$

le coordinate dell'immagine di x sono ottenute dal prodotto $A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ matrice associata.

X

AL - 7.9

$A(u) = u'$

$X = \mathbb{C}^n$ $\quad$ base = base canonica

$\in \mathbb{C}^n \leftarrow A(x) = \underset{\mathbb{C}^n}{Ax}$

$\det(A_1, \dots, A_n) = 0 \Longrightarrow A_1 \dots A_n$ sono dipendenti

DIM (PER ASSURDO)

Siano $A_1, \dots, A_n$: $\det(A_1 \dots A_n) = 0$ ma $A_1 \dots A_n$ sono indip.

Per il th. questi $A_1 \dots A_n$ sono una base di $\mathbb{R}^n$

$$e_i = \alpha_j^i A_j$$

$$1 = \det(e_1, \dots, e_n) = \det(\underbrace{\alpha_{j_1}^1 A_{j_1}}_{e_1}, e_2, \dots, e_n) =$$

$$= \alpha_{j_1}^1 \det(A_{j_1}, e_2, \dots e_n) = \alpha_{j_1}^1 \alpha_{j_2}^2 \det(A_{j_1}, A_{j_2}, e_3 \dots e_n) =$$

$e_2 = \sum_{i=2} \alpha_{j_2}^2 A_{j_2}$

lineare rispetto alla II colonna

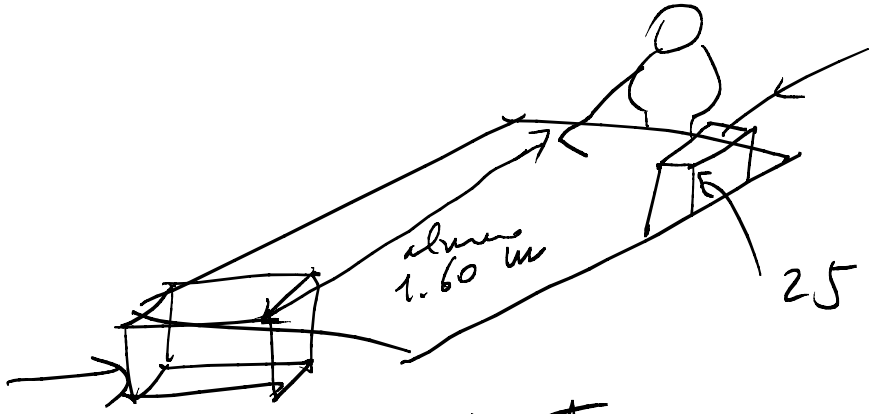
$$= \alpha^{1}_{j_1} \alpha^{2}_{j_2} \cdots \alpha^{n}_{j_n} \det(A_{j_1}, A_{j_2}, \cdots, A_{j_n})$$

$$\sum_{j_1, j_2, \cdots j_n = 1}^{n}$$

$$e_i = \alpha^{j}_{i} A_j$$

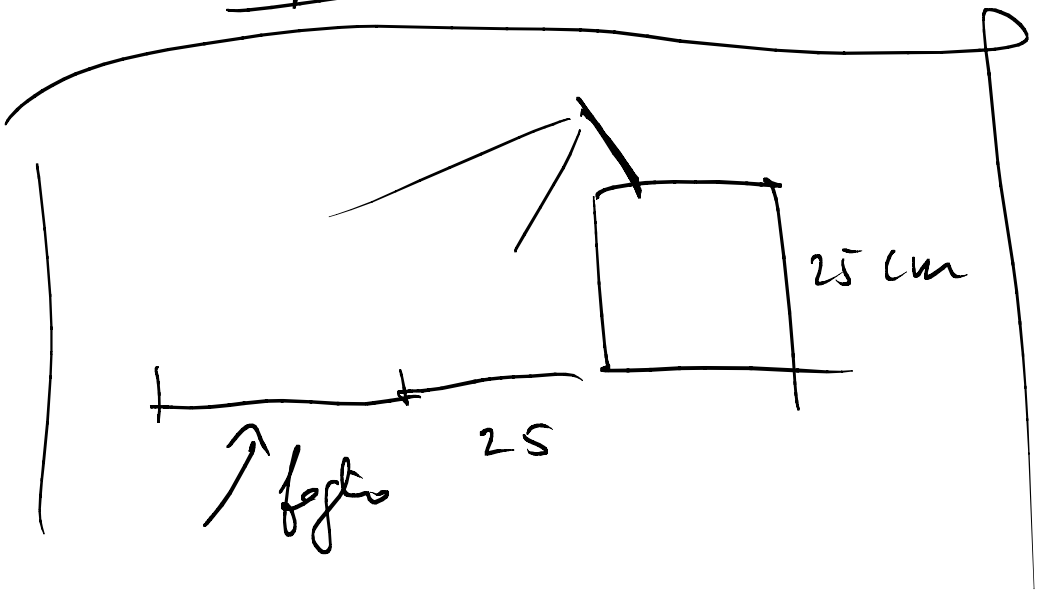
tutti nulli perché
o hanno colonne ripetute,
oppure tutte le colonne sono
diverse ma possono essere
riordinate, scambiando le
cambiando segno al determinante
fino a riportarle nell'ordine iniziale

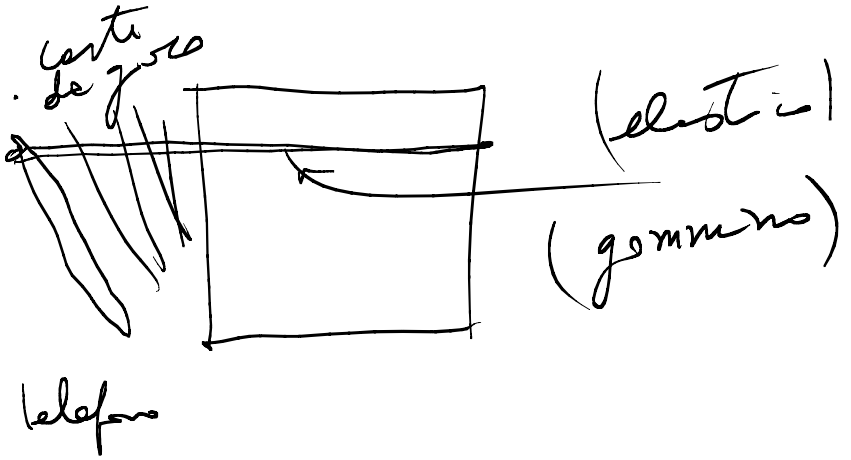
$A_1 \cdots A_n$ per le quali
$\det(A_1, \cdots A_n) \neq 0$ per ipotesi
(assunta)

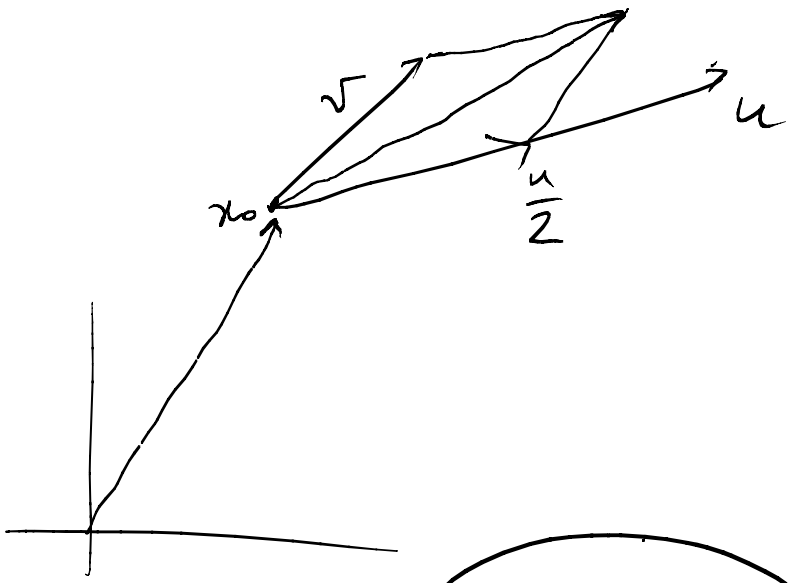


blocchi il telefono
col lato lungo
parallelo al
lato lungo del
foglio
inclinato di 45°
in avanti

display
o
lato con lo schermo sporgente







$x_0 = (1,0,1,1)$

$u = (1,1,1,1)$

$v = (1,0,0,0)$

$|v| = 1$

$\boxed{|u| = 2}$

G_1.2

$\left|\frac{u}{2}\right| = 1 = |v|$

$\frac{u}{2} + v = (1,0,0,0) + \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right) = \left(\frac{3}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right) \parallel (3,1,1,1)$

$x_0 + t\left(\frac{u}{2}+v\right) = \boxed{(1,0,1,1) + t(3,1,1,1)}$

$$L_A(x) = \underbrace{\begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 1 & -4 & -3 \\ 1 & 1 & 2 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}}_{x} = x_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$$

$A e_1 =$ I colonna $\quad A e_2 =$ II colonna ...

Per il Th. gener. dell'immagine

$$\operatorname{Im} A = L_A(\mathbb{R}^4) = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \right\rangle$$

AL_1.1 (II parte) $\begin{matrix} 1 & 2 & 1 & 0 \\ -1 & 1 & -4 & -3 \\ 1 & 1 & 2 & 1 \end{matrix}$ ridurre a scala e contare i pivot

Digito meet.google.com/pxu-bbvz-gfo

o minuscolo, non zero

per la ripresa ambientale

Per la ripresa ravvicinata del foglio col telefono

JITSI MEET a1b2c3