Note Title 12/15/2020

$w_1 \dots w_k$ base di $\ker A$ $(\Rightarrow \dim \ker = k)$

$w_1, \dots, w_k, x_{k+1} \dots x_n$ base di X ottenuta per completamento $(\Rightarrow \dim X = n)$

$$\operatorname{Im} A = \langle A(w_1), A(w_2), \dots, A(w_k), A(x_{k+1}), \dots, A(x_n) \rangle =$$

$$= 0 \quad 0 \quad 0$$

perché $w_i \in \ker A$

$$= \langle A(x_{k+1}), \dots, A(x_n) \rangle$$

Se fossero indipendenti è anche $\dim \operatorname{Im} A = n-k$ da cui segue la tesi.

$$0 = \sum_{k+1}^{m} \alpha_i A(x_i) \underset{\text{lineare di } A}{=\!=} A\left(\sum_{k+1}^{n} \alpha_i x_i\right)$$

$$\sum_{k+1}^{n} \alpha_i x_i \in \ker A = \sum_{1}^{k} \lambda_j w_j$$

$$0 = \sum_{1}^{k} \lambda_j w_j - \sum_{k+1}^{n} \alpha_i x_i \Longrightarrow \boxed{\alpha_i = 0 \ \forall i}$$

$\lambda_j = 0 \ \forall j$

$w_1, \dots, w_k, x_{k+1}, \dots, x_n$ sono indip

$A(x_{k+1}), \dots, A(x_n)$ è una base di $\operatorname{Im} A$

$$x_{\langle u_1 \dots u_n \rangle} = \sum_1^n \alpha_i u_i \quad \text{où } \alpha_i \text{ vérifient}$$

$$\left[x - \left[\sum_1^n \alpha_i u_i \right] \right] u_j = 0 \qquad \forall j = 1 \dots n$$

$$= x_{\langle u_1, \dots, u_n \rangle}$$

$$\dim \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}, \underbrace{\begin{pmatrix}-1\\1\\1\end{pmatrix}}_{X} \right\rangle + \left\langle \underbrace{\begin{pmatrix}0\\1\\2\end{pmatrix}, \begin{pmatrix}0\\3\\2\end{pmatrix}}_{Y} \right\rangle$$

$$\begin{array}{lcccc|l}
\mathrm{I} & 1 & -1 & 0 & 0 & \\
 & 2 & 1 & 1 & 3 & \\
 & 1 & 1 & 2 & 2 & \\
\hline
 & 0 & 3 & 1 & 3 & \mathrm{II} - 2\mathrm{I} \\
 & 0 & 2 & 2 & 2 & \mathrm{III} - \mathrm{I} \\
\mathrm{III} & 0 & 1 & 1 & 1 & \mathrm{III}/2 \\
\mathrm{II} & 0 & 0 & -2 & 0 & \mathrm{II} - 3\mathrm{III}
\end{array}$$

3 pivot

$\dim X + Y = 3$

$$\left\langle \begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \right\rangle^{\perp}$$

(x, y, z, t)

$$\begin{cases} 1\cdot x + 1\cdot y + 1\cdot z + 0\cdot t = 0 \\ 0\cdot x + 0\cdot y + 0\cdot z + 1\cdot t = 0 \end{cases}$$

$$\begin{cases} x + y + z = 0 \\ t = 0 \end{cases}$$

$$\left(\begin{array}{cccc} 1 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} -y-z \\ y \\ z \\ 0 \end{pmatrix} =$$

$x = -y - z$

$t = 0$

$$= y\begin{pmatrix}-1\\1\\0\\0\end{pmatrix} + z\begin{pmatrix}-1\\0\\1\\0\end{pmatrix} = \left\langle \begin{pmatrix}-1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\0\end{pmatrix} \right\rangle$$

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 1 & -4 & -3 \\ 1 & 1 & 2 & 1 \end{pmatrix} \qquad A(x) = Ax$$

$$\langle A(e_1), A(e_2), A(e_3), A(e_4) \rangle$$

$$A_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$$

$$AI = A = (A_1, A_2, \dots A_4)$$

$$(Ae_1 \; Ae_2 \dots Ae_4)$$

x_1	x_2	x_3	x_4	
1	2	1	0	0
−1	1	−4	−3	0
1	1	2	1	0

da ridurre a scala.

$\Rightarrow$ determ. il Ker A equivale risolvere il sistema omogeneo $\boxed{Ax = 0}$

AL_1.1

$X = \langle u_1, \dots, u_n \rangle \qquad Y = \langle v_1, \dots, v_m \rangle$

$Y \subseteq X \iff \boxed{u_1 \dots u_n} \, v_1 \dots v_m$ hanno tutti soluzione

$\Downarrow$

ogni v_j è combinazione di $u_1 \dots u_n$

$\boxed{\text{AL\_1.2}}$

$$p(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \dots + \alpha_n t^n \equiv 0$$

allora $\alpha_0 = \alpha_1 = \dots = \alpha_n = 0$

$p(t) = 0 \ \forall t$ ← allora $0 = p(0) = \alpha_0$

$$p(t) = t\left[\alpha_1 + \alpha_2 t + \dots + \alpha_n t^{n-1}\right]$$

$p_1(t)$

annullamento del prodotto

$$0 \equiv p(t) \Rightarrow p_1(t) = 0 \quad \forall t \neq 0$$

Poiché $p_1(t) = 0 \ \forall t \neq 0 \Rightarrow \lim_{t \to 0} p_1(t) = 0$ = per continuità di $p_1(t)$ = $p_1(0) = \alpha_1$

$$0 \equiv p(t) = t^2 \left[\alpha_2 + \alpha_3 t + \dots + \alpha_n t^{n-2} \right]$$

0 $t \neq 0 \Rightarrow \equiv 0$ per la continuità

AL_3.1 esempi

$\{1, t, \dots, t^{n-1}, t^n\}$ sono indipendenti

$X = \langle u_1, \dots, u_n \rangle \quad Y = \langle v_1, \dots, v_m \rangle$

$w \in X \cap Y$ se e solo se $w \in X$ $\quad w \in Y$

$\exists \alpha_i : w = \sum_1^n \alpha_i u_i \quad \exists \beta_j : w = \sum_1^m \beta_j v_j$

$$\sum_1^n \alpha_i u_i - \sum_1^m \beta_j v_j = 0 \rightarrow \left[\begin{array}{cccc|cccc} \alpha_1 & \cdots & \alpha_n & & \beta_1 & \cdots & \beta_m & \\ \hline u_1 & u_2 & \cdots & u_n & -v_1 & -v_2 & \cdots & -v_m \end{array}\right] 0$$

se $(\bar{\alpha}_1 \cdots \bar{\alpha}_n \bar{\beta}_1 \cdots \bar{\beta}_m)$ è una soluzione di

allora $\underbrace{\boxed{\sum_1^n \bar{\alpha}_i u_i}}_{\in X \cap Y} = \underbrace{\boxed{\sum_1^m \bar{\beta}_j v_j}}_{\in X \cap Y} = w \in X \cap Y$

$$\boxed{\left[x - \sum_1^n \alpha_i u_i\right] u_j = 0 \quad \forall j = 1 \ldots n}$$

$u = {}^{x}X \quad v = {}^{x}X$

$x - u \perp X$

$x - v \perp X$

$(u - v) \perp X$

$\underbrace{\in X \quad \in X}_{\in X}$

$\in X$

$u - v \in X$

$(u - v)w = 0 \quad \forall w \in X$

Scelto $w = u - v$

$(u - v)(u - v) = 0 \Rightarrow |u - v| = 0 \Rightarrow$

$u = v$

AL_2.4

$A \in \mathbb{R}^{n\times n} \exists\, X \in \mathbb{R}^{n\times n} : \quad AX = I$

$$\boxed{X = A^{-1}} \qquad X = (X_1\ X_2\ \dots\ X_n)$$

$$AX = (AX_1\ AX_2\ \dots\ AX_n)$$

$$I = (e_1\ e_2\ \dots\ e_n)$$

$$\begin{cases} AX_1 = e_1 \\ AX_2 = e_2 \\ \vdots \\ AX_n = e_n \end{cases} \iff \underbrace{A_1 A_2 \dots A_n}_{A} \,|\, e_1\ e_2 \dots e_n$$

RISOLVERE G-J

n sistemi lineari

A è invertibile se $A_1 \dots A_n$ è una base di $\mathbb{R}^n$