

$$A = \begin{pmatrix} A_1 \\ \vdots \\ A_m \end{pmatrix}$$

$$B = (B_1 \dots B_k)$$

$$(AB)_{ij} = \sum_j A_{ij}^i B_j^j$$

product
of the
columns

$$A = (1 \ 2 \ 3) \in \mathbb{R}^{1 \times 3}$$

$$B = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \in \mathbb{R}^{3 \times 1}$$

$$AB \in \mathbb{R}^{1 \times 1}$$

$$AB = (1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6) \in \mathbb{R}^{1 \times 1}$$

$$BA \in \mathbb{R}^{3 \times 3}$$

$$BA = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} (1 \ 2 \ 3)$$

$$\begin{pmatrix} 4 \cdot 1 & 4 \cdot 2 & 4 \cdot 3 \\ 5 \cdot 1 & 5 \cdot 2 & 5 \cdot 3 \\ 6 \cdot 1 & 6 \cdot 2 & 6 \cdot 3 \end{pmatrix}$$

$\mathbb{R}^{n \times n} \ni A, B$

$AB, BA \in \mathbb{R}^{n \times n}$

$$\boxed{A(B+C) = AB + AC}$$

DISTRIB

$$(B+C)A = BA + CA$$

$$A(BC) = (AB)C \equiv ABC \text{ ASSOC.}$$

NON VALTE LA PROP. COMMUTATIVA

ALGEBRA (ASSOCATIVA, NON COMMUTATIVA)

Ax

$$A \in \mathbb{R}^{m \times n}$$

$$x \in \mathbb{R}^{n \times 1}$$

$$A = (a_{ij})$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

 \Downarrow

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

 $(Ax)_{1,1}$ (Ax) $\mathbb{R}^{m \times 1}$

$$\begin{aligned} (Ax) &= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{pmatrix} \\ \sum_{i=1}^n x_i A_{i-} &= \end{aligned}$$

"pacchetto" dei I membri
 dim sistema
 dimensione d'
 coeff. ($-a_{i,j-}$)

- $A: \mathbb{R} \rightarrow \mathbb{R}$ $\exists a \in \mathbb{R}: A(x) = ax$ Produkt d' scalar (numer)
- $A: \mathbb{R} \rightarrow \mathbb{R}^n$ $\exists a \in \mathbb{R}^n: A(x) = \overbrace{ax}^{\text{produkt scalar for vector}} a \in \mathbb{R}^n x \in \mathbb{R}$
- $A: \mathbb{R}^n \rightarrow \mathbb{R}$ $\exists a \in \mathbb{R}^n: A(x) = ax$ $a \in \mathbb{R}^n x \in \mathbb{R}^n$ \curvearrowleft produkte scalar
- $A: \mathbb{R}^n \rightarrow \mathbb{R}^m$ $\exists A \in \mathbb{R}^{m \times n}: A(x) = Ax$ $x \in \mathbb{R}^{n \times 1}$

$$(1, 2, 3) \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}^{3 \times 1} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6$$

~~(1, 2, 3)~~ \curvearrowright ~~(4, 5, 6)~~
~~1 \times 3~~ \curvearrowright ~~1 \times 3~~

$$\begin{pmatrix}
 a_1x_1 + \dots + a_nx_n \\
 \vdots \\
 a_mx_1 + \dots + a_nx_n
 \end{pmatrix} \equiv \boxed{\sum_{i=1}^n x_i \cdot A_i} \stackrel{\text{gleich}}{=} Ax$$

\parallel
 (a_{ij})

$$\begin{array}{c}
 \downarrow \\
 a_{11}x_1 + \dots + a_{nn}x_n = (x_1 \dots x_n)(a_{11} \ a_{12} \ \dots \ a_{1n}) \\
 \text{prodotto scalare di vettori} \\
 A \in \mathbb{R}^{m \times n} \quad \leftarrow \quad \left(\begin{array}{c} A^*, A^T \\ \downarrow \end{array} \right) \quad \left(\begin{array}{l} \text{NON} \ \underline{\text{vettori colonne}}, \text{MATICI} \\ \text{ma le righe sono vettori colonna} \end{array} \right) \\
 \left(\begin{array}{c} x_1 \dots x_n \end{array} \right) \left(\begin{array}{c} a_{11} \ \dots \ a_{m1} \\ a_{12} \\ \vdots \\ a_{1n} \end{array} \right) \quad \left(\begin{array}{c} a_{m1} \\ \vdots \\ a_{mn} \end{array} \right) \\
 \boxed{(A^T)_{ij} = a_{ji}} \quad \text{definizione di } A^T
 \end{array}$$

nel caso complesso

$$\boxed{(A^*)_{ij} = \overline{A_{ji}}} \quad \text{definizione di} \\
 \text{matrice adjunta}$$

$$\overline{A_{ji}} = \overline{A_{ji}} \quad \text{e} \quad A \in \mathbb{R}^{m \times n}$$

$$x^T A^T = (a_{11}x_1 + \dots + a_{1n}x_n, \dots, a_{m1}x_1 + \dots + a_{mn}x_n)$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \end{pmatrix}^T = \begin{pmatrix} 2 & 4 \\ 1 & -1 \\ 3 & 0 \end{pmatrix}$$

$$x^T = (x_1 \dots x_n)$$

$$(A^T)^T = A$$

$$\begin{pmatrix} 2 & 4 \\ 1 & -1 \\ 3 & 0 \end{pmatrix}^T = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \end{pmatrix}$$

$$(AB)^T = B^T A^T$$

$$Ax = b \quad (Ax)^T = b^T \Rightarrow x^T A^T = b^T$$

MATRIX 14 IV

$$AB = A(B_1 \dots B_p) =$$

\underbrace{B}_B

$$= \begin{pmatrix} AB_1 & \dots & AB_p \end{pmatrix}$$

↑
 $A = (A_1, A_2, \dots, A_n)$

$$AI = A(e_1 e_2 \dots e_n) =$$

$$= (Ae_1, Ae_2 \dots Ae_n) =$$

$$\sum_{i=1}^n x_i A_i$$

↑
column

$$(1A_1, 1 \cdot A_2, \dots, 1 \cdot A_n) = A$$

I ha lo stesso ruolo del numero 1
in $\mathbb{R} \cup \{\infty\}$ del polinomio 1 in $[RE^x]$

$$A \in \mathbb{R}^{m \times n}$$

$$B_1, B_2, \dots, B_p \in \mathbb{R}^m$$

$$B \in \mathbb{R}^{3 \times p}$$

$$AB \in \mathbb{R}^{m \times p}$$

$$I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

..

$$(e_1 e_2 \dots e_n)$$

$\underbrace{e_1 e_2 \dots e_n}$
base canonica

$$\begin{pmatrix} A^1 \\ A^2 \\ \vdots \\ A^m \end{pmatrix} B = \begin{pmatrix} A^1 B \\ \vdots \\ A^m B \end{pmatrix}$$

$$IA = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{pmatrix} A = \begin{pmatrix} e_1 A \\ e_2 A \\ \vdots \\ e_m A \end{pmatrix} = \begin{pmatrix} A^1 \\ A^2 \\ \vdots \\ A^m \end{pmatrix}$$

$$A \in \mathbb{R}^{m \times n}$$

$$\begin{array}{c} AI \\ IA \\ I \\ = A \end{array} . \quad \begin{array}{l} I_n \in \mathbb{R}^{n \times n} \\ I_m \in \mathbb{R}^{m \times m} \end{array}$$

Dato $A \in \mathbb{R}^{n \times n}$ si dice INVERTIBILE (oppure
REGOLARE oppure NON SINGOLARE)

$\Leftrightarrow \exists X \in \mathbb{R}^{n \times n}$ tali che

$$\begin{cases} AX = I \\ XA = I \end{cases}$$

L'INVERSA X di A
si denota con A^{-1}

Lemme. $\frac{\text{Se } AX = I}{\text{e } YA = I} \text{ allora } X = Y$

DIM. $X = IX = (YA)X = YA(X) = Y(AX) = YI = Y$

$$AX = I$$

$n \times n$ $n \times n$ $n \times n$
 ↓ ↓ ↓
 A X I

colonne dell'ieme (candidate)

||

$$\rightarrow A(X_1 X_2 \dots X_n) = \underbrace{(e_1 e_2 \dots e_n)}_{\text{base canonica}}$$

$$I = \begin{pmatrix} 1 & 0 & \dots \\ 0 & 1 & \dots \\ \vdots & \vdots & \ddots \\ 0 & 0 & \dots \\ e_1 & e_2 & \dots e_n \end{pmatrix}$$

$$(AX_1 \quad AX_2 \quad \dots \quad AX_n) = (e_1 \quad e_2 \quad \dots \quad e_n)$$

Occorre
trovare
Tutti i m^n

$$\begin{cases} AX_1 = e_1 \\ AX_2 = e_2 \\ \vdots \\ AX_n = e_n \end{cases}$$

Le matrici di coefficienti
conducendo tutti con A

$$\underbrace{A_1 \dots A_n}_{A} | e_1 \dots e_n \xrightarrow{\text{G-J.}} e_1 \dots e_n | X_1 \dots X_n$$

$$A \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

$\exists A^{-1}?$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ \hline 0 & -1 & -1 & 1 \end{array} \right] \quad \text{II} - \text{I}$$

$$\left[\begin{array}{cc|cc} 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \\ \hline 0 & 1 & 1 & -1 \end{array} \right] \quad -\text{II}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ \hline e_1 & e_2 & X_1 & X_2 \end{array} \right] \quad \text{I} - 2\text{II}$$

$A^{-1} (= X)$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ \hline e_1 & e_2 & X_1 & X_2 \end{array} \right]$$

$\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
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$$\begin{array}{c|cc|cc} \textcircled{1} & 2 & & 1 & 0 \\ \hline 2 & 4 & & 0 & 1 \\ \hline 0 & 0 & & -2 & 1 \end{array}$$

$\text{II} - 2\text{I}$

NON E'
RISOLUBILE

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ non
E'
INVERTIBILE