Note Title 11/10/2020

Z spazio vettoriale X, Y sottospazi di Z

$X = \langle u_1, u_2, \dots, u_n \rangle \qquad Y = \langle v_1, v_2, \dots, v_m \rangle$

$X \cap Y = \{ w \in Z : w \in X \text{ e } w \in Y \}$

$w \in X \iff \exists\, \alpha_1 \dots \alpha_n \in \mathbb{R} : \; w = \sum_1^n \alpha_i u_i$

$w \in Y \iff \exists\, \beta_1 \dots \beta_m \in \mathbb{R} : \; w = \sum_1^m \beta_j v_j$

$w \in X \cap Y \iff \exists\, \alpha_1 \dots \alpha_n, \beta_1 \dots \beta_m : \; \sum_1^n \alpha_i u_i = \sum_1^m \beta_j v_j \; (= w)$

Se $Z = \mathbb{R}^n$

$$\sum_1^n \alpha_i u_i + \sum_1^m \beta_j (-v_j) = 0$$ ← è un sistema omogeneo

α_1 α_2 ... α_n	β_1 β_2 ... β_m	
u_1 u_2 ... u_n	$-v_1$ $-v_2$ ... $-v_m$	0

È certamente risolubile perché $\alpha_1 = \dots = \alpha_n = \beta_1 = \dots \beta_m = 0$ è una soluzione.

Per ogni soluzione $(\bar\alpha_1, \bar\alpha_2, \dots, \bar\alpha_n, \bar\beta_1, \dots, \bar\beta_m)$ si può ricavare il vettore $w \in X \cap Y$ calcolando $\sum_1^n \bar\alpha_i u_i$ oppure $\sum_1^m \bar\beta_j v_j$

AL_1.2

∨		∨		∨
①	3	1	3	1
0	0	②	1	1
0	0	0	0	②

$k_1 = 1$

$k_2 = 3$

$k_3 = 5$

$k_1 < k_2 < k_3$

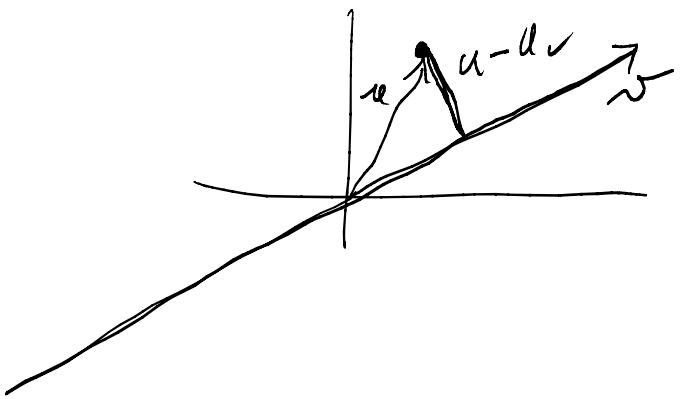
	x_1	x_3	x_5	x_2	x_4	
I	1	1	1	−3	−3	I − II − III
II	0	2	1	0	−1	IV − III
	0	0	2	0	0	III / 2
III	0	0	1	0	0	
II	0	2	0	0	−1	II / 2
II	0	1	0	0	$-\frac{1}{2}$	
I	1	0	0	−3	$-\frac{5}{2}$	

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -3x_2 - \frac{5}{2}x_4 \\ x_2 \\ -\frac{1}{2}x_4 \\ x_4 \\ 0 \end{pmatrix} =$$

$$= x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -\frac{5}{2} \\ 0 \\ -1/2 \\ 1 \\ 0 \end{pmatrix} = \langle \, , \, \rangle$$

$u\,(1, i, -i)$ $\qquad$ $<(i, 1, i)>$

$\underline{v}$



$d = |u - u_v|$

$$u_v = \frac{uv}{|v|^2}\, v = \frac{-i + i - 1}{3}\,(i, 1, i)$$

$$u - u_v = (1, i, -i) + \left(\tfrac{1}{3}i, \tfrac{1}{3}, \tfrac{1}{3}i\right) = \left(1 + \tfrac{1}{3}i, \tfrac{1}{3} + i, -\tfrac{2}{3}i\right)$$

$$|u - u_v| = \left|\left(1 + \tfrac{1}{3}i, \tfrac{1}{3} + i, -\tfrac{2}{3}i\right)\right|$$

Z spazio vettoriale X, Y sottospazi di Z

(1) $X+Y = \{w \in Z : \exists x \in X, y \in Y \quad w = x+y\} \subseteq Z$

$x+y \in Z$
$\forall x \in X \subseteq Z$
$\forall y \in Y \subseteq Z$

(2) $w, w' \in X+Y \Rightarrow w+w' \in X+Y$

$\Updownarrow$

$\exists x \in X, y \in Y : w = x+y$

$\exists x' \in X, y' \in Y : w' = x'+y'$

$\Rightarrow w+w' = \underbrace{(x+x')}_{\in X} + \underbrace{(y+y')}_{\in Y} \Rightarrow$

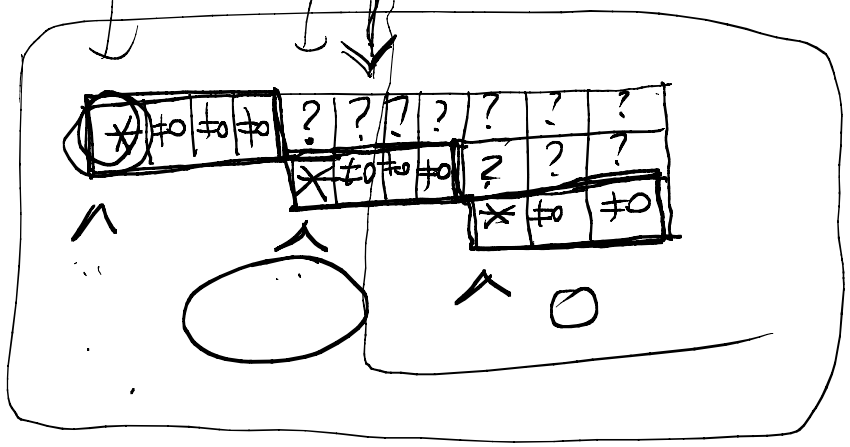
$$\sum_{\text{pivot}} x_i A_i = -\sum_{\text{non pivot}} x_j A_j$$

fissato $\bar{j}$ e posto $x_{\bar{j}} = -1$ e $x_j = 0 \ \forall j \neq \bar{j}$ si ottiene

$$\boxed{\sum_{\text{pivot}} x_i A_i = A_{\bar{j}}} \iff A_{\bar{j}} \in \langle A_i, \ i \text{ pivot} \rangle$$

pivot

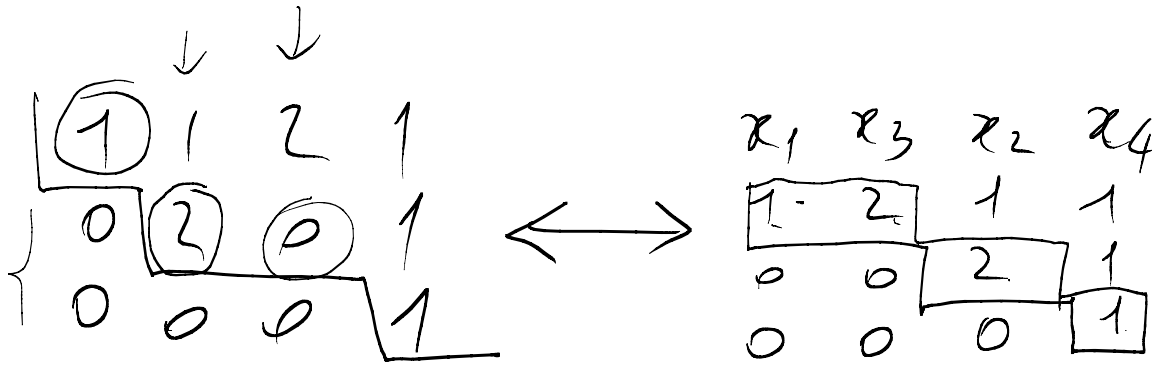
non pivot



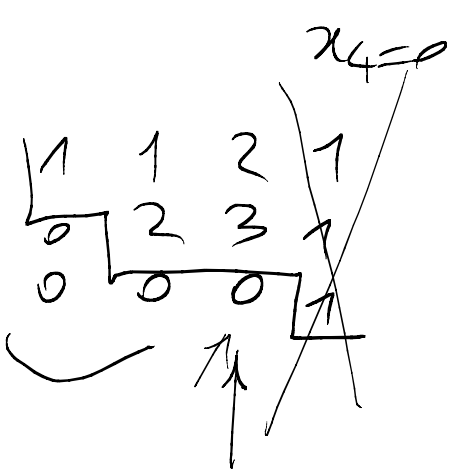
"sulla scala" $\neq 0$

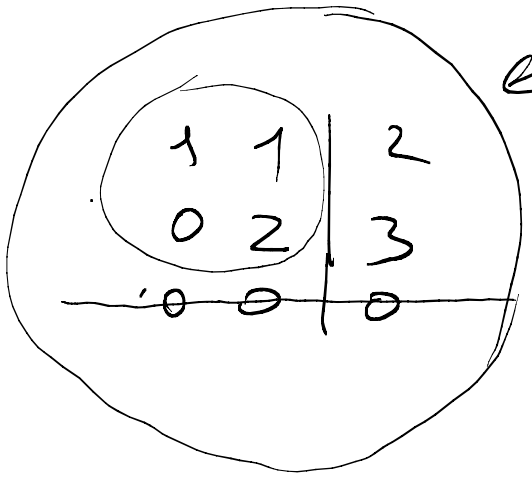
"sotto la scala" $= 0$

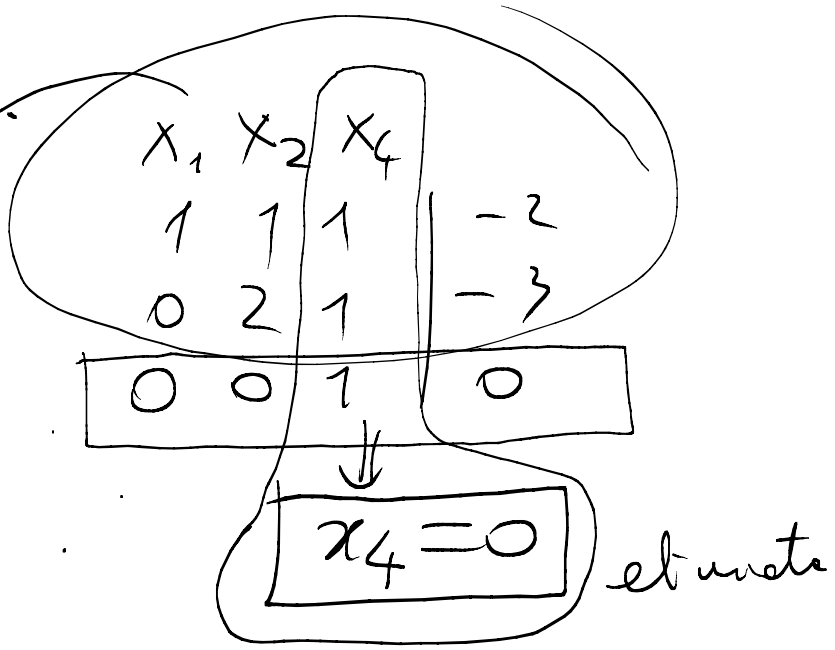
"sopra la scala" ?



$$\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$







$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 1 & -4 & -3 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

x_1 x_2

II $\quad 0 \quad 3 \quad -3 \quad -3$

II $\quad 0 \quad 1 \quad -1 \quad -1$

III $\quad 0 \quad -1 \quad 1 \quad 1$

II $\quad 0 \quad 0 \quad 0 \quad 0$

x_1 x_2

$$\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \end{array}$$

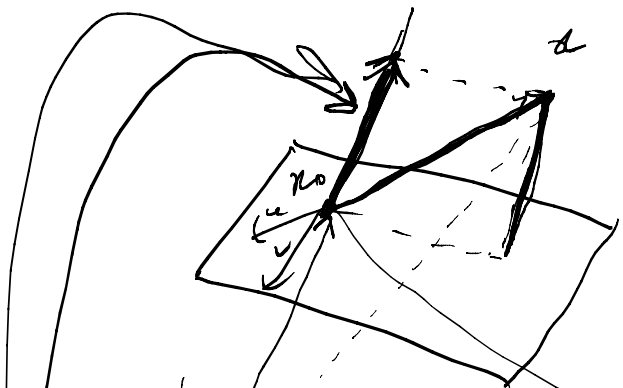
$$\left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$A(u) = Au$$

$$A(u) = A\left(\sum_1^n u_i e_i\right) = \sum u_i A(e_i)$$

$$\operatorname{Im} A = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \right\rangle$$

$$\operatorname{Ker} A = \begin{cases} A(u) = 0 \\ Au = 0 \end{cases}$$



$$ax + by + cz = d$$

$(x_0, y_0, z_0) \in$

$a \neq 0$

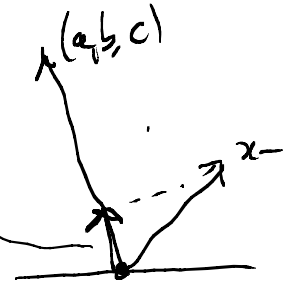
$ax = d - by - cz$

$x = \frac{d}{a}$ $\quad$ $y=0,\ z=0$

$\left(\frac{d}{a}, 0, 0\right) \in$ piano

(a,b,c) vettore normale al piano

$(x - x_0)_{(a,b,c)}$



$$d\left(x, \{ax+by+cz=d\}\right) =$$

$$= \left| (x - x_0)_{(a,b,c)} \right|$$

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \dots + a_{mn}x_n = b_m \end{cases}$$

$$\Downarrow$$

$$x_1 \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + x_n \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \left\langle \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix}, \dots, \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix} \right\rangle$$

$$\begin{cases} x + 2y + 3z = 4 \\ y - z = 2 \\ z = 1 \end{cases}$$

1	2	3	4
0	1	-1	2
0	0	3	1

1	2	0	-1	1
0	0	1	2	0
0	0	0	0	1

non-zero

x_1	x_2	x_3	
1	1	4	-3
0	2	2	-3

x_1	x_2	x_3		
1	1		4	-3
0	2		-3	-2

	x_1	x_2	x_3	
I	1	1	4	-3
II	0	1	-3/2	-1
I	1	0	11/2	-2

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{11}{2} - 2x_3 \\ -3/2 - x_3 \\ x_3 \end{pmatrix} =$$

$$= \begin{pmatrix} 1/2 \\ -3/2 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

retta affine