Note Title 11/9/2020

$X = \langle u_1 \dots u_n \rangle$ $\quad Y = \langle v_1, \dots, v_m \rangle$

$\boxed{X \subseteq Y}$ ogni elemento di X appartiene a Y

$u_1, u_2, \dots, u_n \in Y$ accade se il sistema

$\left[\begin{array}{c|c} v_1 \dots v_m & u_1 \; u_2 \dots u_n \end{array}\right]$ tutti gli n sistemi hanno soluzione.

È facile provare, risolvendo se i generatori di X appartengono a Y

LEMMA Se uno spazio vettoriale Y contiene $u_1, u_2, \dots u_n$ allora contiene tutto $\langle u_1 \dots u_n \rangle$.

DIM. $Y \ni u_1, \dots, u_n$ allora $Y \ni \alpha_1 u_1, \alpha_2 u_2 \dots, \alpha_n u_n$ $\forall \alpha_i \in \mathbb{R}$

perché Y è chiuso per multipl scalare.

e inoltre, se contiene $\alpha_1 u_1, \alpha_2 u_2, \dots, \alpha_n u_n \Rightarrow$ contiene la loro somma $\sum_1^n \alpha_i u_i$

LEMMA CNS perché $\langle u_1, \dots, u_n \rangle \subseteq \overline{\langle v_1, \dots, v_m \rangle}^{Y}$ è che $u_i \in \langle v_1 \dots v_m \rangle \ \forall i = 1 \dots n$.

$X = Y \iff X \subseteq Y$ e $Y \subseteq X$

$X = \left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \right\rangle$ $\quad$ $Y = \left\langle \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix} \right\rangle$

$\Downarrow ?$

$X = Y$

$Y \subseteq X$

$$\begin{array}{cc|cc} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 \end{array}$$

$$1\begin{pmatrix}1\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$$

$$-1\begin{pmatrix}1\\0\end{pmatrix} + 1\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$$

$X \subseteq Y$

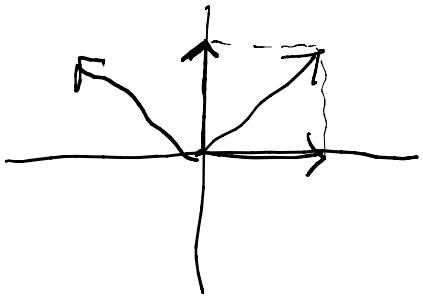
$\left\langle \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} \right\rangle \overset{?}{\subseteq} \left\langle \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix} \right\rangle$

$$\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}$$

$\Rightarrow Y \subseteq X$

① −1	1	0	I + II
1 1	0	1	II − I
0 2	−1	1	II / 2
0 1	$-\frac{1}{2}$	$\frac{1}{2}$	II

1 0	$\frac{1}{2}$	$\frac{1}{2}$
0 1	$-\frac{1}{2}$	$\frac{1}{2}$



AL_1.2

$$\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}-1\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-1\\1\end{pmatrix}$$

$\Rightarrow$ $X \subseteq Y$ $\Downarrow$ $X = Y$

$X = \langle u_1, \dots, u_n \rangle$ $\quad Y = \langle v_1, \dots, v_m \rangle$

$X \cap Y = ?$

$w \in X \cap Y \Leftrightarrow w \in X$ e $w \in Y$

$w = \sum_1^n \alpha_i u_i$ e $w = \sum_1^m \beta_j v_j \Longrightarrow$

$\Rightarrow$ $\sum_1^n \alpha_i u_i = \sum_1^m \beta_j v_j$ $\quad \alpha_i, \beta_j$ incognite

w $\qquad$ w

$u_1 \dots u_n \; v_1 \dots v_m$ noti

$$\sum_1^n \alpha_i u_i - \sum_1^m \beta_j v_j = 0$$

In $\mathbb{R}^n$ è un sistema lineare omogeneo.

α_1	α_2	$\dots$	α_n	β_1	β_2	$\dots$	β_m	
u_1	u_2	$\dots$	u_n	$-v_1$	$-v_2$	$\dots$	$-v_m$	0

Se $(\bar\alpha_1, \bar\alpha_2, \dots, \bar\alpha_n \mid \bar\beta_1, \dots, \bar\beta_m)$ è una qualunque soluzione di quella

$$\bar w = \sum_1^n \bar\alpha_i u_i = \sum_1^m \bar\beta_j v_j$$

— un punto dell'intersezione.

$$X = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\rangle \qquad Y = \left\langle \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

u_1 u_2 $\overline{v_1}$ v_2

α_1	α_2	β_1	β_2	
(1)	1	−2	−1	0
1	−1	−0	−1	0 II − I
1	1	−2	−2	0 III − I
II 0	−2	2	0	II/(−2)
III 0	0	0	−1	III/(−1)
II 0	1	−1	0	

$(\alpha_1, \alpha_2, \beta_1, \beta_2)$

α_1	α_2	β_1	β_2
(1)	1	−2	−1
0	(1)	−1	0
0	0	0	(1)

α_1	α_2	β_2	β_1
1	1	−1	2
0	1	0	1
0	0	1	0
1	0	0	1

I + III − II

$$X\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = \begin{pmatrix}\beta_1\\ \beta_1\end{pmatrix} = \beta_1\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$Y\begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix} = \begin{pmatrix}\beta_1\\ 0\end{pmatrix} = \beta_1\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\alpha_1 u_1 + \alpha_2 u_2 = \beta_1\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} + \beta_1\begin{pmatrix}1\\ -1\\ 1\end{pmatrix} = \beta_1\begin{pmatrix}2\\ 0\\ 2\end{pmatrix}$$

$$\text{"} < \begin{pmatrix}1\\ 0\\ 1\end{pmatrix} >$$

$$\beta_1 v_1 + \beta_2 v_2 = \beta_1\begin{pmatrix}-2\\ 0\\ -2\end{pmatrix} + 0\begin{pmatrix}1\\ 1\\ 2\end{pmatrix}$$

$$!!$$

$$< \begin{pmatrix}1\\ 0\\ 1\end{pmatrix} >$$