Note Title 11/5/2020

x	y		
(2)	-3	1	2

$\Leftrightarrow$

$2x-3y=1 \quad | \quad 2x-3y=2$

EQUAZIONI SCORRELATE

PIVOT

x		y	
2	1	2	3

x		y	
1	$\frac{1}{2}$	1	$\frac{3}{2}$

$2x = 1+3y$

scelgo a priori y e calcolo x

$x = \frac{1}{2} + \frac{3}{2}y$

$\forall y \in \mathbb{R} \ \exists x$ coppia

$(\frac{1}{2}+\frac{3}{2}y, y)$ tutte e sole soluzioni

$2x = 2+3y$

$x = 1+\frac{3}{2}y$

$(1+\frac{3}{2}y, y)$

$\forall y \in \mathbb{R}$

$AX = B$

A, X, B matrici

Le soluzioni dell'equazioni (due)

$$\begin{array}{cc|cc} x & y & & \\ \hline 2 & -3 & 1 & 2 \end{array}$$

sono

$$\begin{array}{c|cc} x & & \\ \hline 1 & \frac{1}{2}+\frac{3}{2}y & 1+\frac{3}{2}y' \end{array}$$

al variare di y e y' in $\mathbb{R}$

si ottengono tutte le soluzioni del sistema (con 2 termini, noti)

$$\begin{array}{c|cccc} x_1 \dots x_k & & & x_{k+1} & \dots x_n \\ \hline \text{triang.} & b_1 & d_1 & \vdots & \vdots \\ & \vdots & \vdots & & \\ & b_k & d_k & & \end{array}$$

$\Downarrow$ GAUSS-JORDAN

$$\begin{array}{c|cccc} x_1 \dots x_k & & & x_{k+1} & \dots x_n \\ \hline 1\ 0 \dots 0 & b'_1 & d'_1 & \vdots & \\ 0\ 1 \dots 0 & \vdots & \vdots & & \\ 0\ 0 \dots 1 & b'_k & d'_k & & \end{array}$$

I sistema

$$\begin{array}{ccc|ccccc} x_1 & x_2 & x_3 & & & & x_4 & x_5 \\ \hline 1 & 0 & 0 & 1 & 0 & 1 & -3 & 1 \\ 0 & 1 & 0 & 2 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 & -1 & 1 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 - 3x_4 + x_5 \\ 2 - 2x_4 + x_5 \\ -1 - x_4 + x_5 \\ x_4 \\ x_5 \end{pmatrix}$$

I colon.

II sistema

$$\begin{pmatrix} 0 & -3x_4' + x_5' \\ 1 & -2x_4' + x_5' \\ 1 & -x_4' + x_5' \\ x_4' & \\ x_5' & \end{pmatrix}$$

III sist

$$\begin{pmatrix} 1 & -3x_4'' + x_5'' \\ 0 & -2x_4'' + x_5'' \\ 0 & -x_4'' + x_5'' \\ x_4'' & \\ x_5'' & \end{pmatrix}$$

I colonna

II colonna

PIVOT

x	y	z	
(1)	1	2	0 1
2	-2	3	1 1

$\xrightarrow{II - 2I}$

x	y	z	
(1)	1	2	0 1
0	(-4)	-1	1 -1

SCALA

NON PIVOT e X membro

x	y		z
1	1	0 1	-2
0	-4	1 -1	1

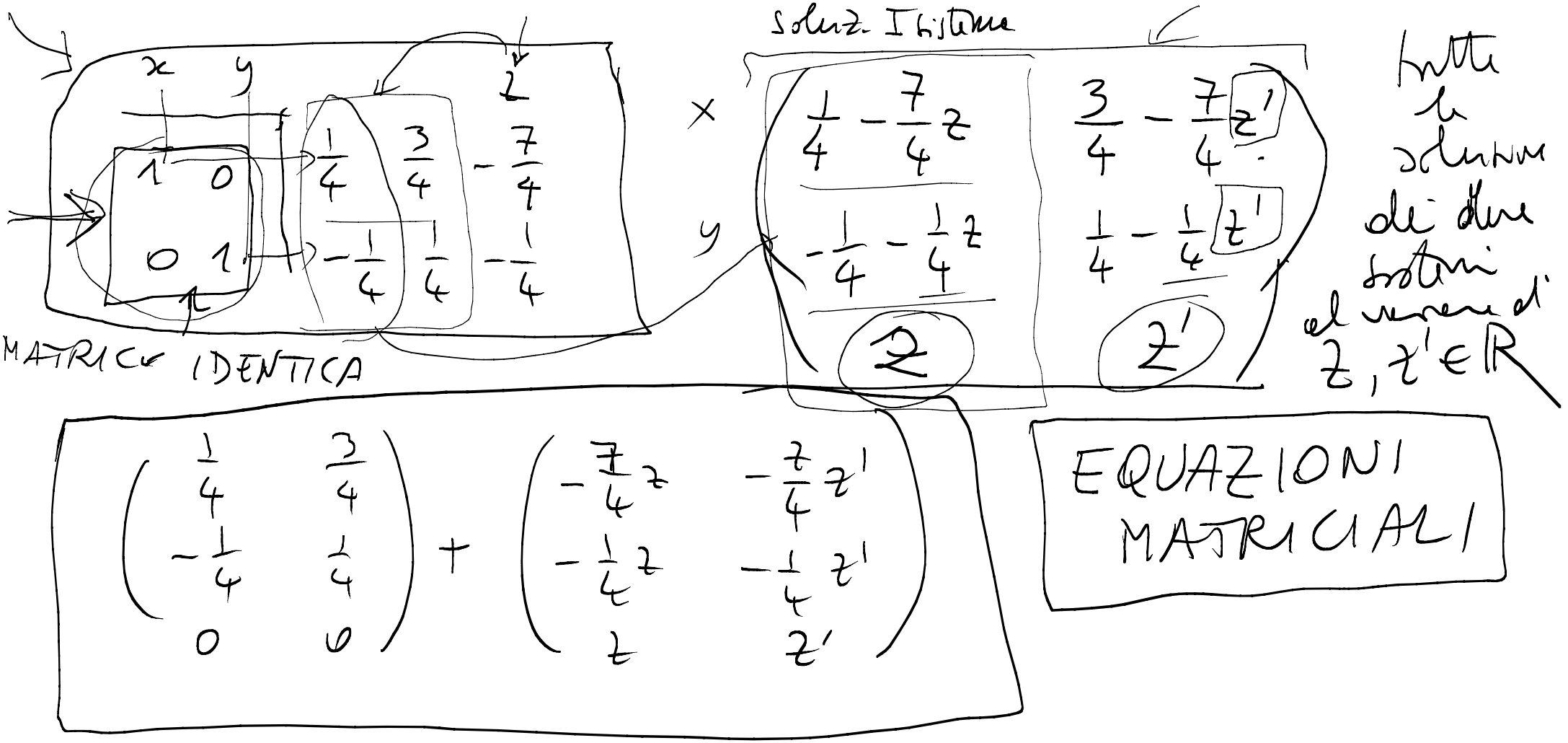
$\xrightarrow{II/(-4)}$

x	y		z
1	(1)	0 1	-2
0	1	$-\frac{1}{4}$ $\frac{1}{4}$	$-\frac{1}{4}$

$\xrightarrow{I - II}$

$x + y = 0 - 2x$ ||| $x + y = 1 - 2z$

MURO DI BERLINO


x y z
1 0
0 1
1/4 3/4 -7/4
-1/4 1/4 -1/4
MATRICE IDENTICA
x
y
Soluz. Sistema
1/4 - 7/4 z
-1/4 - 1/4 z
z
3/4 - 7/4 z'
1/4 - 1/4 z'
z'
tutte le soluzioni di due sistemi al variare di z, z' ∈ R
(1/4 3/4 / -1/4 1/4 / 0 0) + (-7/4 z -7/4 z' / -1/4 z -1/4 z' / z z')
EQUAZIONI MATRICIALI

	x	y	z		
I	1	−2	1	1	I − III + 2II
II	2	1	2	−1	II − 2I
III	1	3	3	1	III − I
II	0	5	0	−3	II / 5
III	0	5	2	0	III − II
III	0	0	2	3	III / 2
III	0	0	1	3/2	
II	0	1	0	−3/5	
I	1	0	0	$1-\frac{3}{2}-\frac{6}{5}=-\frac{17}{10}$	

1	−2	1
0	5	0
0	0	2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{17}{10} \\ -3/5 \\ 3/2 \end{pmatrix}$$

$$
\begin{array}{c|ccc|c|l}
 & 1 & 2 & 0 & -1 & \\
 & -1 & 1 & -3 & -2 & \mathrm{II}+\mathrm{I} \\
 & 2 & -1 & 5 & 3 & \mathrm{III}-2\mathrm{I} \\
\mathrm{II} & 0 & 3 & -3 & -3 & \mathrm{II}/3 \\
\mathrm{II} & 0 & 1 & -1 & -1 & \\
\mathrm{III} & 0 & -5 & 5 & 5 & \mathrm{III}+5\mathrm{II} \\
\mathrm{III} & 0 & 0 & 0 & 0 &
\end{array}
$$

$$
\begin{array}{c|ccc|c}
\mathrm{I} & 1 & 2 & 0 & -1 \\
\mathrm{II} & 0 & 1 & -1 & -1
\end{array}
$$

si può sopprimere perché verificato $\forall\, x, y, z$.

$$
\begin{array}{c|cc|cc|l}
 & x & y & & z & \\
\mathrm{I} & 1 & 2 & -1 & 0 & \mathrm{I}-2\mathrm{II} \\
\mathrm{II} & 0 & 1 & -1 & 1 & \\
\mathrm{I} & 1 & 0 & 1 & -2 &
\end{array}
$$

$1-2z$

$$
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-2z \\ -1+z \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}_{x_0} + z \overbrace{\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}}^{\langle(-2,1,1)\rangle}
$$

$$
\boxed{\cos^2 x \in \left\langle \frac{3}{4}, 2\cos 2x \right\rangle} \qquad \boxed{\cos 2x = 2\cos^2 x - 1}
$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 1 & -2 & 3 & 1 & 0 \end{array}\right) \quad \begin{array}{l} \\ \mathrm{II}+\mathrm{I} \\ \mathrm{III}-\mathrm{I} \end{array}$$

$$\begin{array}{l} \mathrm{II} \\ \mathrm{III} \\ \mathrm{III} \\ \mathrm{III} \end{array} \quad \begin{array}{cccc} 0 & 1 & -1 & 2 \\ 0 & -4 & 4 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{array} \quad \begin{array}{l} \\ \mathrm{III}/(-4) \\ \mathrm{III}-\mathrm{II} \\ \mathrm{III}/(-2) \end{array}$$

$$x\begin{pmatrix}1\\-1\\1\end{pmatrix}+y\begin{pmatrix}2\\-1\\-2\end{pmatrix}+z\begin{pmatrix}-1\\0\\3\end{pmatrix}+\cancel{w\begin{pmatrix}1\\1\\1\end{pmatrix}}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

ha le stesse soluzioni di

$$x\begin{pmatrix}1\\-1\\1\end{pmatrix}+y\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}-1\\0\\3\end{pmatrix}$$

$$\begin{array}{cccc|c} & & & w & \\ 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}$$

z NON PIVOT

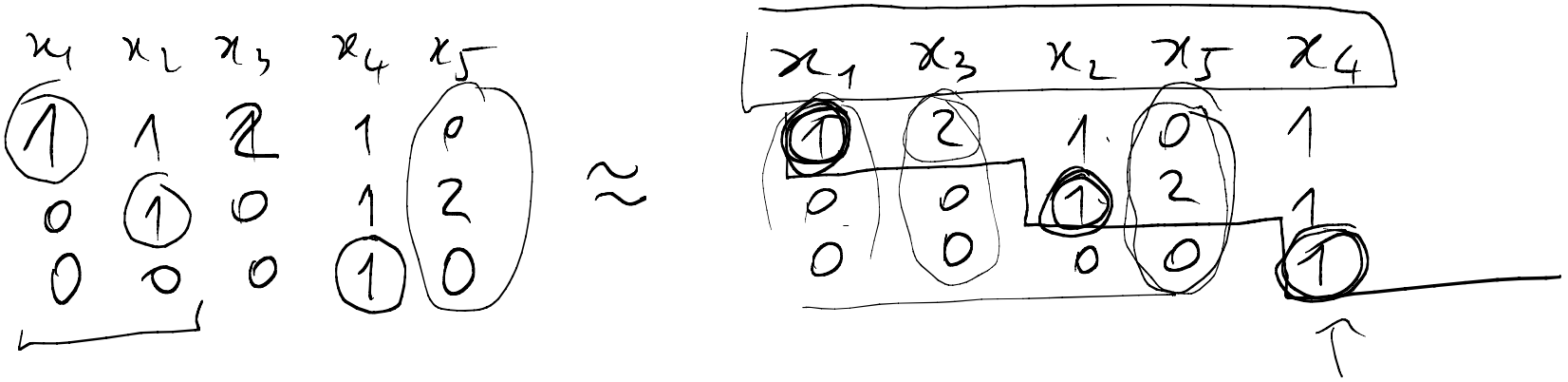
$w=0$

$$\begin{pmatrix}-1\\-1\\0\end{pmatrix}\in\left\langle\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}2\\1\\0\end{pmatrix}\right\rangle$$

$x \quad y \quad z$

(-1)

$$\begin{pmatrix}-1\\0\\3\end{pmatrix}\in\left\langle\begin{pmatrix}1\\-1\\1\end{pmatrix},\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right\rangle$$


x1 x2 x3 x4 x5
1 1 2 1 0
0 1 0 1 2
0 0 0 1 0
≃
x1 x3 x2 x5 x4
1 2 1 0 1
0 0 1 2 1
0 0 0 0 1