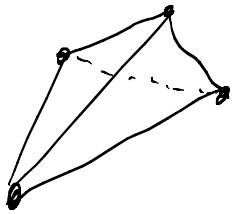
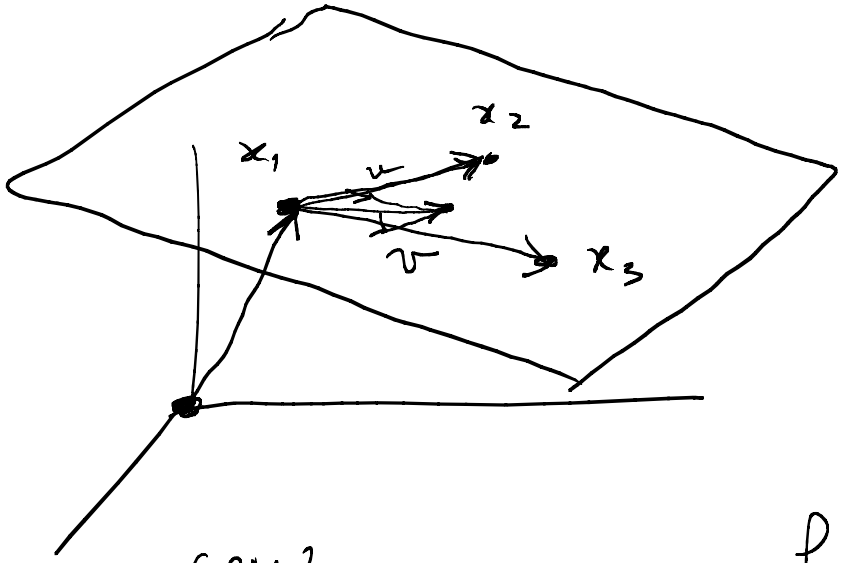
Note Title 11/3/2020

comb. affine $\sum_{1}^{n} \alpha_i u_i \quad : \quad \sum_{1}^{n} \alpha_i = 1$

comb. lineari +



comb. lineari $\supseteq$ comb. affini $\supseteq$ coniche $\supseteq$ convesse



$x_1 + \alpha u + \beta v$

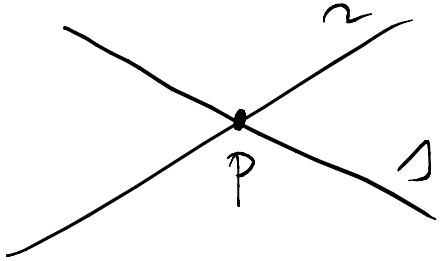
$\begin{cases} u = x_2 - x_1 \\ v = x_3 - x_1 \end{cases}$

$f(\alpha,\beta) = x_1 + \alpha(x_2 - x_1) + \beta(x_3 - x_1) =$

$= (1-\alpha-\beta)x_1 + \alpha x_2 + \beta x_3$ comb. affini di x_1, x_2, x_3

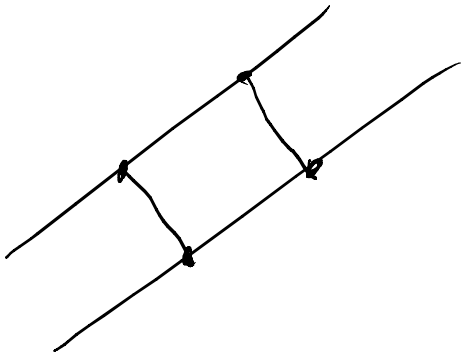
comb.

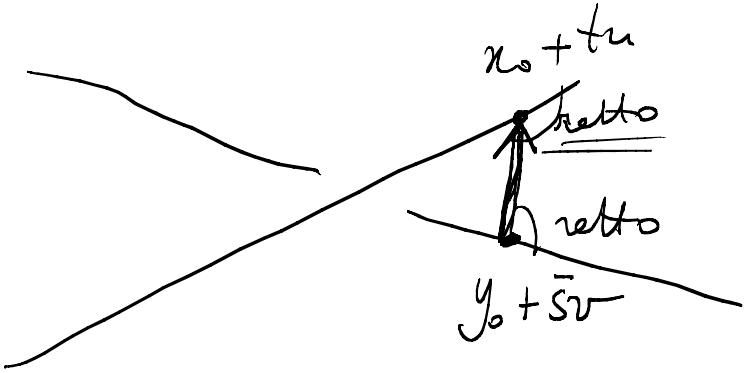
$\boxed{(1-\alpha-\beta) + \alpha + \beta = 1}$



$P \in r \qquad P \notin s$

$d(P,P) = 0$





$$\begin{cases} [x_0 + tu - y_0 - sv]\, u = 0 \\ [x_0 + tu - y_0 - sv]\, v = 0 \end{cases}$$

sistema di due equazioni nelle due incognite t, s

$[\ ]$ 1 u, v

$$|x_0+tu-y_0-sv|^2 =$$

$$= |x_0+tu-(x_0+\bar{t}u)+(x_0+\bar{t}u)-(y_0+\bar{s}v)+(y_0+\bar{s}v)-y_0-sv|^2 =$$

$$= |(t-\bar{t})u + [x_0+\bar{t}u-(y_0+\bar{s}v)] + (\bar{s}-s)v|^2 \underset{\text{P. Pitag.}}{=}$$

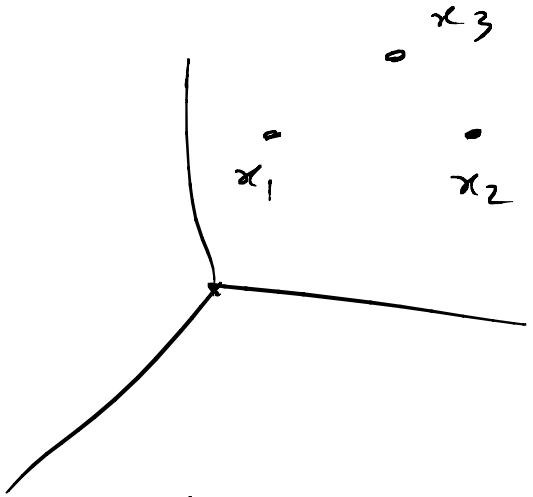
$[x_0+\bar{t}u-(y_0+\bar{s}v)] \perp u, v \Rightarrow \perp \langle u,v \rangle$

$(\bar{s}-s) \in \langle u,v \rangle$

$$= |x_0+\bar{t}u-y_0-\bar{s}v|^2 + \underbrace{|(t-\bar{t})u+(\bar{s}-s)v|^2}_{\geq 0} \geq |x_0+\bar{t}u-y_0-\bar{s}v|^2$$

$$\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\end{pmatrix}, \begin{pmatrix}2\\0\\-3\end{pmatrix}$$

x_1 x_2 x_3



Piano affine per x_1, x_2, x_3 è l'insieme delle combin. affini

$$f(\alpha,\beta) = (1-\alpha-\beta)\begin{pmatrix}1\\1\\2\end{pmatrix} + \alpha\begin{pmatrix}1\\-1\\-1\end{pmatrix} + \beta\begin{pmatrix}2\\0\\-3\end{pmatrix} =$$

$$= \begin{pmatrix}1-\alpha-\beta+\alpha+2\beta\\1-\alpha-\beta-\alpha\\2(1-\alpha-\beta)-\alpha-3\beta\end{pmatrix} = \begin{pmatrix}1+\beta\\1-2\alpha-\beta\\2-3\alpha-5\beta\end{pmatrix}$$

$\alpha, \beta \in \mathbb{R}$.

AL_2.1 $\mathbb{R}^n$

Schwarz
$|xy| \leq |x||y|$

$$|x+y| \leq |x|+|y|$$

$$|x+y|^2 = (x+y)(x+y) = |x|^2+|y|^2+2xy \leq$$

$$\leq |x|^2+|y|^2+2|xy| \leq |x|^2+|y|^2+2|x||y| =$$

(Schwarz)

$$= (|x|+|y|)^2$$

$$\Downarrow$$

$$|x+y| \leq |x|+|y|$$

x	y	z	
1	−1	2	−2
0	2	1	0
0	0	3	−3

$\Leftrightarrow$

$$\begin{cases} x - y + 2z = -2 \\ 2y + z = 0 \\ \boxed{3z = -3} \end{cases}$$

1 eq. I grado in 1 incognita ha soluzione unica perché $a_{33} = 3 \neq 0$

$\boxed{z = -1}$

AL_1.1

$\boxed{z = -1}$

$$\begin{cases} x - y + 2(-1) = -2 \\ 2y + (-1) = 0 \end{cases}$$

$$\begin{cases} x - y = 0 \\ \boxed{2y = 1} \end{cases}$$

$x = \frac{1}{2}$

$y = \frac{1}{2}$

sistema triangolare di $n-1$ equazioni in $n-1$ incognite $(x_1 \dots x_{n-1})$

$(x, y, z) = \left(\frac{1}{2}, \frac{1}{2}, -1\right)$

FUNZIONE (APPLICAZIONE, OPERATORE) LINEARE

$f: X \to Y$, X, Y spaz. vettoriali

(Additività) 1) $f(x+x') = f(x) + f(x') \quad \forall x, x' \in X$

(Omogeneità) 2) $f(\lambda x) = \lambda f(x) \quad \forall \lambda \in \mathbb{R} \quad \forall x \in X$

fissato $u \neq 0$

$$f(x) = x_u = \frac{xu}{|u|^2} u$$

$f(x,y) = x+y \quad f: \mathbb{R}^2 \to \mathbb{R}$

$f((x,y)) = \underbrace{x+y}_{\in \mathbb{R}}$
$\in \mathbb{R}^2$

LA PROIEZIONE È LINEARE.

funzione BILINEARE

$f : X \times X \to Y$ $\quad$ X, Y spazi vettoriali

f si dice bilineare se

a) $\forall \bar{u} \in X$ fissato $g(v) = f(\bar{u}, v)$ è lineare

b) $\forall \bar{v} \in X$ fissato $h(u) = f(u, \bar{v})$ è lineare

$u(v+w) = uv + uw$	$(u+v)w = uw + vw$
$u(\lambda v) = \lambda(uv)$	$(\lambda u)w = \lambda(uw)$
lineare rispetto al secondo fattore	lineare rispetto al I fattore

$$(x+y)(x+y) = x(x+y) + y(x+y) =$$

bloccando il secondo fattore linearità rispetto al I

bloccando i due primi fattori

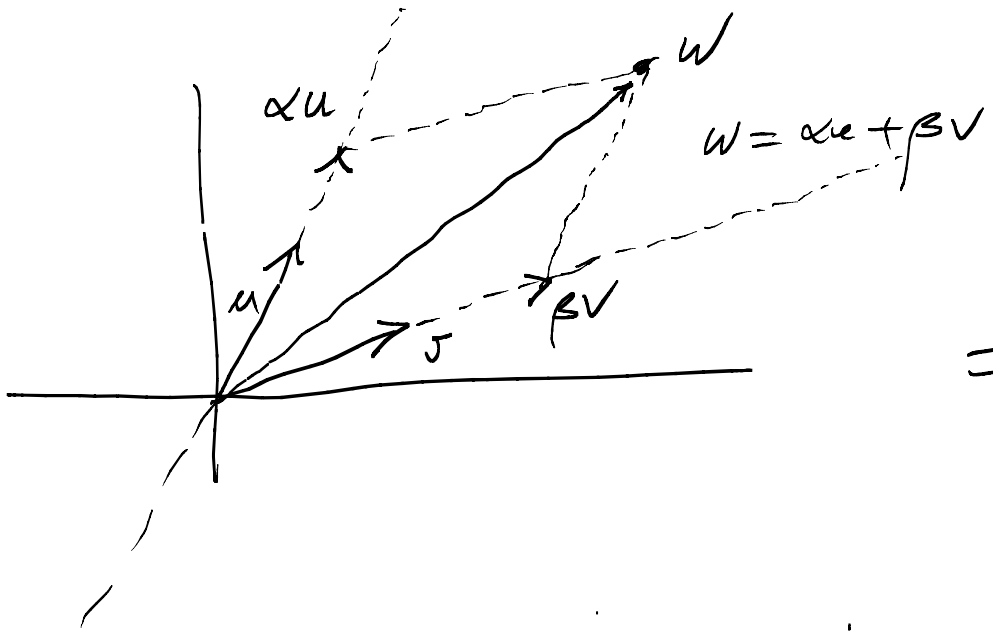
$$x \cdot x + \underbrace{xy + y \cdot x}_{} + y \cdot y \overset{\text{simmetria}}{=} |x|^2 + |y|^2 + 2xy$$

ESEMPIO di funzione NON LINEARE

$$f(x) = 2x+1 \qquad f(2x) = 2(2x) + 1 = 4x+1$$

$$2f(x) = 2(2x+1) = 4x+2$$

$\boxed{f(x) = ax}$ $a \in \mathbb{R}$ UNICHE f LINEARI



$$\langle u, v \rangle = \{ w : w = \alpha u + \beta v \quad \exists \alpha, \beta : \}$$

In uno spazio vettoriale X

Span o spazio generato

$$\langle x_1, x_2, \dots x_n \rangle = \left\{ \sum_{1}^{n} \alpha_i x_i : \alpha_i \in \mathbb{R} \right\}$$

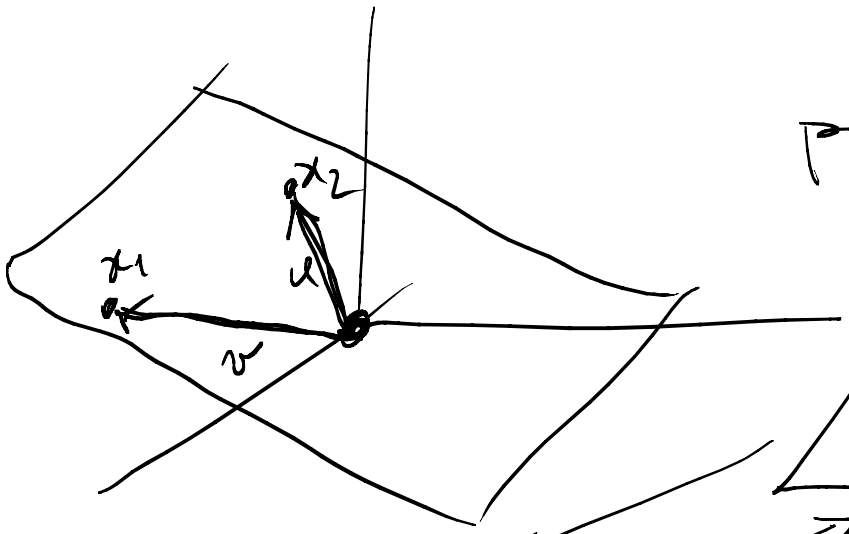
generatori

AL_3.1

$$\left\langle \begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}2\\-1\\-2\end{pmatrix} \right\rangle = \left\{ \alpha \begin{pmatrix}1\\1\\2\end{pmatrix} + \beta \begin{pmatrix}2\\-1\\-2\end{pmatrix},\ \alpha,\beta \in \mathbb{R} \right\} =$$

$$= \left\{ \begin{pmatrix}\alpha+2\beta\\ \alpha-\beta\\ 2\alpha-2\beta\end{pmatrix} : \alpha,\beta \in \mathbb{R} \right\}$$

(Piano parametrico passante per $\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}2\\-1\\-2\end{pmatrix}$)

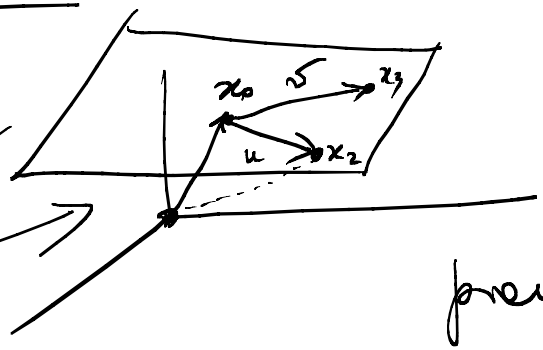


piano per l'origine e per x_1 e x_2

$\Leftrightarrow \langle u, v \rangle$

$u = x_1 - 0 = x_1$

$v = x_2 - 0 = x_2$



u, v non allineati

$$\begin{cases} u = x_2 - x_0 \\ v = x_3 - x_0 \end{cases}$$

piano affine $= x_0 + \langle u, v \rangle$

$$\|x+y\|^2+\|x-y\|^2 \neq 2\left(\|x\|^2+\|y\|^2\right)$$

FALSA L'IDENTITÀ DEL PARALLELOGRAMMA

$$\|x\| = \max_{1..n} |x_i|$$

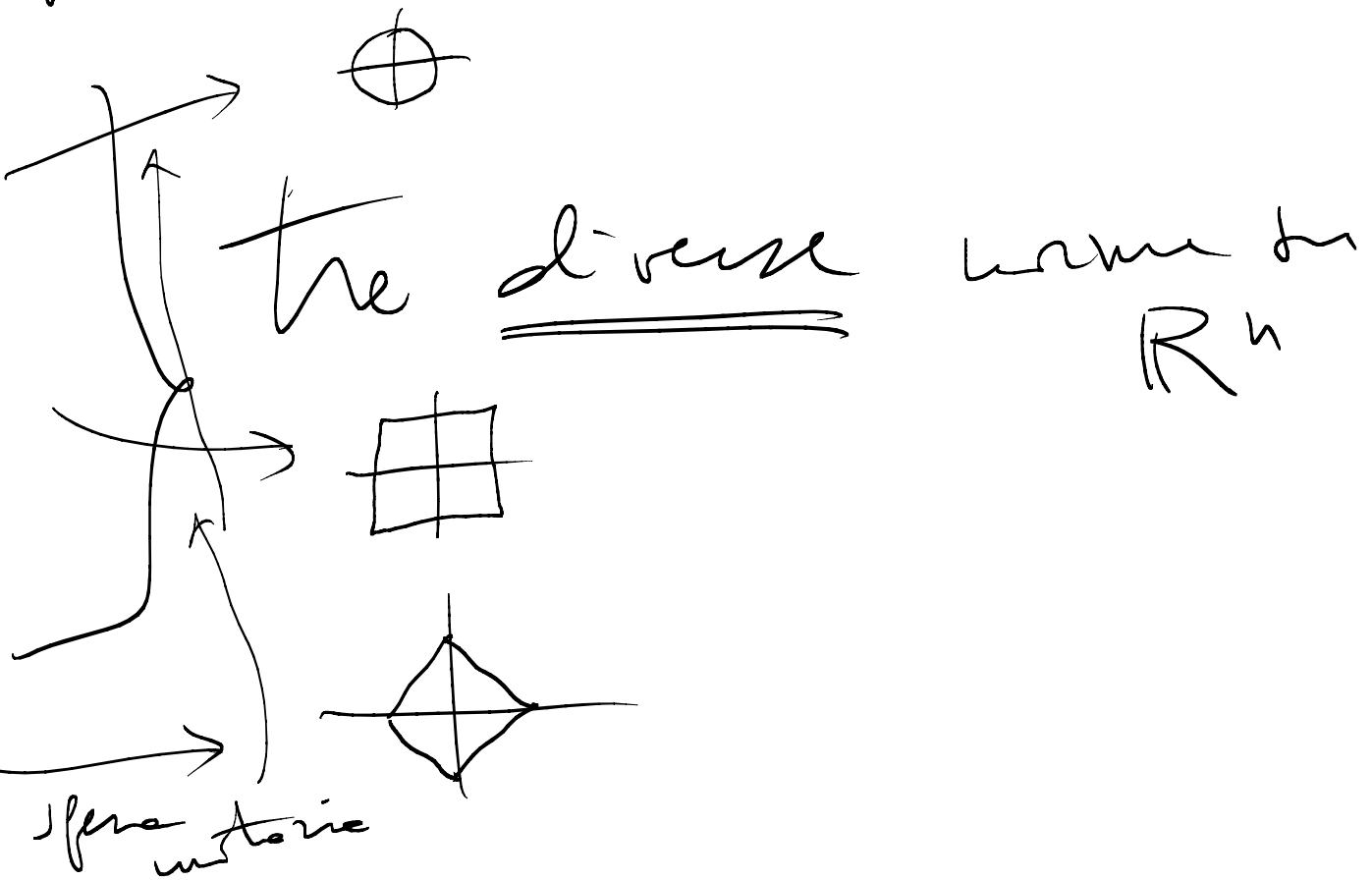
$\{\mathbb{R}^n, |||\ |||\}$ spazio normato "esotico"

$\{\mathbb{R}^n, |\ |\}$ spazio euclideo

$$|x| = \sqrt{\sum_{1}^{n} x_i^2}$$

$$\|x\| = \max_{1..n} |x_i|$$

$$|||x||| = \sum_{i=1}^{n} |x_i|$$



$$\hat{x} = \frac{1}{|x|} x$$

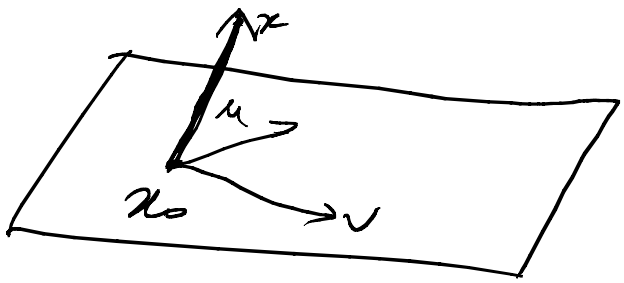
$7\hat{x}$ direzione e verso come x
e lunghezza 7

$\alpha > 0$ $\alpha\hat{x}$ ha lunghezza α e stessa direzione e verso di x

$$|\alpha\hat{x}| = |\alpha|\,|\hat{x}| = |\alpha| \left|\frac{1}{|x|} x\right| = |\alpha| \frac{1}{|x|} |x| = |\alpha| = \alpha$$

$\alpha > 0$

$$f(\alpha,\beta) = x_0 + \alpha u + \beta v$$



$$x : \begin{cases} (x-x_0)\,u = 0 \\ (x-x_0)\,v = 0 \end{cases}$$ da risolvere

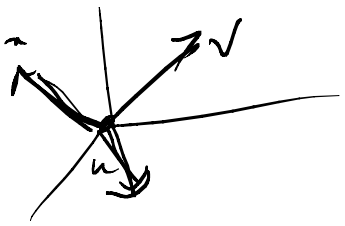
$$\begin{cases} \left[(x,y,z)-(x_0,y_0,z_0)\right] u = 0 \\ \left[\qquad\qquad\qquad\right] v = 0 \end{cases}$$

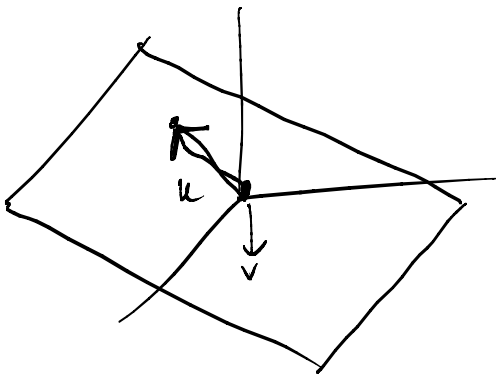
$$\begin{cases} 0 = u_1(x-x_0) + u_2(y-y_0) + u_3(z-z_0) \\ 0 = v_1(x-x_0) + v_2(y-y_0) + v_3(z-z_0) \end{cases}$$

incognito

dati

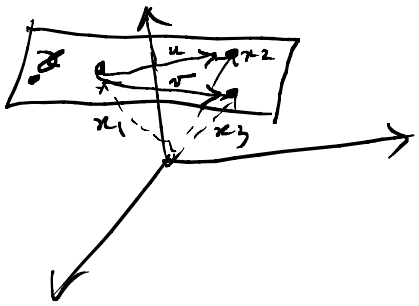
$$\begin{cases} x\,u = 0 \\ x\,v = 0 \end{cases}$$





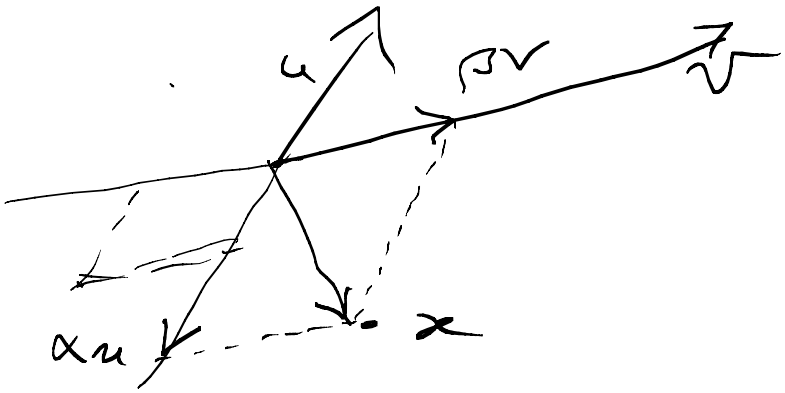
prodotto = $\langle u, v \rangle$
lineare

meet.google.com/
pxu-bbv2-gfu
jitsi



$u = x_2 - x_1$

$v = x_3 - x_1$



$$x(\alpha,\beta) = x_1 + \alpha u + \beta v = x_1 + \alpha(x_2 - x_1) + \beta(x_3 - x_1) =$$

$$= (1-\alpha-\beta)x_1 + \alpha x_2 + \beta x_3$$

combin. affini perché $(1-\alpha-\beta)+\alpha+\beta = 1$