SOLUZIONE DI SISTEMI CON TERMINI NOTI MULTIPLI

Note Title 11/2/2020

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & -2 \\ 1 & 3 & 1 & -1 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 2 & -1 & 1 & 0 \\ 1 & 3 & 1 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & -2 \\ 2 & -1 & 1 & -2 & 0 \\ 1 & 3 & 1 & -1 & -3 \end{array}\right)$$

$$\left\langle \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ -3 \end{pmatrix} \right\rangle \subseteq$$

$$\subseteq \left\langle \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$\xrightarrow[\text{III}-\text{I}]{\text{II}-2\text{I}} \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & -2 \\ 0 & -3 & -1 & -2 & 4 \\ 0 & 2 & 0 & -1 & -1 \end{array}\right)$$

$$\begin{array}{c|c} x+y+z=0 & x+y+z=-2 \\ 1 \quad 1 \quad 1 \quad 1 & 1 \end{array}$$

$$\xrightarrow{\text{III} \leftrightarrow \text{II}} \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & -2 \\ 0 & 2 & 0 & -1 & -1 \\ 0 & -3 & -1 & -2 & 4 \end{array}\right) \longrightarrow$$

$$\xrightarrow{\text{II}/2} \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -3 & -1 & -2 & 4 \end{array}\right) \xrightarrow{\text{III}+3\text{II}} \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & -\frac{7}{2} & 5/2 \end{array}\right) \xrightarrow{\text{I}+\text{III}}$$

$$\left(\begin{array}{ccc|cc} 1 & 1 & 0 & -7/2 & 1/2 \\ 0 & 1 & 0 & -1/2 & -1/2 \\ 0 & 0 & -1 & -7/2 & 5/2 \end{array}\right) \xrightarrow{\text{I}-\text{II}} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1/2 & -1/2 \\ 0 & 0 & -1 & -7/2 & 5/2 \end{array}\right) \xrightarrow{\text{III}(-1)}$$

$$\longrightarrow \begin{array}{ccc|cc} x & y & z & & \\ \hline 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1/2 & -1/2 \\ 0 & 0 & 1 & 7/2 & -\frac{5}{2} \end{array}$$

le soluzioni dei due due sistemi iniziali.

La soluzione del I sistema è $\left(-3, -\frac{1}{2}, \frac{7}{2}\right)$

" " del II " è $\left(1, -\frac{1}{2}, -\frac{5}{2}\right)$

(x, y, z)

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x	y			
1	-2	1	-1	0
0	1	1	2	1
3	0	1	-2	0
-2	-1	1	3	0

$$\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\2\\-2\\3\end{pmatrix}, \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \in ?$$

$$? \in \left\langle \begin{pmatrix}1\\0\\3\\-2\end{pmatrix}, \begin{pmatrix}-2\\1\\0\\-1\end{pmatrix} \right\rangle$$

$\text{III} - 3\text{I}$

$\text{IV} + 2\text{I}$

	1	-2	1	-1	0
→	0	1	1	2	1
	0	6	-2	1	0
	0	-5	3	1	0

→

$\text{III} - 6\text{II}$

$\text{IV} + 5\text{II}$

$$\left(\begin{array}{cc|ccc} 1 & -2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 2 & 1 \\ \hline 0 & 0 & -8 & -11 & -6 \\ ? & ? & ? & ? & ? \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right)$$

NESSUNO DEI TRE SISTEMI È RISOLUBILE!

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

$$x_1\begin{pmatrix} a_{11} \\ a_{21} \\ a_{m1} \end{pmatrix} + x_2\begin{pmatrix} a_{12} \\ a_{22} \\ a_{m2} \end{pmatrix} + \cdots + x_n\begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \left\langle \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix} \cdots \begin{pmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{pmatrix} \right\rangle$$