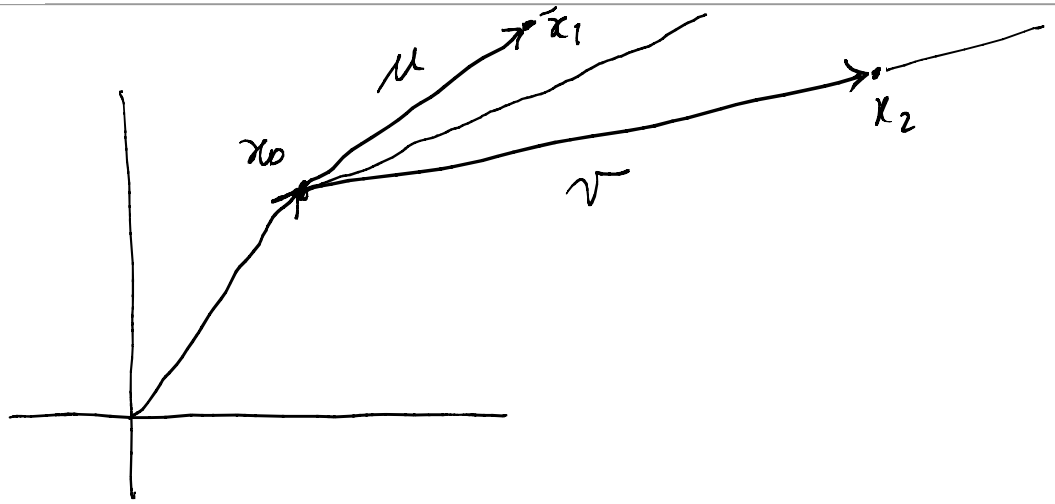
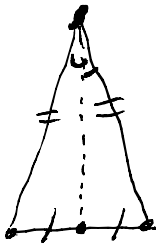
Note Title 10/22/2020



$$\begin{cases} u = x_1 - x_0 \\ v = x_2 - x_0 \end{cases}$$



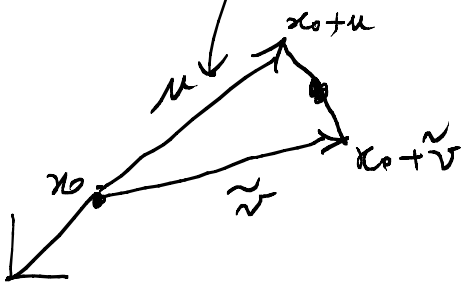
triangolo isoscele

bisettrice = mediana = altezza

$$\tilde{v} = \boxed{\frac{|u|}{|v|}}^{\lambda>0} v \Rightarrow |\tilde{v}| = |u| \qquad \boxed{|\lambda u| = |\lambda||u|}$$

$|u| \frac{v}{|v|}$, $\hat{v}$

isoscele



bisettrice

calcolo del punto medio di un segmento x_1 x_2

$$x_1 + \frac{1}{2}(x_2 - x_1) = \left(1 - \frac{1}{2}\right)x_1 + \frac{1}{2}x_2 =$$

$$= \frac{x_1 + x_2}{2}$$

$$\frac{1}{2}\left(x_0+u+x_0+\tilde{v}\right) = x_0 + \frac{u+\tilde{v}}{2}$$

$2x_0+u+\tilde{v}$

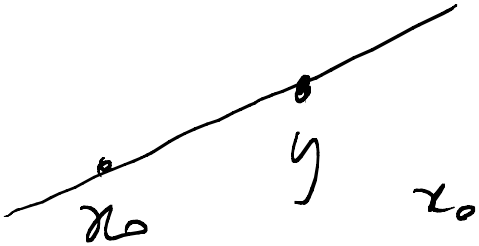
Mediana (= bisettrice) è la retta congiungente x_0 e

$\boxed{x_0 + \frac{u+\tilde{v}}{2}}$, cioè $x_0 + t\left(x_0 + \frac{u+\tilde{v}}{2} - x_0\right) =$

$= \boxed{x_0 + \frac{t}{2}(u+\tilde{v})} = \boxed{x_0 + s(u+\tilde{v})}$

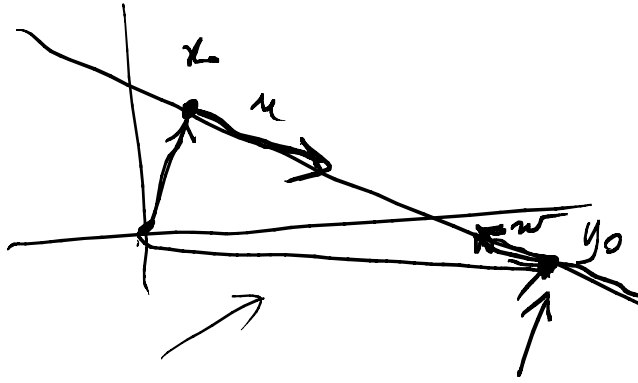
$t \in \mathbb{R}$ $\quad s = \frac{t}{2}$

eq. param. della bisettrice dell'angolo



$x_0 + t(y - x_0)$

$y = x_0 + \frac{u+\tilde{v}}{2}$

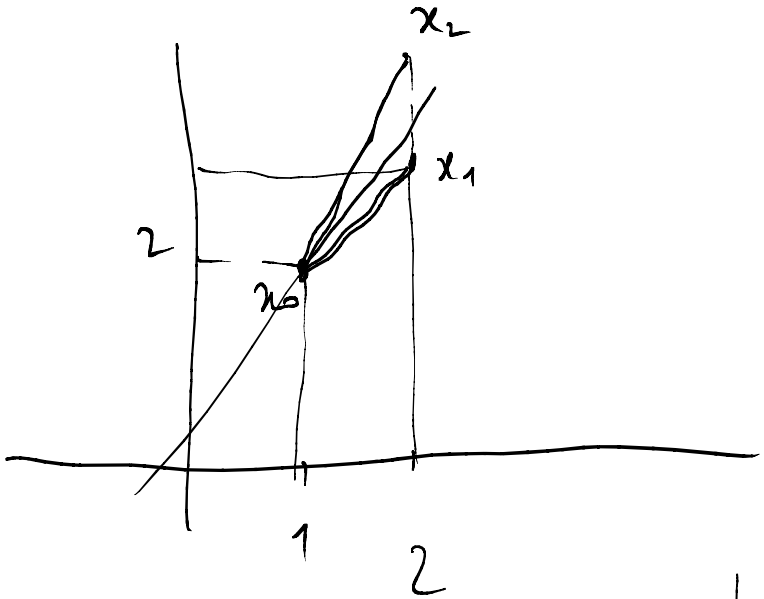


le rette

$x_0 + tu$ $\quad$ $y_0 + sv$ $\quad$ $y_0 + rw$ con moduli u, v, w

$(1,2)$ vertice dell'angolo $\quad (2,3), (2,4)$

$x_0 \qquad x_1, x_2$

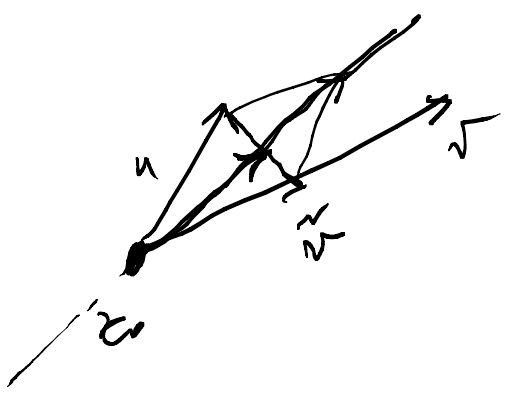


$u = x_1 - x_0 = (2,3) - (1,2) = (1,1)$

$v = x_2 - x_0 = (2,4) - (1,2) = (1,2)$

$|u| = \sqrt{2} \qquad |v| = \sqrt{5}$

$\tilde{v} = \frac{v}{|v|}(1,2) = \left(\sqrt{\frac{1}{5}}, \sqrt{\frac{8}{5}}\right)$



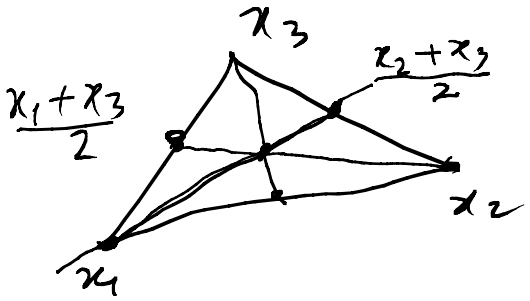
bisettrice è $\boxed{x_0 + t(u+\tilde{v}) \quad t \in \mathbb{R}}$ $(t \in \mathbb{R}^+$ semiretta$)$

$$\boxed{(1,2) + t\left(1+\sqrt{\frac{1}{5}}, 1+\sqrt{\frac{8}{5}}\right)}$$

$u+\tilde{v}$

$x_0 + tw$ e $x_0 + \frac{t}{2}w$ sono la stessa retta

Baricentro



baricentro $=$

$$= \boxed{\frac{x_1 + x_2 + x_3}{3}}$$

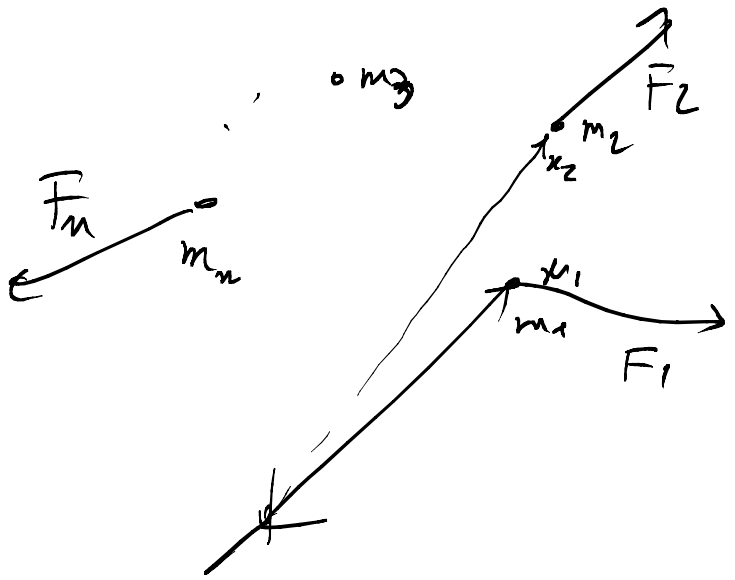
$$x_1 + t\left(\frac{x_2+x_3}{2} - x_1\right) = (1-t)x_1 + \frac{t}{2}(x_2+x_3)$$

$$x_2 + s\left(\frac{x_1+x_3}{2} - x_2\right) = (1-s)x_2 + \frac{s}{2}(x_1+x_3) =$$

$$\boxed{s = \frac{2}{3}}$$

$$= \left(1-\frac{2}{3}\right)x_2 + \frac{2}{3}\cdot\frac{1}{2}(x_1+x_3) = \frac{1}{3}(x_1+x_2+x_3)$$

Centro di massa



$$F = ma$$

$$\boxed{F(x) = m\ddot{x}}$$

n punti materiali di posizioni $x_1 \dots x_n$ e masse $m_1 \dots m_n$, soggetti a forze $F_1 \dots F_n$

$$F_i = m_i \ddot{x}_i$$

$$\boxed{\begin{aligned} F_1 &= m_1 \ddot{x}_1 \\ &\vdots \\ F_n &= m_n \ddot{x}_n \end{aligned}}$$

$$\boxed{\sum_1^n F_i} = \sum_1^n m_i \ddot{x}_i = \left(\sum_1^n m_i x_i\right)^{\cdot\cdot} =$$

Risultante delle forze $F_1 \dots F_n$

$$= \left(\sum_1^n m_i\right)\left(\sum_1^n \frac{m_i}{\sum m_i} x_i\right)^{\cdot\cdot}$$

Massa totale $M = \sum m_i$

posizione x

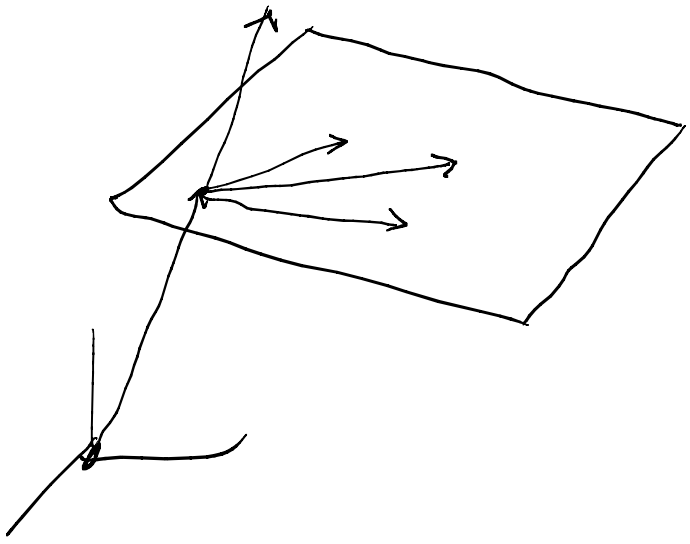
C.M. del sistema $(x_1, m_1) \dots (x_n, m_n)$

$$\boxed{x_{C.M.} = \frac{1}{\sum m_i} \sum m_i x_i}$$

$(1,2)$ massa 3 $(2,1)$ massa 7 $(2,3)$ massa 12

$M = 3+7+12 = 22$

$$x_{C.M.} = \frac{3}{22}(1,2) + \frac{7}{22}(2,1) + \frac{12}{22}(2,3) =$$



$$3x - 2y + z = 8$$

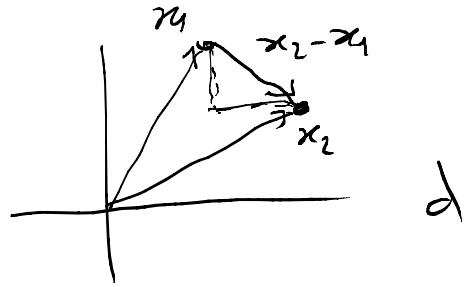
un vettore normale è $(3,-2,1) = u$

La perpendicolare per il punto $(1,1,1) = x_0$ al piano $3x - 2y + z = 8$

$$r \quad x_0 + tu = (1,1,1) + t(3,-2,1)$$

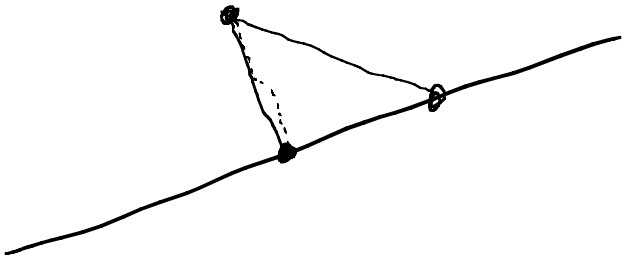
DISTANZA

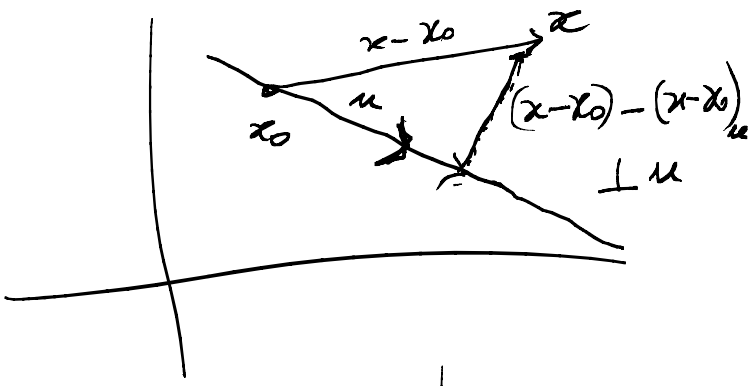
$$d(x_1, x_2) \equiv |x_1 - x_2|$$



$$d((2,1),(1,2)) =$$

$$= |(2,1) - (1,2)| = |(1,-1)| = \sqrt{2}$$





distanza (minima) da x alla retta $x_0 + tu$

$$d = \left| (x - x_0) - (x - x_0)_u \right|$$

$(1,1,2)$ $\quad (-1,1,3) + t\underbrace{(1,1,1)}$ $\quad r$

$x \qquad x_0 \qquad\qquad u$

$x - x_0 = (2,0,-1)$

$d(x,r) = \left|(2,0,-1) - \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right| =$

$(x-x_0)_u = \frac{2\cdot 1 + 0\cdot 1 + (-1)1}{1+1+1} \cdot (1,1,1)$

$= \left|\left(\frac{5}{3}, -\frac{1}{3}, -\frac{4}{3}\right)\right| = \frac{1}{3}\left|(5,-1,-4)\right| =$

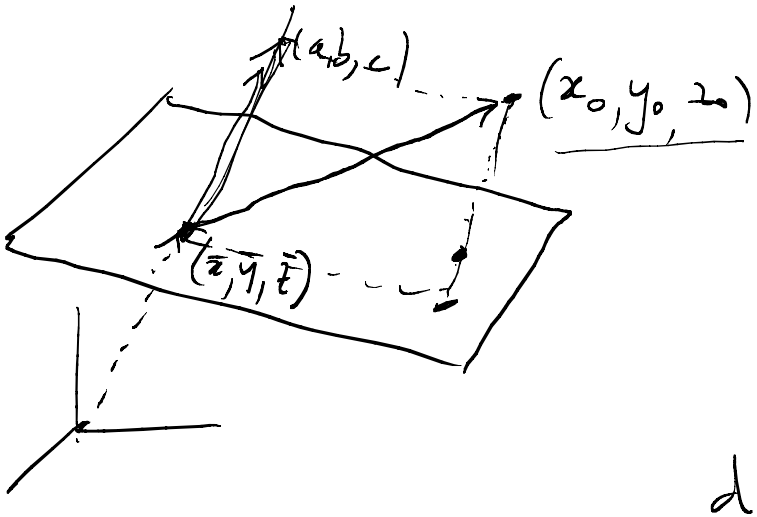
$= \frac{1}{3}(1,1,1) = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

$= \frac{1}{3}\sqrt{25+1+16}$ ..

$d(x, \text{piano implicito})$

$ax+by+cz=d$ TT piano implicito

(x_0, y_0, z_0)



(a,b,c) è perpendicolare a Π

$(x_0, y_0, z_0) + t(a,b,c)$ retta perpend. Π

$$d\left((x_0,y_0,z_0), \Pi\right) = \left| \left[(x_0,y_0,z_0) - (\bar{x},\bar{y},\bar{z}) \right]_{(a,b,c)} \right|$$

$(\bar{x},\bar{y},\bar{z})$ qualsiasi punto di Π

$ax + by + cz = d$ $\quad (\bar{x},\bar{y},\bar{z}) \in \Pi$

almeno uno dei numeri a, b, c deve essere $\neq 0$

Supp. sia a

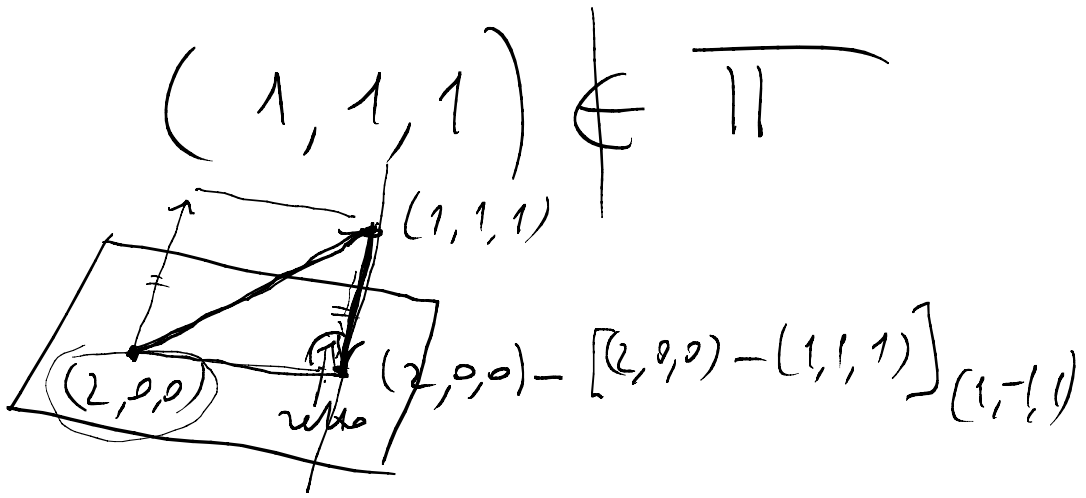
$y=0 \; z=0 \Rightarrow x = \frac{d}{a}$

$\bar{x} = d/a \quad \bar{y}=0 \quad \bar{z}=0$

π $\boxed{x - y + z = 2}$

$(1,1,1) \notin \pi$

$\nu = (1,-1,1) \quad (\equiv (a,b,c))$



Un punto qualunque di π si può trovare, per esempio, ponendo $y=0$, $z=0$ e ricavando $x=2$

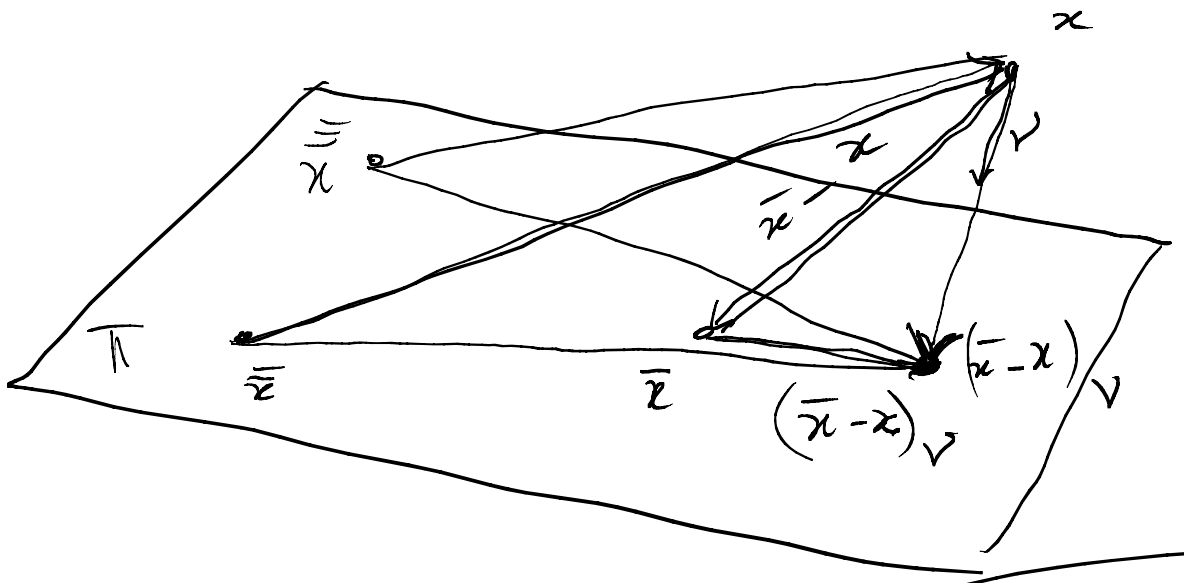
$(\bar{x}, \bar{y}, \bar{z}) = (2,0,0)$

$(1,1,1) - (2,0,0) = (-1,1,1)$

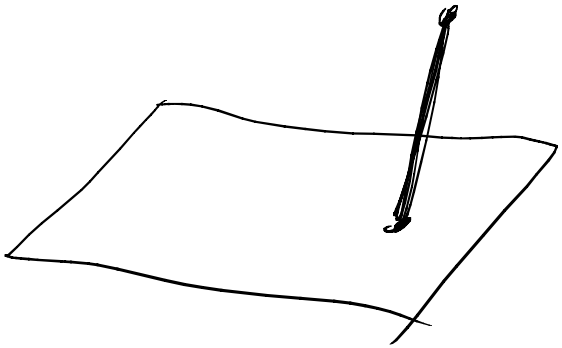
$(x_0, y_0, z_0) \qquad (\bar{x}, \bar{y}, \bar{z})$

$$(-1,1,1)_{(1,-1,1)} = \frac{-1-1+1}{1+1+1}(1,-1,1) = -\frac{1}{3}(1,-1,1)$$

la sua norma è $\frac{1}{3}|(1,-1,1)| = \frac{\sqrt{3}}{3} = \boxed{\frac{1}{\sqrt{3}}}$



Scomponendo per tre volte di punti su Π ($\bar{x}$, $\bar{\bar{x}}$ e $\bar{\bar{\bar{x}}}$)



$$\left(x-\bar{\bar{\bar{x}}}\right)_v = \left(x-\bar{x}+\underbrace{\bar{x}-\bar{\bar{x}}}_{\perp v}+\bar{\bar{x}}-\bar{\bar{\bar{x}}}\right)_v$$

e quindi la proiezione non cambia al variare del punto $\bar{x}$

NOTA