Note Title 10/20/2020

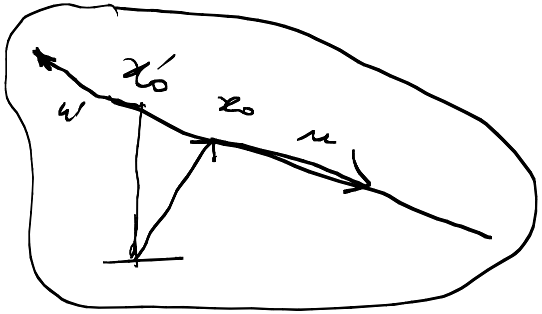
$$3x - y = 4 \qquad r \qquad 6x - 2y = 8$$

considere due punti distinti di r

Ponendo $x = 0$ ne segue $-y = 4$ $P_1 = (0, -4) \in R$

$y = 0$ ... $x = \frac{4}{3}$ $P_2\left(\frac{4}{3}, 0\right) \in R$

$\underline{x_0} + t\underline{u}$ $\quad u = P_2 - P_1 = \left(\frac{4}{3}, 4\right) \parallel \underline{\left(\frac{1}{3}, 1\right)}$



$$\left(\frac{4}{3}, 0\right) + t\left(\frac{1}{3}, 1\right)$$ retta parametrica che ha per immagine la retta implicita $3x - y = 4$

$X = \langle u_1 \dots u_n \rangle \quad Y = \langle v_1, \dots, v_m \rangle$

$$\left.\begin{array}{l} \boxed{X \subseteq Y} \\ Y \subseteq X \end{array}\right\} X = Y$$

$\forall x \in X \Rightarrow x \in Y \; ; \; \exists \alpha_i : \boxed{x = \sum_{1}^{m} \alpha_i v_i}$

$u_i \in \underline{\langle v_1 \dots v_m \rangle} \quad \forall i = 1 \dots n \Rightarrow$ Ogni combin. di $u_i \in Y \Rightarrow X \subseteq Y$

$v_1 v_2 \dots v_m \mid u_1 u_2 \dots u_n$ Se questi sistemi hanno soluzione $\forall u_i$ allora ogni u_i è combinazione di $v_1 \dots v_m$ e quindi $X \subseteq Y$

$\boxed{AL\_1.2}$

$$\boxed{\langle (1,0),(0,1)\rangle} \subseteq \langle (1,0),(1,1)\rangle$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right)$$

$$\boxed{\left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right)}$$

ho sempre soluzione?
se sì l'inclusione è vera
se no, no.

$$-1(1,0)+1(1,1)=(0,1)$$

$$\langle (1,0,0),(0,1,0)\rangle \supseteq \langle (1,0,0),(0,1,0),(0,0,1)\rangle$$

$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

AL_3.7

$$X+Y = X\oplus Y \iff X\cap Y=\{0\}$$

$$w\in X\cap Y \iff \begin{matrix} w\in X \\ w\in Y \end{matrix} \iff \begin{matrix} w=\sum_1^n \alpha_i u_i \\ w=\sum_1^m \beta_j v_j \end{matrix}$$

$$\boxed{\sum_1^n \alpha_i u_i - \sum_1^m \beta_j v_j = 0}$$

sistema lineare omogeneo (GAUSS)

$$\boxed{u_1\, u_2 \dots u_n\, v_1 \dots v_m \,|\, 0}$$

ha la soluzione $\alpha_i=\beta_j=0\ \forall i,j$. Potrebbe avere altre soluzioni. Per ognuna d'esse, ad es. $\bar\alpha_1, \bar\alpha_2, \dots, \bar\alpha_n, \bar\beta_1, \dots, \bar\beta_m$ si può ricostruire $\bar w\in X\cap Y$ calcolando o $\sum_1^n \bar\alpha_i u_i$ oppure $\sum_1^m \bar\beta_j v_j$

$$X = \sum_{1}^{n} X_i \qquad X = \bigoplus_{1}^{n} X_i \ ?$$

$$x_i \in X_i \qquad \boxed{\sum_{1}^{n} x_i = 0} \Rightarrow x_i = 0 \ \forall i$$

$$x_i \in X_i \Leftrightarrow x_i \in \langle u^i_1, u^i_2, \dots, u^i_{k_i} \rangle \qquad \boxed{x_i = \sum_{1}^{k_i} \alpha^i_{j_i} u^i_{j_i}}$$

GAUSS

$$\boxed{\underbrace{\sum_{1}^{k_1} \alpha^1_{j_1} u^1_{j_1}}_{x_1} + \dots + \underbrace{\sum_{1}^{k_n} \alpha^n_{j_n}}_{x_n} u^n_{j_n} = 0}$$

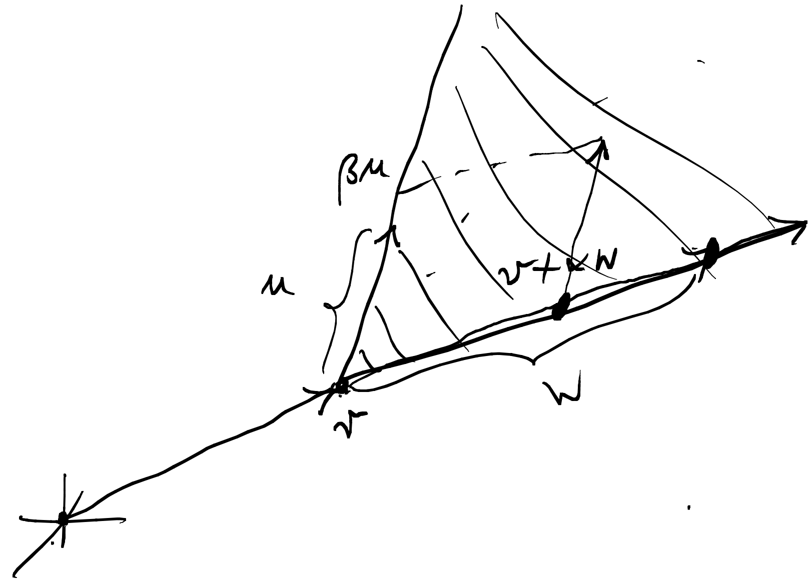
sistema lineare omogeneo

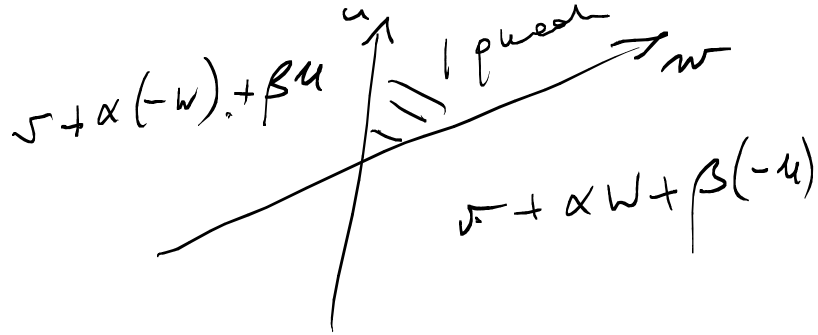
deve verificarsi che

$$\underbrace{\sum_{1}^{k_i} \bar{x}^i_{j_i} u^i_{j_i}}_{x_i} = 0$$

Per ogni soluzione $(\underbrace{\bar{x}^1_1 \dots \bar{x}^1_{k_1}}\ \underbrace{\bar{x}^2_1 \dots \bar{x}^2_{k_2}} \dots \underbrace{\bar{x}^n_1 \dots \bar{x}^n_{k_n}})$ di

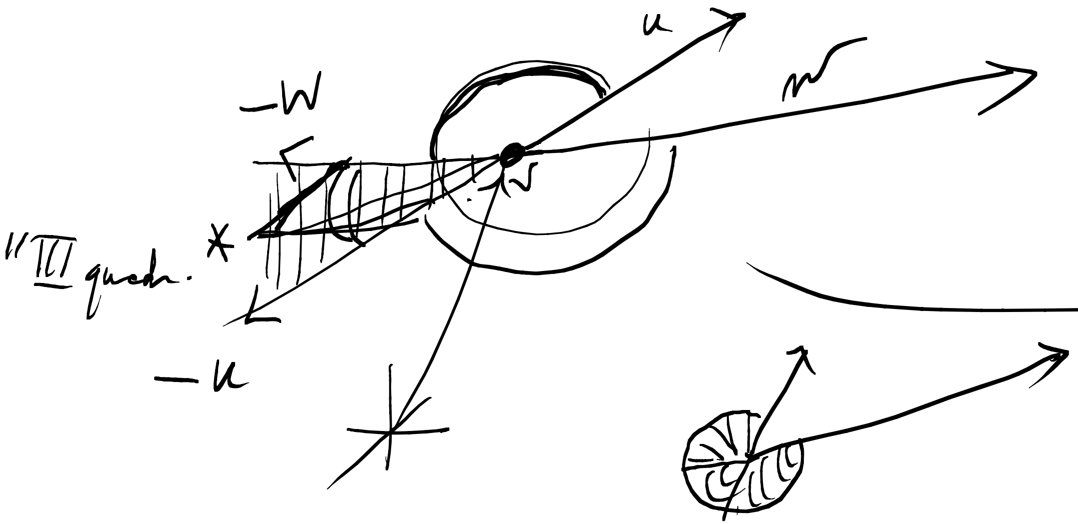
$$\sum_{1}^{k_1} \bar{x}^1_{j_1} u^1_{j_1} = 0 \dots$$



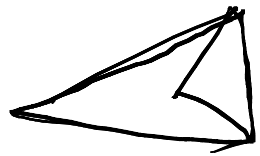


$$\boxed{v + \alpha w + \beta u}$$

$$\alpha \geq 0 \quad \beta \geq 0$$



$$v + \alpha(-w) + \beta(-u)$$



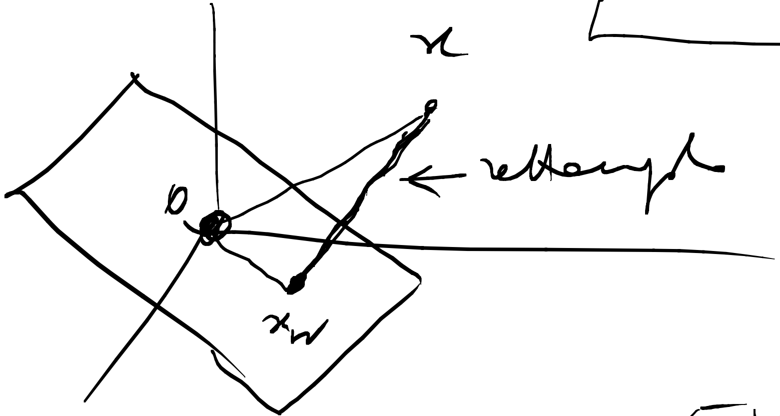
$x \qquad \langle u_1, u_2, \dots, u_k \rangle = W$

$x_{\langle u_1, \dots, u_k \rangle}$ (Proiezione di x su W) è definita dalle proprietà

1) $x_{\langle u_1 \dots u_k \rangle} \in \langle u_1, \dots, u_k \rangle = W$

Th. proiezione $\rightarrow$ $\boxed{2) \left[x - x_{\langle u_1 \dots u_k \rangle} \right] u_j = 0 \quad \forall j = 1 \dots k}$

in $\mathbb{R}^n$ sono un sistema di equazioni



$(1,2,3) \quad \langle \underset{u_1}{(1,1,2)}, \underset{u_2}{(1,-1,1)} \rangle$

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \underbrace{\left[\alpha \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right]}_{\text{proiezione}} \right\} \begin{matrix} (1,1,2) = 0 \\ (1,-1,1) = 0 \end{matrix}$$

due equazioni nelle due incognite α, β

$\begin{pmatrix} \bar{\alpha} \\ \bar{\beta} \end{pmatrix}$ le soluzione di allora $\boxed{x_W = \bar{\alpha} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \bar{\beta} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}$

1) $\sum_1^n \alpha_i x_i \quad \alpha_i \in \mathbb{R}$ qualunque COMB LINEARI

2) $\boxed{\sum_1^n \alpha_i x_i \quad \alpha_i \in \mathbb{R} \quad \sum_1^n \alpha_i = 1}$ AFFINI

oppure $\boxed{\underbrace{\left(1-\sum_2^n \alpha_i\right)}_{\alpha_1} x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n \quad \forall \alpha_2 \dots \alpha_n \in \mathbb{R}}$

3) $\left(1-\sum_2^n \alpha_i\right) x_1 + \sum_2^n \alpha_i x_i \quad \alpha_i \geq 0$ CONICHE

oppure $\left(\sum_1^n \alpha_i x_i \quad \alpha_2, \dots, \alpha_n \geq 0 \quad \sum_1^n \alpha_i = 1\right.$

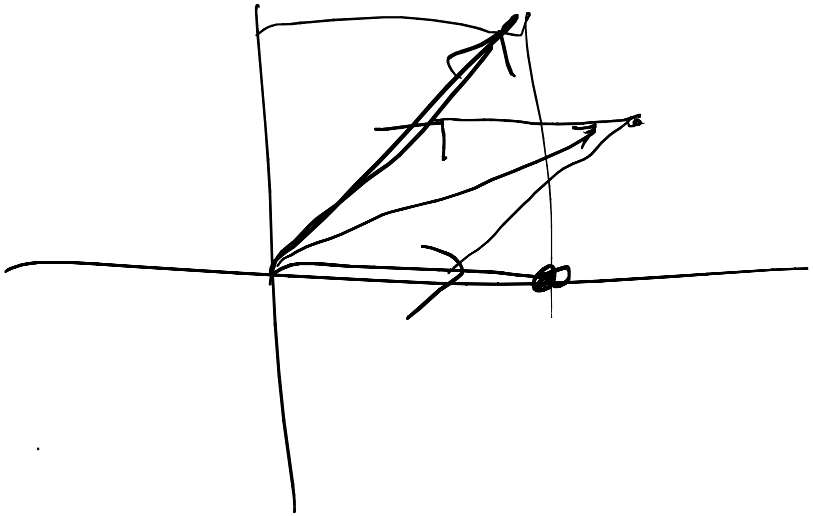
4) $\sum_1^n \alpha_i x_i \quad \alpha_i \in [0,1] \quad \sum \alpha_i = 1$ CONVESSE

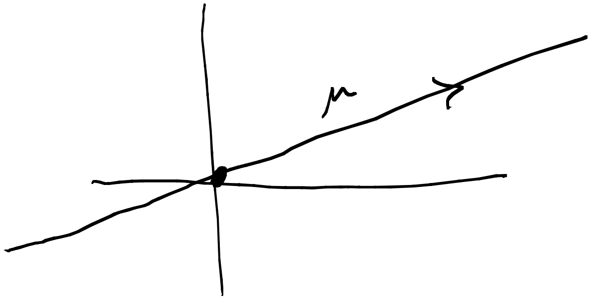
$$\left\{ \underset{u_1}{(1,0)}, \underset{u_2}{(1,1)} \right\} \qquad \langle (1,0), (1,1) \rangle = \mathbb{R}^2$$

$$\boxed{u_1 u_2 = 1}$$

$$u_1 u_1 = 1$$

$$u_2 u_2 = 2$$

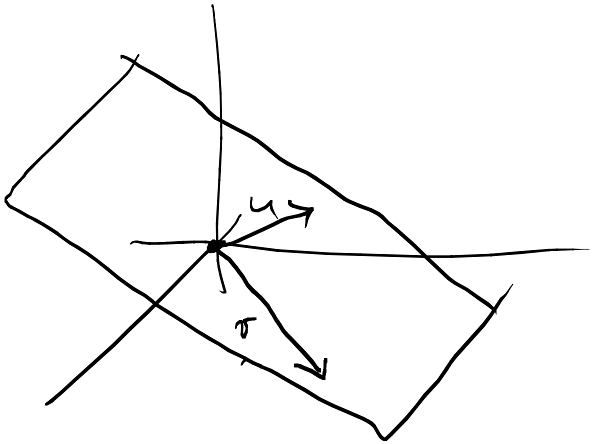




$$\gamma(t) = tu$$

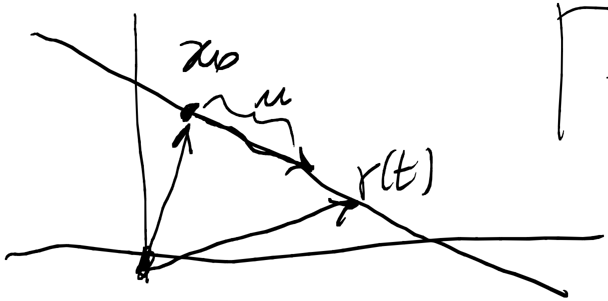
$$\{tu : t \in \mathbb{R}\} = \langle u \rangle$$

retta per l'origine

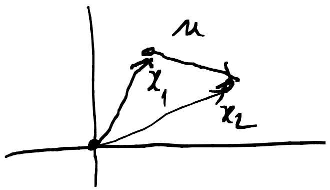


$$\varphi(\alpha, \beta) = \alpha u + \beta v$$

piano per l'origine

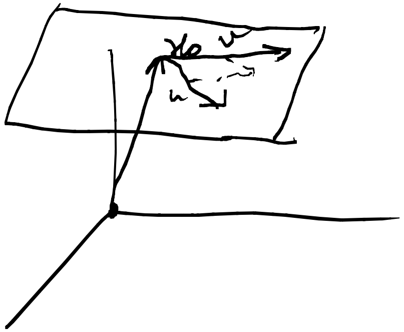


$\boxed{\gamma(t) = x_0 + t\underline{\underline{u}}}$ retta per x_0 in direzione di u



$$u = x_2 - x_1$$

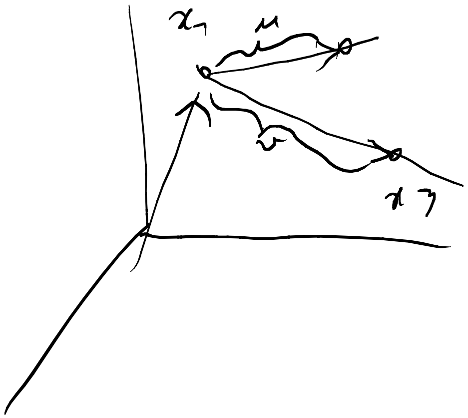
$$\gamma(t) = x_1 + t(\underbrace{x_2 - x_1}_{u}) = \overbrace{(1-t)x_1 + t x_2}^{\text{combin. affi}}$$



$\mathbb{R}^3$

$$\boxed{\varphi(\alpha,\beta) = x_0 + \alpha u + \beta v}$$

u e v spostamenti del piano non allineati



$$\varphi(\alpha,\beta) = x_1 + \alpha\underbrace{(x_2 - x_1)}_{u} + \beta\underbrace{(x_3 - x_1)}_{v} =$$

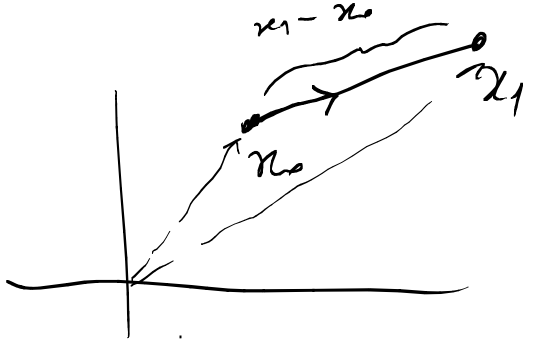
$$= (1-\alpha-\beta)x_1 + \alpha x_2 + \beta x_3$$

combinaz. affine

$$u = x_2 - x_1$$

$$v = x_3 - x_1$$

$$(1-\alpha-\beta) + \alpha + \beta = 1$$



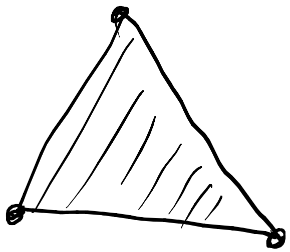
$$\gamma(t) = x_0 + t(x_1 - x_0) \qquad t \in [0,1]$$

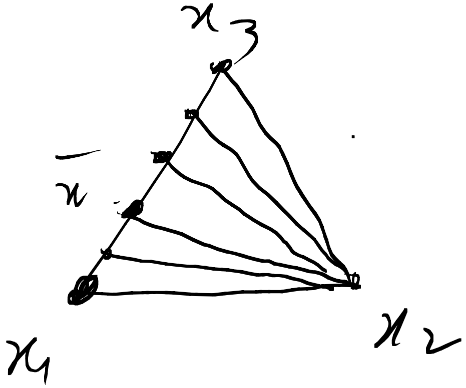
$$\| $$

$$\underbrace{(1-t)}_{\in[0,1]}\, x_0 + t x_1 \qquad t \in [0,1]$$

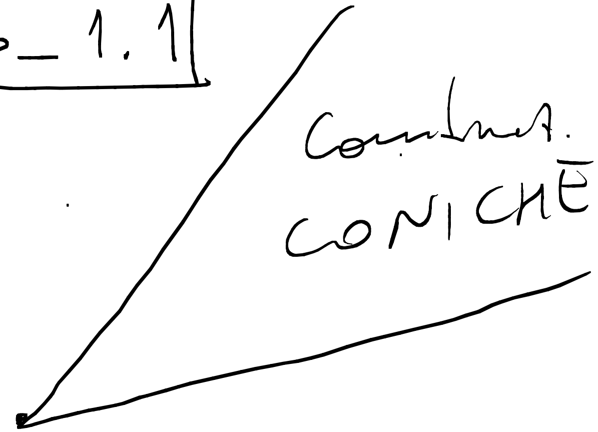
$\boxed{\text{Combin. convesse} \qquad \sum_1^n \alpha_i x_i \qquad \alpha_i \in [0,1], \quad \sum_1^n \alpha_i = 1}$

$\boxed{\text{G\_1.1}}$

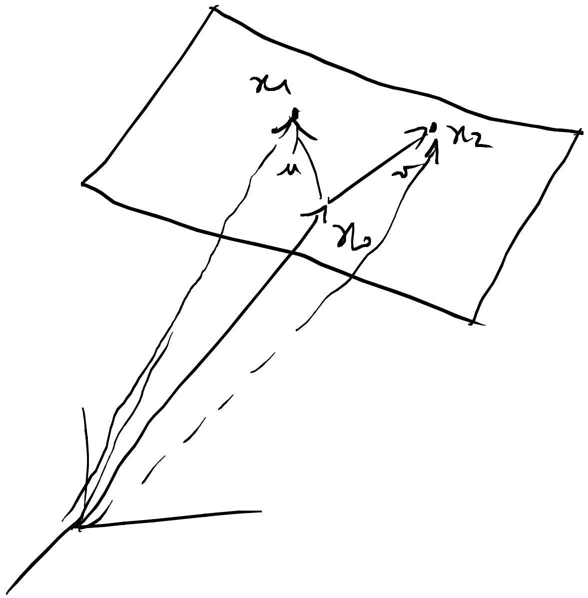




contorno



Combinaz.
CONICHE



$$\boxed{\varphi(\alpha, \beta) = x_0 + \alpha u + \beta v} =$$

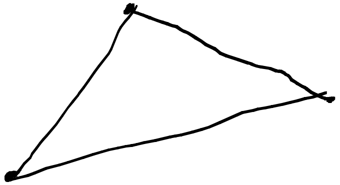
$$u = (x_1 - x_0)$$

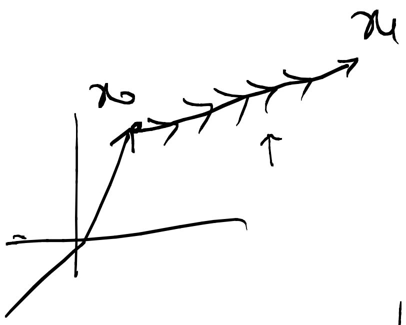
$$v = (x_2 - x_0)$$

$$= x_0 + \alpha(x_1 - x_0) + \beta(x_2 - x_0) =$$

$$= \boxed{(1 - \alpha - \beta) x_0 + \alpha x_1 + \beta x_2}$$

$$\boxed{(1 - \alpha - \beta) + \alpha + \beta = 1}$$





$t \in [0,1]$

$x_0 + t(x_1 - x_0) =$

$= (1-t)x_0 + t x_1$

$(1-t) + t = 1$

$\varphi(\alpha,\beta) = (1-\alpha-\beta)x_1 + \alpha x_2 + \gamma x_3$

$\alpha, \beta \in [0,1]$ $\quad (1-\alpha-\beta)+\alpha+\beta = 1$

$\varphi : \underset{\alpha}{[0,1]} \times \underset{\beta}{[0,1]} \to \mathbb{R}^3$

al variare di α e β in $[0,1]$ si ottengono tutti i punti del triangolo