RIEPILOGO DELLE DEFINIZIONI

Note Title 10/19/2020

Dati $x_1, x_2, \dots, x_k \in \mathbb{R}^n$ si definiscono le <u>COMBINAZIONI</u>

- $\boxed{\text{LINEARI: } \sum_1^k \alpha_i x_i \,,\ \alpha_i \in \mathbb{R}}$ (o $\mathbb{C}$, negli spazi complessi)

  $\left\{ \sum_1^k \alpha_i x_i : \alpha_i \in \mathbb{R} \right\} \equiv \langle x_1, \dots, x_k \rangle$ span o sottospazio generato da $x_1 \dots x_k$

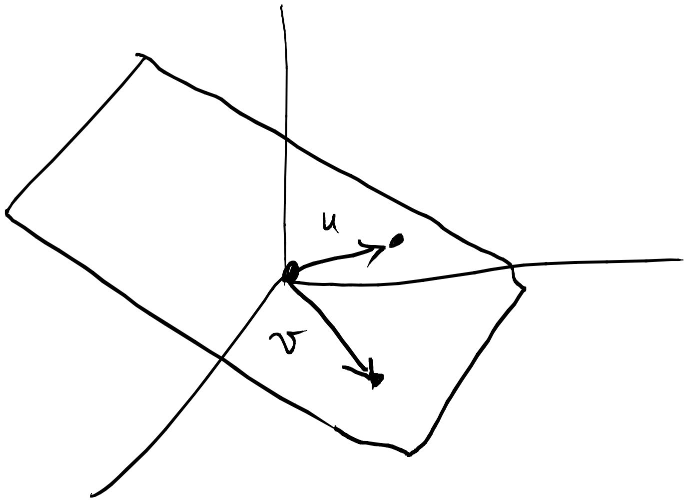
- $\boxed{\text{AFFINI: } \sum_1^k \alpha_i x_i \,,\ \alpha_i \in \mathbb{R} \text{ e } \sum_1^k \alpha_i = 1}$

  $\left\{ \sum_1^k \alpha_i x_i \,,\ \alpha_i \in \mathbb{R}\ \sum_1^k \alpha_i = 1 \right\}$ è lo spazio affine generato da $x_1 \dots x_n$

  $\boxed{\text{Alternative}}$ $\left(1 - \sum_2^k \beta_j\right) x_1 + \sum_2^k \beta_j x_j \quad \beta_j \in \mathbb{R}$ (<u>la somma dei coeff. è 1</u>)

- $\boxed{\text{CONICHE: } \left(1 - \sum_2^k \alpha_i\right) x_1 + \sum_2^k \alpha_i x_i \,,\ \alpha_i \in \mathbb{R}\ \alpha_i \geq 0}$ Il loro insieme è un "CONO" di vertice x_1. <u>La somma dei coeff. è 1.</u>

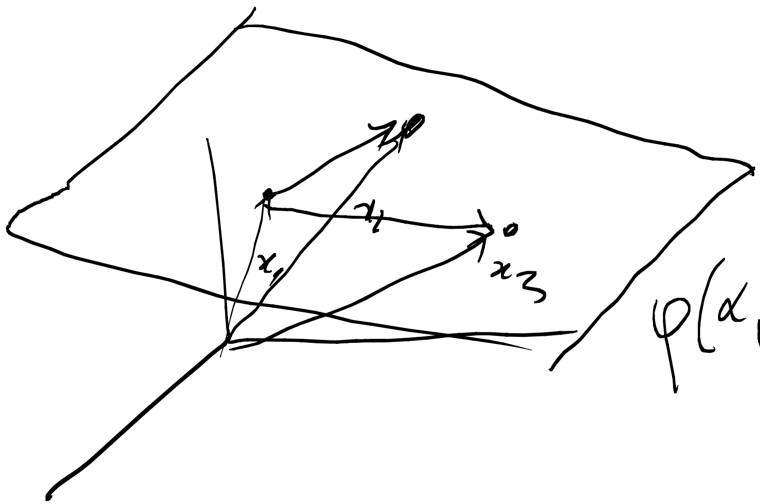
- $\boxed{\text{CONVESSE: } \sum_1^k \alpha_i x_i \,,\ \alpha_i \in [0,1]\ \sum_1^k \alpha_i = 1}$ Il loro insieme è il più piccolo poliedro convesso che contiene $x_1 \dots x_k$



$\langle u, v \rangle = \{\alpha u + \beta v\}$

piano per l'origine

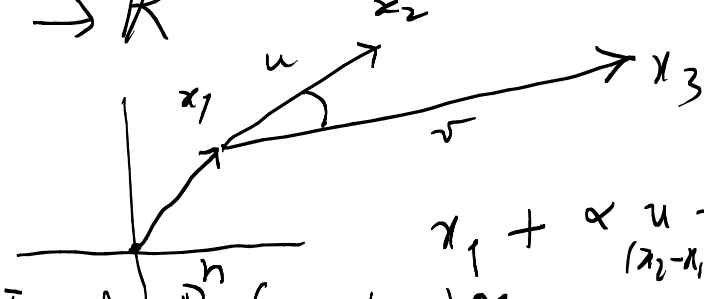
$\langle \overset{u}{(x_2 - x_1)}, \overset{v}{(x_3 - x_1)} \rangle$



$$\overbrace{x_1 + \alpha (x_2 - x_1) + \beta (x_3 - x_1)}$$

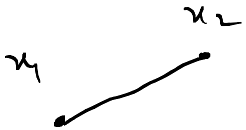
$$\varphi(\alpha, \beta) = \boxed{(1 - \alpha - \beta) x_1 + \alpha x_2 + \beta x_3}$$

$\varphi : \mathbb{R}^2 \to \mathbb{R}^n$

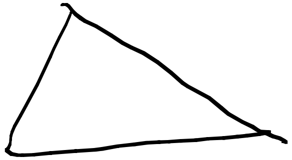


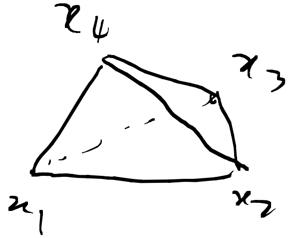
$x_1 + \alpha \underset{(x_2 - x_1)}{u} + \beta \underset{(x_3 - x_1)}{v}$ $\quad \alpha, \beta \geq 0$

$\varphi : I$ quadr. $\to \mathbb{R}^n \quad (1 - \alpha - \beta) x_1 + \alpha x_2 + \beta x_3$

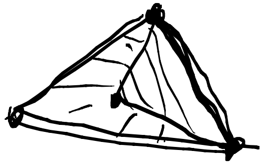


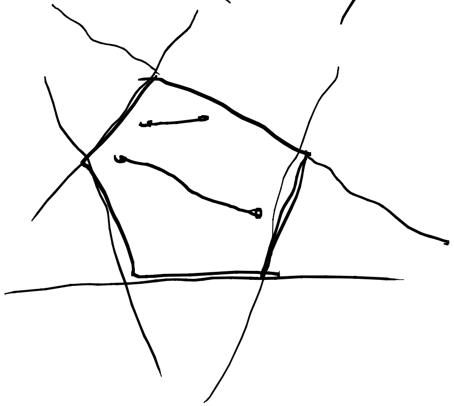
$$(1-\lambda)x_1+\lambda x_2$$





$$(1-\lambda-\mu)x_1+\lambda x_2+\mu x_3$$





Intersezioni di semipiani.

$$(1,1,1)$$

$$\{(1,0,0),\ (1,1,0)\}$$

$$(1,1,1)_{\langle \underset{u}{(1,0,0)},\ \underset{v}{(1,1,0)} \rangle}$$

$$\boxed{v - v_u} \perp u$$

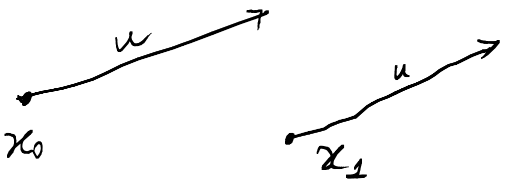
$$v_u = \frac{(1,1,0)(1,0,0)}{|(1,0,0)|^2}(1,0,0) = (1,0,0)$$

$$v - v_u = (1,1,0) - (1,0,0) = (0,1,0)$$

$$\langle (1,0,0),\ (1,1,0) \rangle \overset{\subseteq}{\underset{\supseteq}{=}} \langle \underbrace{(1,0,0),\ (0,1,0)}_{\text{base canonica del piano}} \rangle$$

$$(1,1,1)_{\langle \underset{u}{(1,0,0)},\ \underset{v}{(1,1,0)} \rangle} = (1,1,1)_{\langle \underset{u}{(1,0,0)},\ \underset{v - v_u}{(0,1,0)} \rangle}$$

facile da calcolare perché i generatori sono $\perp$



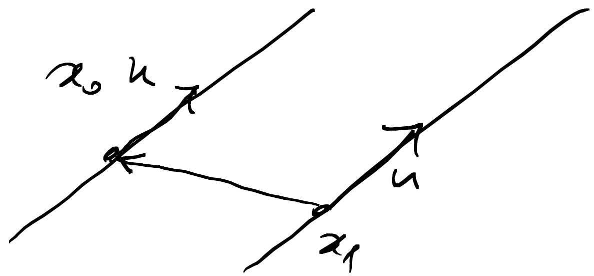
$x_0 + t\underline{u} \quad \underline{t \in \mathbb{R}} \quad (r)$

La parallela ad r per x_1 è $\underline{\varphi(s) = x_1 + s\underline{u}} \quad \underline{s \in \mathbb{R}}$

Se $\exists\ \bar{s}, \bar{t} \in \mathbb{R} \ : \ x_0 + \bar{t}u = x_1 + \bar{s}u = \bar{x}$

$\bar{x}$ è un punto per cui passano le due rette.

$$\boxed{x_0 - x_1 = (\bar{s} - \bar{t})u} \quad \neq$$



$x_0 = x_1 + (\bar{s} - \bar{t})u$ e quindi $x_0 \in \varphi$ $\qquad x_1 = x_0 + (\bar{t} - \bar{s})u$

$x_1 \in r$

le due rette φ ed z hanno 2 punti comuni x_0 ed x_1

e quindi coincidono

$$x_1 = x_0 + (\bar{t} - \bar{s})u$$

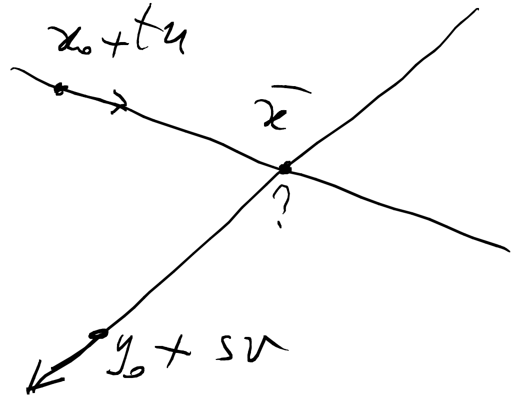
$$x_1 + su = x_0 + \underbrace{(\bar{t} - \bar{s})u + su}_{\text{un multiplo di } u} = x_0 + \alpha u$$

$$\boxed{x_1 - x_0 \neq \text{multiplo di } u}$$

La retta per $\underset{x_0}{(1,1,0,1)}$ parallela a $t\underbrace{(1,1,1,1)}_{u} = \gamma(t)$

$x_0 + tu$ cioè $(1,1,0,1) + s(1,1,1,1)$

$\exists\, t,s \;:\; x_0 + t\underline{u} = y_0 + s\underline{v} = \bar{x}$

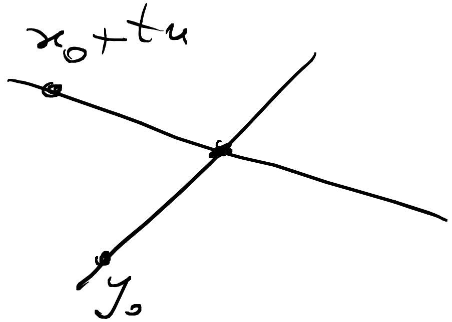


$\bar{x} \in$ entrambe le rette

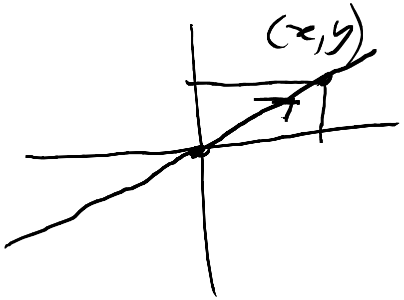
Intersezioni

$\neq$

$\underline{x_0 + tu} = \underline{y_0 + tv}$



COLLISIONI



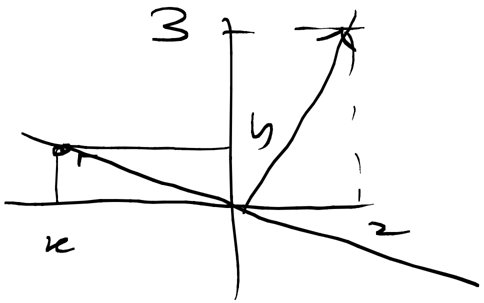
$ax + by = 0 \longrightarrow \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$\boxed{\{(x,y) \in \mathbb{R}^2 : ax + by = 0\}}$

(a,b) è ortogonale a (x,y)

La retta implicita $ax + by = 0$ è il luogo di tutti i vettori ortogonali da (a,b)

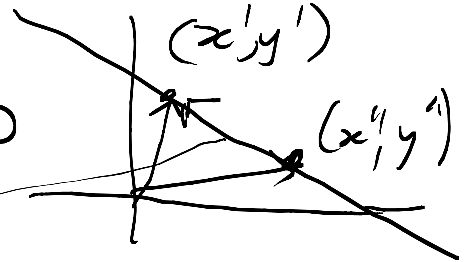
$$2x+3y=0$$



$(x',y'),\ (x'',y'')$ appartengono a $2x+3y=4$

$$2x'+3y'=4$$

$$2(x'-x'')+3(y'-y'')=0$$



$$2x''+3y''=4$$

$$\underline{\underline{(2,3)}}\,(x'-x'',\ y'-y'')$$

$$\underbrace{\begin{pmatrix}2\\3\end{pmatrix}}\underbrace{\begin{pmatrix}x'-x''\\y'-y''\end{pmatrix}}=0$$

vettore differenza (spostamento) tra i due punti distinti della retta $2x+3y=4$

$$x_0+\bar t u-(x_0+\bar s u)=\underline{\underline{(\bar t-\bar s)u}}$$