Note Title 10/16/2020

$x \quad \langle u_1, u_2, \ldots, u_k \rangle = W$

$\underline{x_{\langle u_1, \ldots, u_k \rangle}}$ proiezione di x su W

1) $\underline{x_{\langle u_1 \ldots u_k \rangle}} \in W = \langle u_1, \ldots, u_k \rangle$

la proiezione è combin. lineare di $u_1 \ldots u_k$.

2) $(x - \underline{x_{\langle u_1 \ldots u_k \rangle}})\, u_j = 0 \quad \forall j = 1 \ldots k$

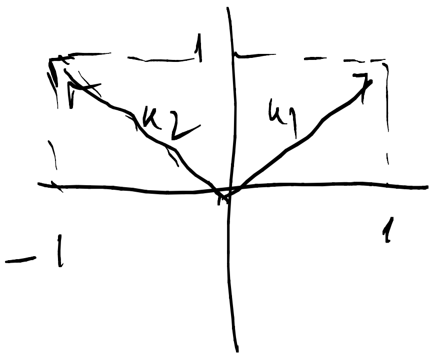
$\{u_1, \ldots u_k\}$ si dice SISTEMA ORTOGONALE se si verifica

$$\begin{cases} u_i u_j = 0 & \text{se } \underline{i \neq j} \\ \underline{u_i u_j > 0} & \text{se } \underline{i = j} \end{cases} \quad |u_i|^2 > 0$$

LA BASE CANONICA verifica tali proprietà.

se $|u_i| = 1$ $\forall i = 1 .. k$ allora il sistema si dice ORTONORMALE. (sistema ortogonale di versori)

esattamente la base canonica.



$(1,1)$ e $(-1,1)$ sono ortogonali, perché

$1\cdot(-1) + 1\cdot 1 = 0$ ma non ortonormali

perché $|(1,1)| = |(-1,1)| = \sqrt{2} > 0$ ← $u_i u_i > 0$

$$\{ \underset{e_1}{(1,0 \dots 0)}, \underset{e_2}{(0,1,0 \dots 0)}, \dots, \underset{e}{(0 \dots 0,1)} \}$$

FISICA

$\hat{x}, \hat{y}, \hat{z}$

Supposto che $u_1, \dots, u_k$ sistema ORTOGONALE

si definisce $x_{\langle u_1, \dots, u_k \rangle} = \sum_{i=1}^{k} x_{u_i}$

È vero che:

1) $\sum_1^k x_{u_i} \in \langle u_1, \dots, u_k \rangle$?

2) $\left(x - \sum_1^k x_{u_i}\right) u_j = 0 \quad \forall j = 1..k$?

(Th. della proiezione)

1) Sì, perché x_{u_i} è un multiplo di u_i

$$\sum_1^k x_{u_i} = \sum_1^k \frac{x u_i}{|u_i|^2} u_i \in \langle u_1, \dots, u_k \rangle \longleftarrow \text{combinazione lineare}$$

j è fissato ad arbitrio

scalari

2) $\left(x - \sum_{i=1}^k \frac{x u_i}{|u_i|^2} u_i\right) u_j = x u_j - \left(\sum_{i=1}^k \frac{x u_i}{|u_i|^2} u_i\right) u_j = x u_j - \sum_{i=1}^k \frac{x u_i}{|u_i|^2} u_i u_j =$

La sommatoria contiene i prodotti $u_i u_j$.

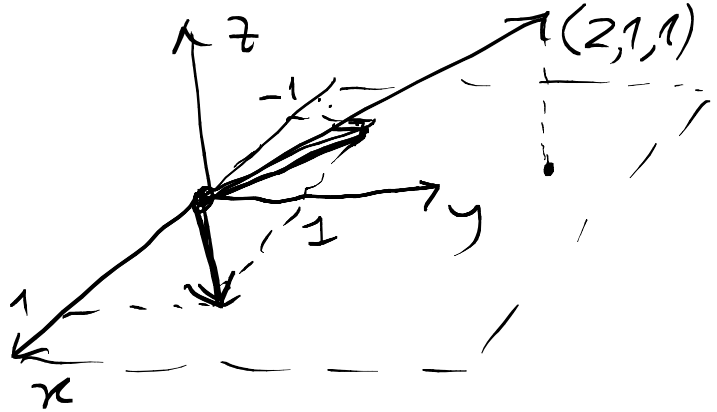
$$u_j u_j = |u_j|^2$$

$$= x u_j - \cancel{\frac{x u_j}{|u_j|^2} u_j u_j} = 0$$

$$\boxed{x_{\langle u_1, \dots, u_k \rangle} = \sum \frac{x u_i}{|u_i|^2} u_i \quad \text{DEF DI PROIEZ. su } \langle u_1 \dots u_k \rangle \text{ - ort. ortog.}}$$

Se $\{u_1 \dots u_k\}$ sono ortonormali allora la formula diventa

$$\boxed{x_{\langle u_1 \dots u_k \rangle} = \sum (x u_i) u_i} \quad \text{ORTONORMALE}$$



$(1,1,0)$

$(-1,1,0)$

$x \to (2,1,1)$ $\quad \langle \underset{u_1}{(1,1,0)}, \underset{u_2}{(-1,1,0)} \rangle$

u_1, u_2 non nulli

$|u_1| = |(1,1,0)| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$|u_2| = \sqrt{2}$

$u_1 u_2 = (1,1,0)(-1,1,0) =$
$= 1\cdot(-1) + 1\cdot 1 + 0\cdot 0 = 0$

$\{u_1, u_2\}$ sono un sistema ortogonali (non ortonormali) in $\mathbb{R}^3$

$$x_{\langle u_1, u_2 \rangle} = \frac{x u_1}{|u_1|^2} u_1 + \frac{x u_2}{|u_2|^2} u_2 = \frac{2\cdot 1 + 1\cdot 1 + 1\cdot 0}{2}(1,1,0) + \frac{2\cdot(-1) + 1\cdot 1 + 1\cdot 0}{2}(-1,1,0) =$$

$$= \frac{3}{2}(1,1,0) - \frac{1}{2}(-1,1,0) = \left(\frac{3}{2}, \frac{3}{2}, 0\right) + \left(\frac{1}{2}, -\frac{1}{2}, 0\right) =$$

$$= (2,1,0)$$

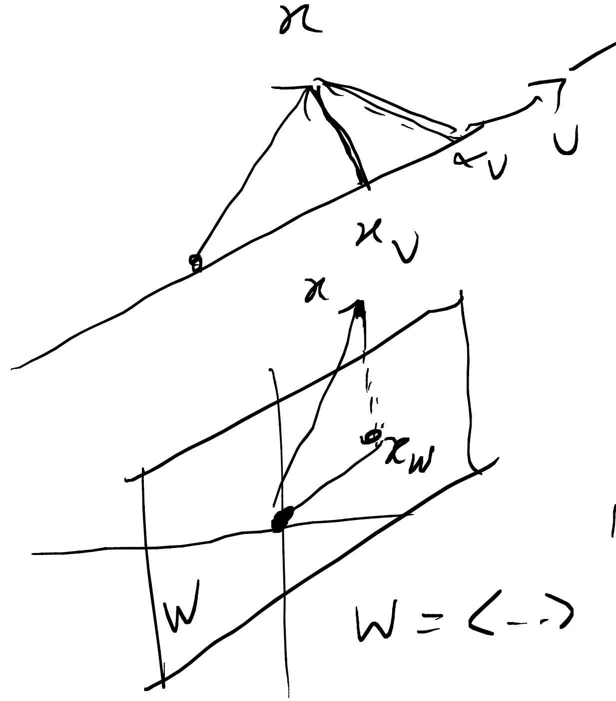
$\underset{u_1}{(1,0)}, \underset{u_2}{(1,1)}$ non è ortogonale, perché

$$\boxed{\underset{u_1}{(1,0)}\underset{u_2}{(1,1)} = 1 \neq 0}$$

$$u_1 u_1 = 1 \neq 0$$

$$u_2 u_2 = 2 \neq 0$$

$$\left| x - \underbrace{\sum_{1}^{k} \alpha_i u_i}_{\in W} \right| \geq \left| x - x_{\langle u_1, \dots, u_k \rangle} \right|$$



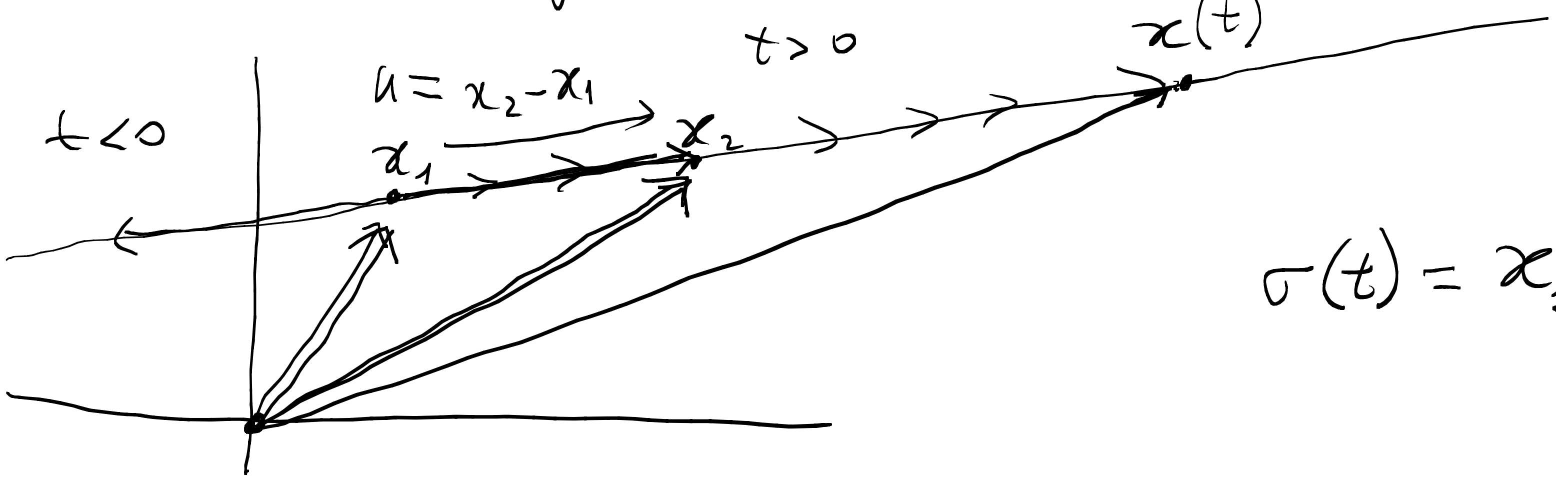
$$\Big|\underbrace{x - \sum_{1}^{k}\alpha_i u_i}_{\in W}\Big|^2 = \Big|\underbrace{x - x_{\langle u_1,\dots,u_k\rangle}}_{\text{ortogonale ad } u_i\ \forall i} + \underbrace{\underbrace{x_{\langle u_1,\dots,u_k\rangle}}_{\in W} - \underbrace{\sum_{1}^{k}\alpha_i u_i}_{\in W}}_{\in W = \langle u_1,\dots,u_k\rangle}\Big|^2 \underset{\text{Pitag.}}{=}$$

$$= \Big|\underbrace{x - x_{\langle u_1,\dots,u_k\rangle}}_{\text{ortog. a } u_1, u_2, \dots u_k}\Big|^2 + \underbrace{\Big| \qquad \Big|^2}_{\geq 0} \geq \Big| x - x_{\langle u_1,\dots,u_k\rangle}\Big|^2$$

Equaz. param. retta per 2 punti: G_1.1

x_0 punto in $\mathbb{R}^n$, $u \in \mathbb{R}$ (velocità)

la retta per x_0 in direzione u è $x_0 + tu$ $\left(= \gamma(t)\right.$ $\left.\begin{pmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{pmatrix}\right)$



$$\sigma(t) = x_1 + t(x_2 - x_1)$$

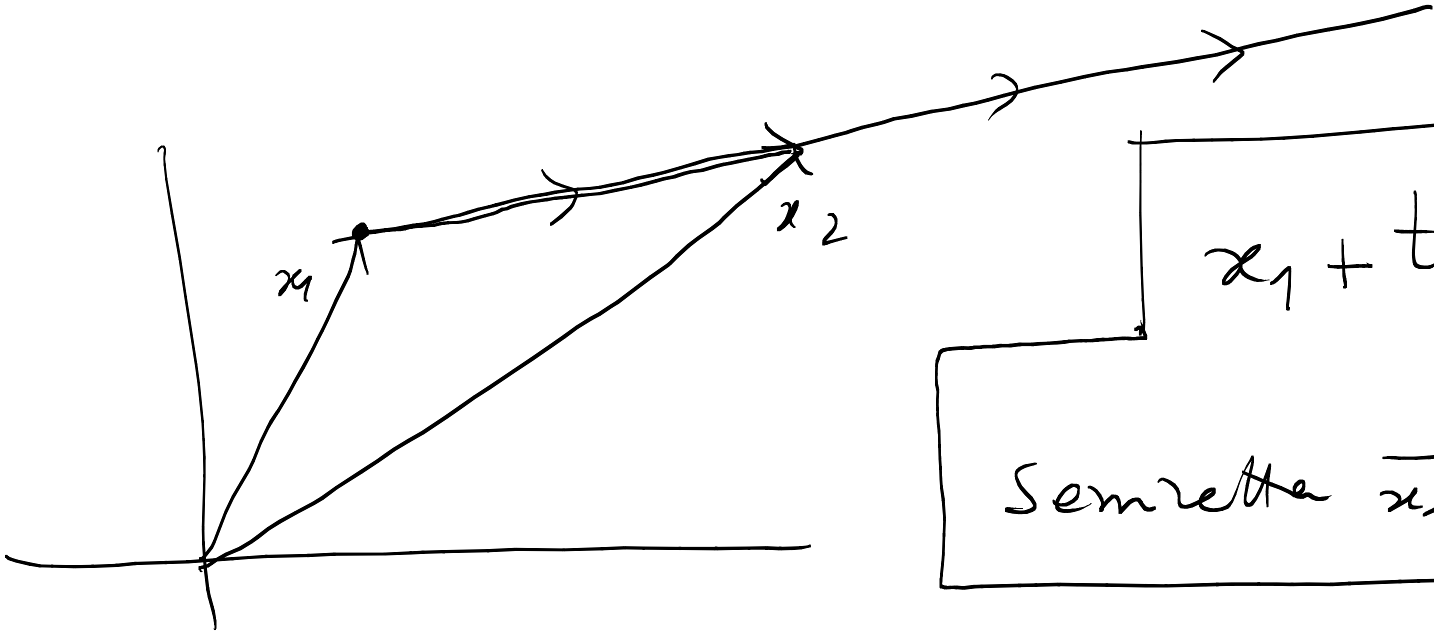
$t \in \mathbb{R}$

$$x_1 + t(x_2 - x_1) = (1-t)x_1 + t x_2$$

I punti della retta sono pertanto combinazioni lineari per cui $(1-t)+t=1$

$$\sum_{1}^{n} \alpha_i x_i \quad : \quad \sum \alpha_i = 1$$

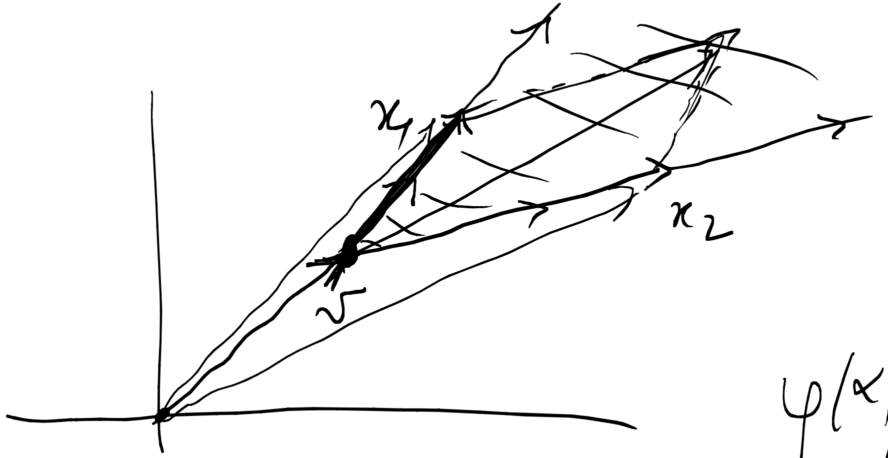
COMBINAZIONI AFFINI



$$x_1 + t(x_2 - x_1)$$

$t \geq 0$

Semiretta $\overrightarrow{x_1 x_2}$



Angolo di vertice v
e lati x_1 e x_2

$\varphi(\alpha,\beta)$

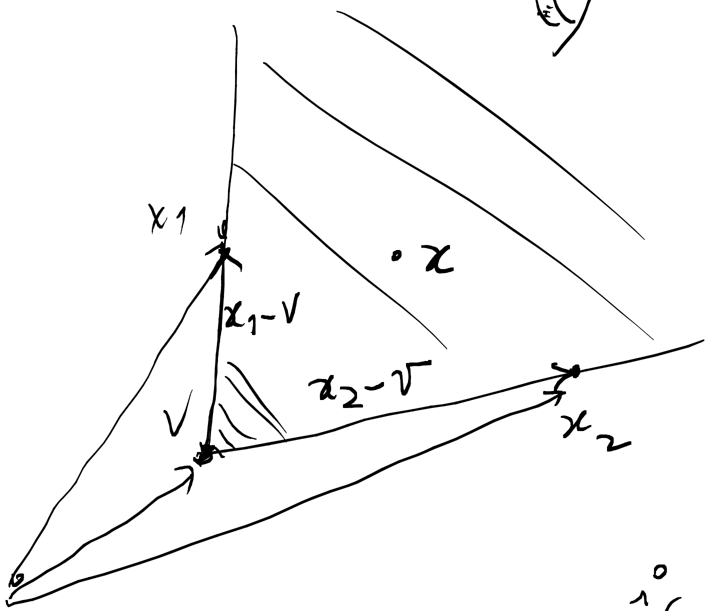
$$v + \alpha(x_1 - v) + \beta(x_2 - v)$$

$\alpha,\beta > 0$ $\quad \alpha,\beta \in \mathbb{R}$

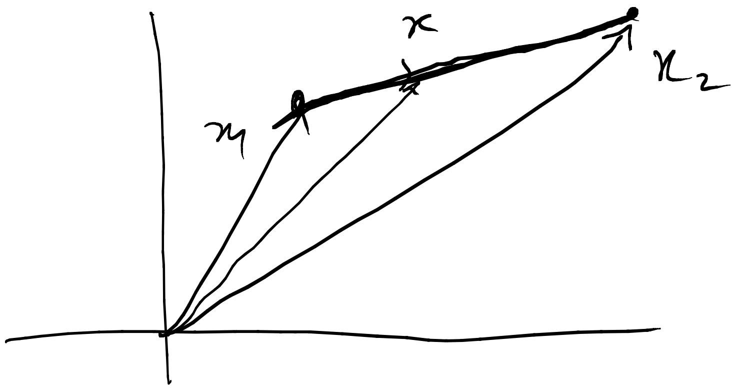
$$\varphi(\alpha,\beta) = \boxed{(1-\alpha-\beta)v + \alpha x_1 + \beta x_2}$$

$\alpha,\beta > 0$

$$\boxed{(1-\alpha-\beta) + \alpha + \beta = 1}$$



$$x = v + \mathring{\hat{\alpha}}(x_1 - v) + \mathring{\hat{\beta}}(x_2 - v) \qquad \alpha,\beta \geq 0$$



COMBIN.
CONVESSE

$$x = x_1 + t(x_2 - x_1)$$

$$t \in [0,1] \qquad 0 \le t \le 1$$

$$= (1-t)x_1 + \underbrace{t}_{\ge 0} x_2 \Rightarrow$$

$$t \ge 0 \Rightarrow 1-t \le 1$$

$$t \le 1 \Rightarrow 1-t \ge 0$$

$(1-t) + t = 1$ combinazioni affini

$\sum^{k} \alpha_i x_i$ si dirà combinazione convessa di $x_1 \dots x_k$ se

$\sum \alpha_i = 1 \qquad \alpha_i \ge 0 \Rightarrow \alpha_i \le 1 \quad \forall i = 1..k$

$(1,2,-1,4)$ $(0,1,2,-3)$ segmento

x_1 x_2

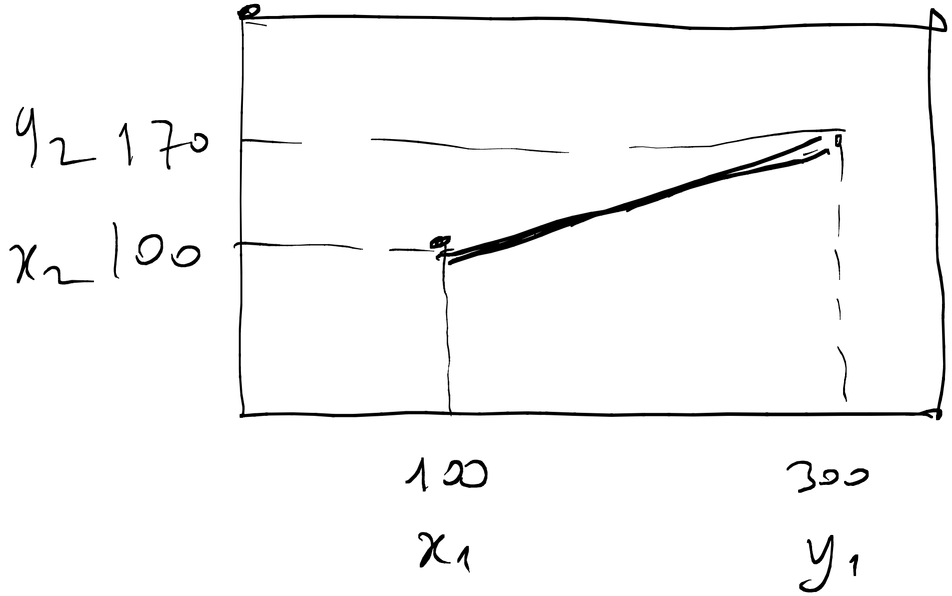
$$\gamma(t) = x_1 + t(x_2 - x_1) =$$

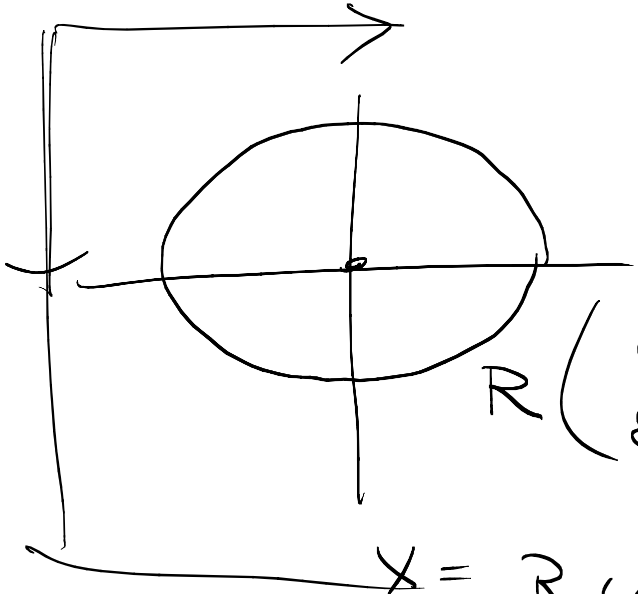
$$= (1,2,-1,4) + t(0-1,\ 1-2,\ 2-(-1),\ -3-4) =$$

$$= (1 + (-t),\ 2 + t(-1),\ -1+3t,\ 4+(-7t)) =$$

$$= (1-t,\ 2-t,\ -1+3t,\ 4-7t) \qquad t \in [0,1]$$

(x_1, x_2) (y_1, y_2) $\sigma(t) = (x_1, x_2) + t(y_1 - x_1, y_2 - x_2) \Leftrightarrow \boxed{\begin{array}{l} x = x_1 + t(y_1 - x_1) \\ y = x_2 + t(y_2 - x_2) \end{array}}$ $t \in [0,1]$

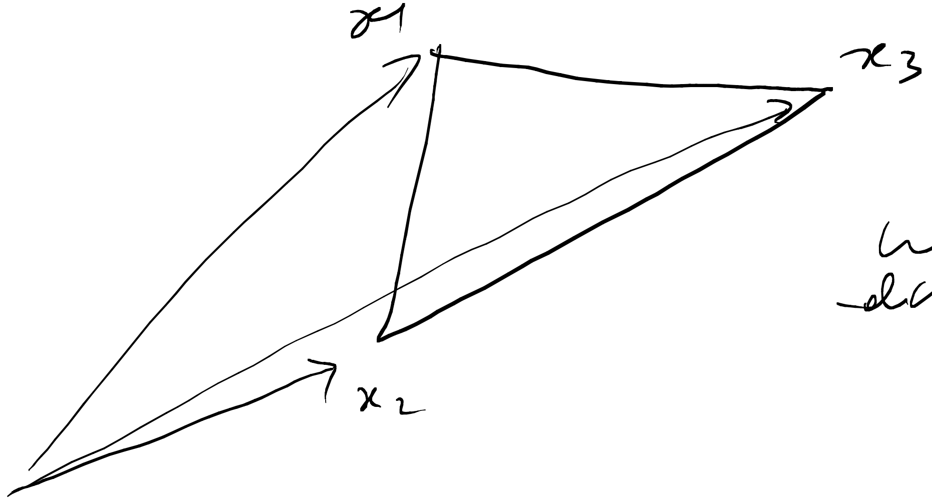


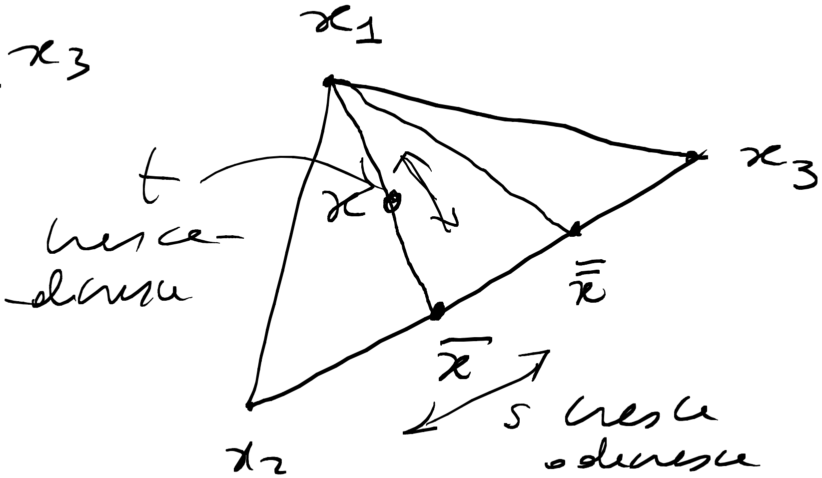


$$R\begin{pmatrix}\cos t\\ \sin t\end{pmatrix}$$

$$x = R\cos t + x_0$$

$$y = R\sin t + y_0$$





Il segmento fra x_1 e $\bar{x}$ $\quad x = x_1 + t(\bar{x} - x_1)$ $\quad t \in [0,1]$

$$\bar{x} = x_2 + s(x_3 - x_2)$$

$$x = x_1 + t\Big(\underbrace{[x_2 + s(x_3 - x_2)]}_{\bar{x}} - x_1\Big) =$$

$s \in [0,1]$

$$= (1-t)x_1 + \underbrace{t x_2 + ts(x_3 - x_2)}_{} =$$

$$t x_2 + ts\, x_3 - ts\, x_2$$

$$t(1-s)x_2 + ts\, x_3$$

$$= (1-t)x_1 + t(1-s)x_2 + ts\, x_3$$

$1-t \in [0,1]$

$t(1-s) \in [0,1]$

$ts \in [0,1]$

SONO COMBINAZIONI CONVESSE PERCHE'

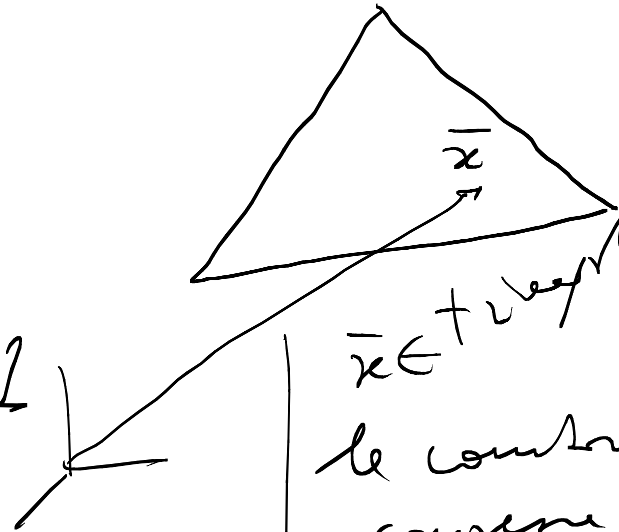
$$(1-t)+t(1-s)+ts =$$

$$= 1-\cancel{t}+\cancel{t}-\cancel{ts}+\cancel{ts} = 1$$

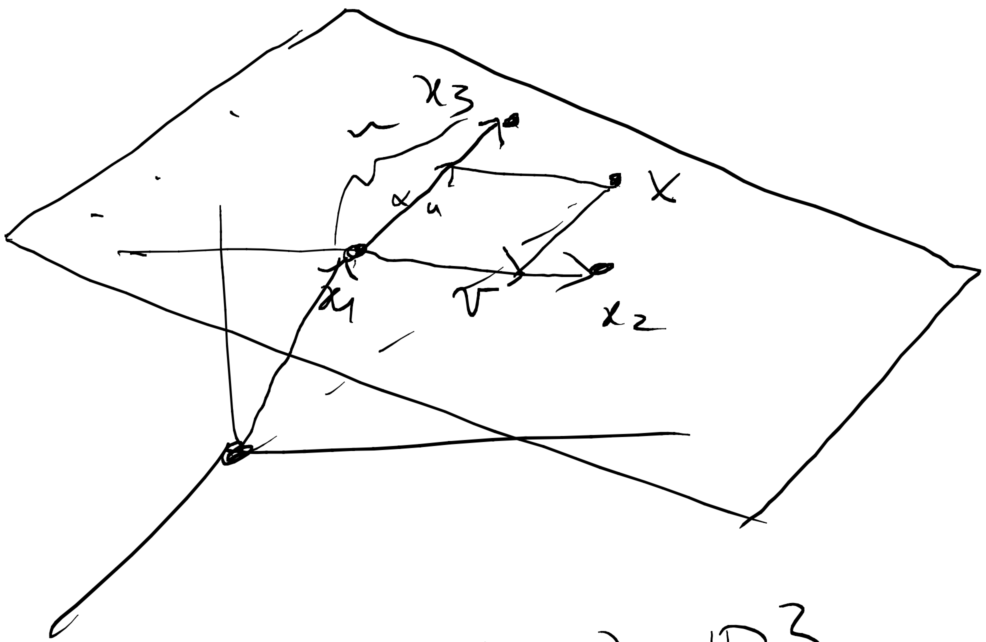
quindi la somma dei coeff. vale 1

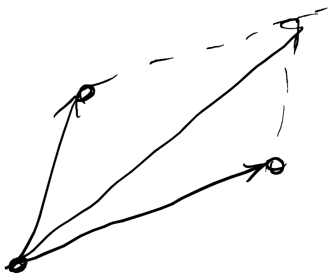
e in più ognuno dei coeff.

$\in [0,1]$



$\bar{x} \in$ triangolo sono tutte le combinazioni convesse di vertici





$$u = x_3 - x_1$$
$$v = x_2 - x_1$$

$$(\alpha,\beta) \in \mathbb{R}^2$$

$$\varphi(\alpha,\beta) = x_1 + \underbrace{\alpha u + \beta v}_{} = x_1 + \alpha(\overbrace{x_3 - x_1}^{u}) + \beta(\overbrace{x_2 - x_1}^{v})$$

$$\alpha, \beta \in \mathbb{R}$$

$$= \boxed{\overbrace{(1-\alpha-\beta)}^{\gamma_1} x_1 + \overbrace{\alpha}^{\gamma_2} x_3 + \overbrace{\beta}^{\gamma_3} x_2}$$

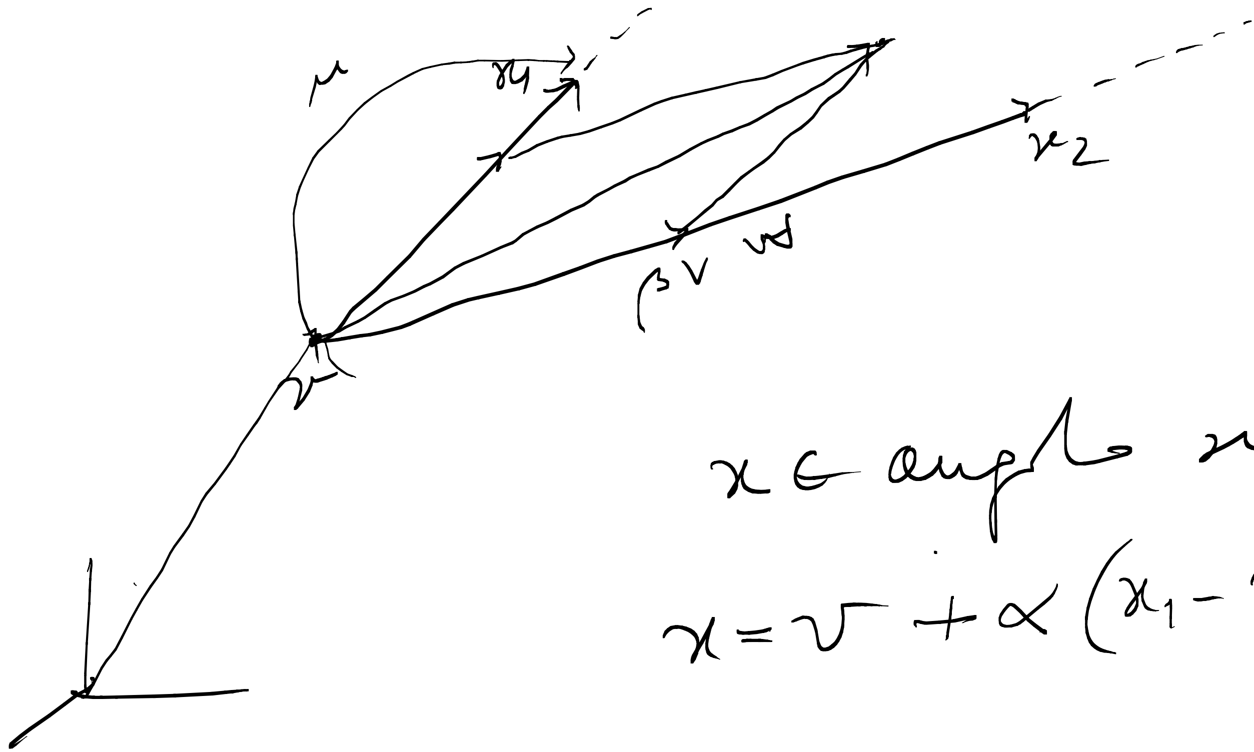
$$\gamma_1 + \gamma_2 + \gamma_3 = 1$$

combinazioni affini

$$\in \mathbb{R}^3$$

$$\varphi : \mathbb{R}^2 \to \mathbb{R}^3$$

$$x_1, x_2, x_3 \in \mathbb{R}$$



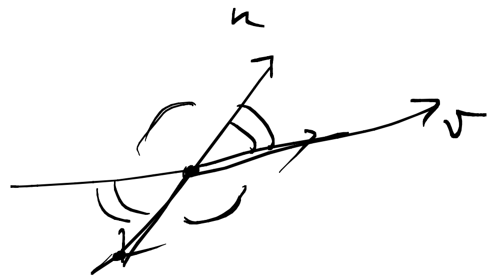
$x \in$ angolo v

$$x = v + \alpha(x_1 - v) + \beta(x_2 - v) =$$

$$\underline{\alpha, \beta > 0}$$

$$= \underbrace{(1 - \alpha - \beta)v + \underset{\geq 0}{\alpha} x_1 + \underset{\geq 0}{\beta} x_2}_{\text{combinazione conica}}$$

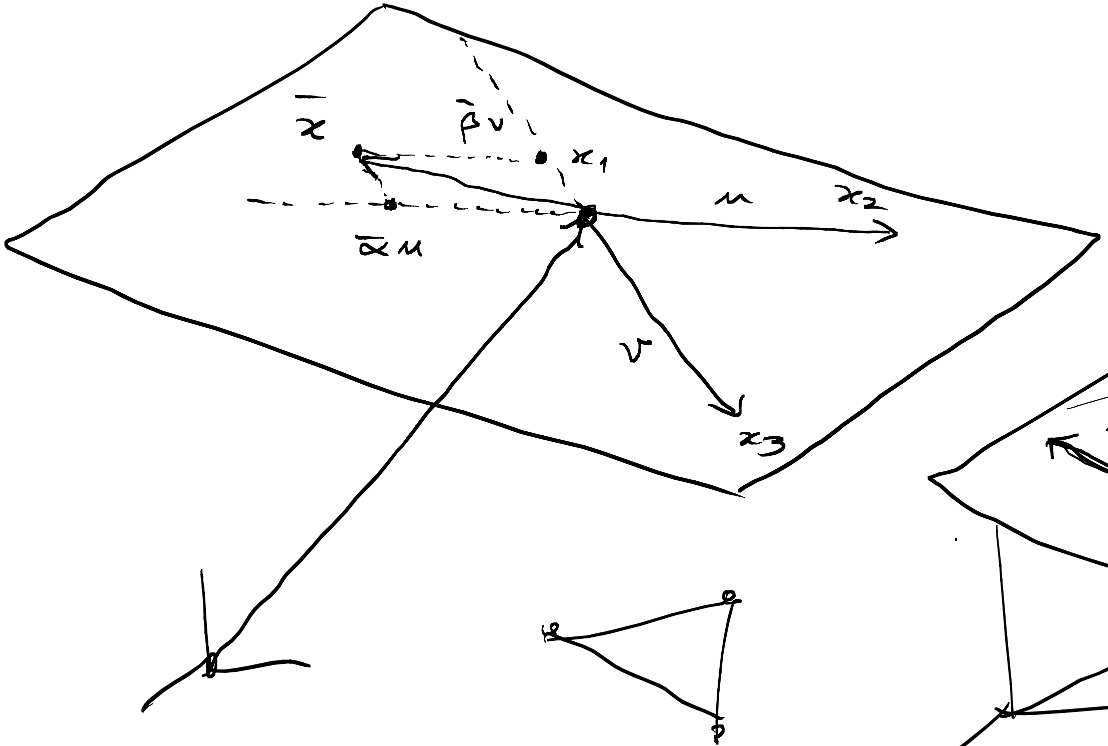
$$(1 - \alpha - \beta) + \alpha + \beta = 1$$



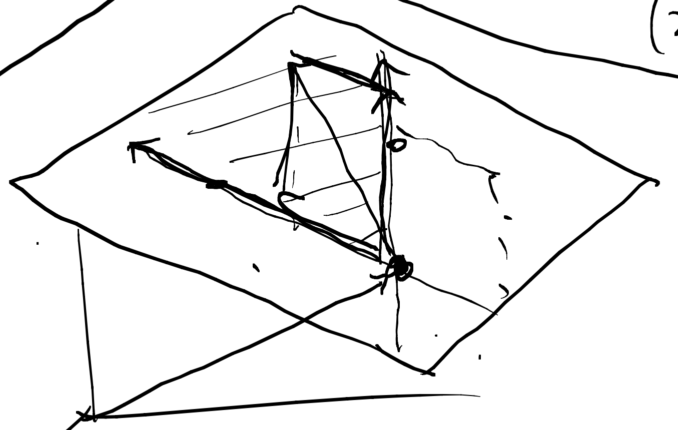
$x_1 \dots x_n$

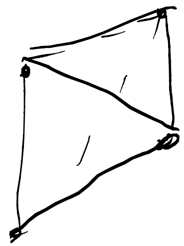
$$\sum \alpha_i x_i \quad ; \quad \sum \alpha_i = 1$$

tutte le α_i meno una sono ≥ 0



$$\bar{x} = x_1 + \bar{\alpha} \underbrace{u}_{(x_2 - x_1)} + \bar{\beta} \underbrace{v}_{(x_3 - x_1)}$$

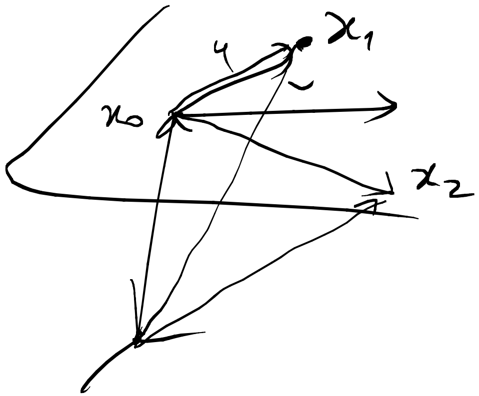




$t \in [0,1]$
$s \in [0,1]$
$r \in [0,1]$

$$(1-\lambda-\mu)x_1 + \lambda x_2 + \mu x_3 \qquad \lambda, \mu \in [0,1]$$

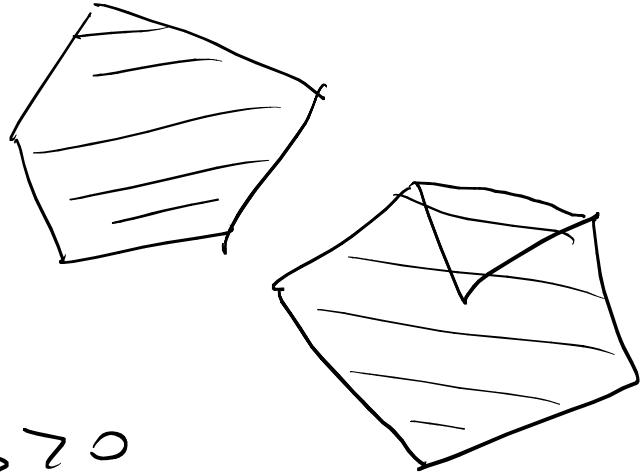
$$\boxed{x_0 + \alpha u + \beta v} =$$

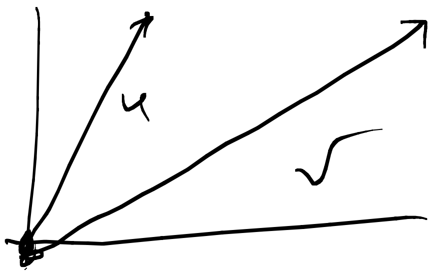


$$u = x_1 - x_0$$

$$v = x_2 - x_0$$

$$= \underbrace{(1 - \alpha - \beta) x_0 + \alpha x_1 + \beta x_2}_{\text{combinazione affine}}$$





$$\alpha u + \beta v \qquad \alpha, \beta \geq 0$$