Note Title 10/15/2020

$x_v \;,\; v \neq 0,$ $\boxed{x_v = \dfrac{x v}{|v|^2}\, v}$ $\quad (\underbrace{x - x_v}_{\text{"resto" della proiezione}})\, v = 0$

$$\langle v_1, \dots, v_k \rangle = \left\{ \sum_{1}^{k} \alpha_i v_i \;:\; \alpha_i \in \mathbb{R} \;\; \forall i = 1..k \right\}$$

$x_{\langle v_1, \dots, v_k \rangle}$ = Proiezione di $x$ su $\langle v_1, \dots, v_k \rangle$

1) $x_{\langle v_1, \dots, v_k \rangle} \in \langle v_1, \dots, v_k \rangle$ (cioè $\exists\, \bar{\alpha}_i \in \mathbb{R} : x_{\langle v_1, \dots, v_k \rangle} = \sum_{1}^{k} \bar{\alpha}_i v_i$)

2) $(x - x_{\langle v_1, \dots, v_k \rangle})\, w = 0 \quad \forall w \in \langle v_1, \dots, v_k \rangle$

generatori dello span

CNS perché $u \perp \langle v_1 .. v_k \rangle$ è che $u v_i = 0 \;\; \forall i = 1..k$

Lemma

$u \in X \qquad u w = 0 \;\; \forall w \in \langle v_1, \dots, v_k \rangle \iff \boxed{u v_i = 0 \;\; \forall i = 1..k}$

$$uw = 0 \ \forall w \in \langle v_1, \dots, v_k \rangle \Longrightarrow u v_i = 0 \ \forall i = 1 \dots k$$

<u>Dim</u> In particolare $v_1, v_2, \dots v_k \in \langle v_1, v_2, \dots, v_k \rangle$

CN

$$\boxed{v_1 = 1 \cdot v_1 + 0 v_2 + \dots + 0 v_k}$$

Scegliendo $w = v_1$
$= v_2$
$\vdots$
$= v_k$

si ottiene la tesi

$C$ è incon.

$F \Rightarrow C$

$C \Rightarrow F$
c conf

CS

$$\underbrace{\boxed{u v_i = 0 \ \forall i = 1 \dots k}}_{\text{ipotesi}} \Rightarrow \underbrace{\boxed{uw = 0 \ \forall w \in \langle v_1, \dots, v_k \rangle}}_{\text{Tesi}}$$

$$\forall w \in \langle v_1, \dots, v_k \rangle \ \exists \alpha_1 \dots \alpha_k : w = \sum_{1}^{k} \alpha_i v_i$$

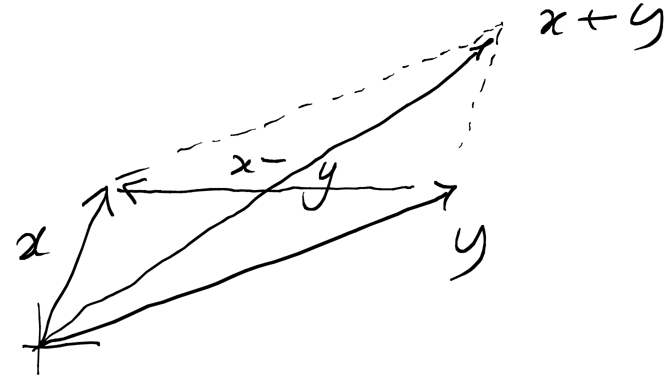
$$uw = u\left(\sum_{1}^{k} \alpha_i v_i\right) = \underset{\text{fissato}}{u}\left(\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k\right) = u(\alpha_1 v_1) + u(\alpha_2 v_2) + \dots + u(\alpha_k v_k) =$$

$$= \alpha_1 u v_1 + \alpha_2 u v_2 + \dots + \alpha_k u v_k = \sum_{1}^{k} \alpha_i \underbrace{(u v_i)}_{=0} = 0$$

IDENTITA' DEL PARALLELOGRAMMA:

In uno spazio euclideo si verifica

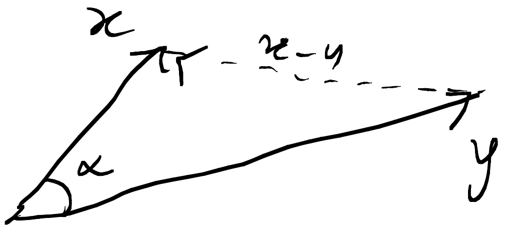
$$|x+y|^2+|x-y|^2=2\left(|x|^2+|y|^2\right)$$




$$\| \qquad\qquad\qquad\qquad\qquad //$$

$$|x|^2+\cancel{2xy}+|y|^2+|x|^2-\cancel{2xy}+|y|^2$$

In un parallelogramma la somma dei quadrati delle diagonali è uguale alla somma dei quadrati dei lati.




$$\boxed{\begin{aligned}|x-y|^2&=|x|^2+|y|^2-2xy=\\&=|x|^2+|y|^2-2|x||y|\cos\alpha\end{aligned}}$$

Th.
CARNOT

$$\boxed{xy = \frac{1}{4}\left(|x+y|^2 - |x-y|^2\right)}$$

$$|x| = (xx)^{1/2}$$

$$\frac{1}{4}\left(\cancel{|x|^2} + 2xy + \cancel{|y|^2} - \cancel{|x|^2} - \cancel{|y|^2} + 2xy\right) = \frac{1}{4}(4xy) = xy$$

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ESEMPIO DI SPAZIO NORMATO

$$X = \mathbb{R}^n = \{ (x_1, \dots, x_n) : x_i \in \mathbb{R} \ \forall i = 1..n \}$$

$$\|x\| = \max\{ |x_1|, |x_2|, \dots, |x_n| \}$$

DIVERSA DA QUELLA SOLITA.

$$\|(1,0)\| = 1$$
$$\|(-1,1)\| = 1$$
$$\|(0,-4)\| = 4$$

1) $\|x\| \geq 0$ SI'   2) $\|x\| = 0 \Rightarrow |x_i| = 0 \ \forall i \Rightarrow x = 0$   3) $\|\lambda x\| = \max\{\dots, |\lambda x_i|, \dots\} =$

$$= \max\{|\lambda||x_1|, |\lambda||x_2|, \dots, |\lambda||x_n|\} = |\lambda| \max\{|x_1|, |x_2|, \dots, |x_n|\}$$

$a < b \quad c > 0$

$ac < bc$

4) $$\|x+y\| = \max\{\dots, |x_i+y_i|, \dots\} \leq$$

$\leq |x_i| + |y_i|$

$$\leq \max\{\dots, |x_i| + |y_i|, \dots\} \leq$$

$$\leq \max\{|x_1| + \max\{|y_i|\}, |x_2| + \max\{|y_i|\}, \dots, |x_n| + \max\{|y_i|\}\}$$

$$\leq \max\{|x_i|\} + \max\{y_i\} \leq \|x\| + \|y\|$$

$b < c$

$a+b < a+c$

Se esistesse un prodotto scalare tale che $\|x\| \overset{?}{=} (x \cdot x)^{1/2}$ varrebbe l'identità del parallelogramma.

Se si pone $x=(0,1)$, $y=(1,0)$ si ha

$$\boxed{\mathbb{R}^2}$$

$$\|x\|=1 \qquad \|y\|=1 \qquad \|x+y\|=\|(1,1)\|=1$$

$$\|x-y\|=\|(-1,1)\|=1$$

$$\|x+y\|^2+\|x-y\|^2=1+1=2$$

FALSA!

$$2\left(\|x\|^2+\|y\|^2\right)=2(1+1)=4$$

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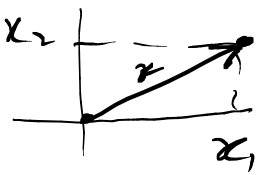
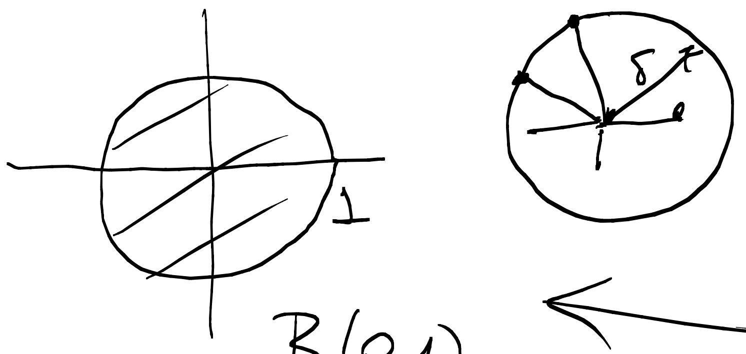
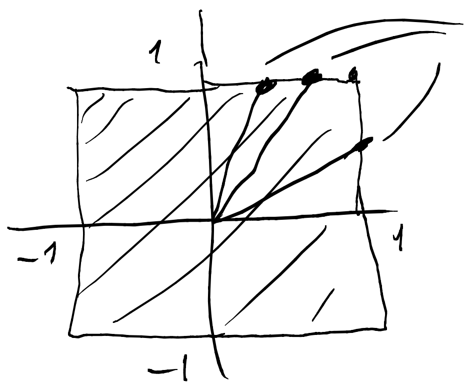
Sfera di centro l'origine e <u>raggio $\delta$</u> è l'insieme

$$B(0,\delta) = \{x\in X : \underbrace{\|x\|}_{\|x-0\|}\le \delta\}$$





$B(0,1)$
usando la norma euclidea




$|x| \leq 1$

$|y| \leq 1$

$\Downarrow$

$(x,y) \in B(0,1)$
con la norma del massimo

$\|x\| = \max\{|x_1|, |x_2|\}$

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Fissato $i$ $\quad |x_i + y_i| \leq |x_i| + |y_i| \leq \max\limits_{1..n} |x_i| + \max\limits_{1..n} |y_i| = \|x\| + \|y\|$

4) valori assoluti in $\mathbb{R}$ $\Downarrow$

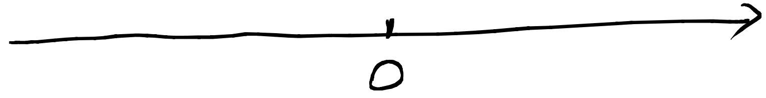
$\forall\, i$ fissato $\quad |x_i + y_i| \leq \|x\| + \|y\|$

$$\|x+y\| = \max_{1..n} |x_i + y_i| = |x_j + y_j| \quad \text{per qualche } j = 1..n$$

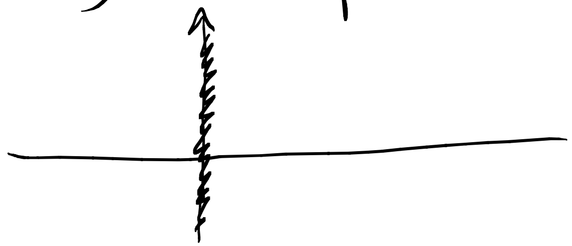
$$\|$$

$$|x_j + y_j| \leq \|x\| + \|y\|$$

$x = 0$ 1) rappresenta l'origine sulla retta




2) rappresenta l'asse $y$ nel piano



3) rappresenta il piano $yz$ nello spazio

$$\boxed{\text{Figure} = \left\{ (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : f(x_1, x_2, \dots, x_n) = 0 \right\}}$$

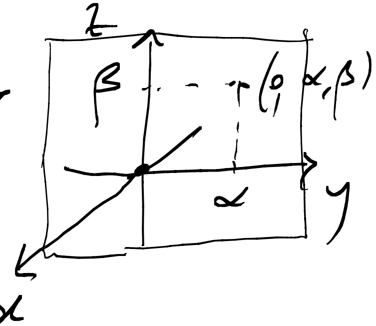
Geometria algebrica

1) $\{ x \in \mathbb{R}^1 : x = 0 \} = \{0\}$

Figure = luogo di soluzioni di equazioni

2) $\{(x_1,x_2)\in\mathbb{R}^2 : x=0\} = \{(0,\alpha) : \alpha\in\mathbb{R}\}$ asse y

3) $\{(x_1,x_2,x_3)\in\mathbb{R}^3 : x=0\} = \{(0,\alpha,\beta) : \alpha,\beta\in\mathbb{R}\}$




$2x+3y=3$ eq. di una retta

$(x,y)\in$ retta $\Longleftrightarrow \boxed{2x+3y=3}$

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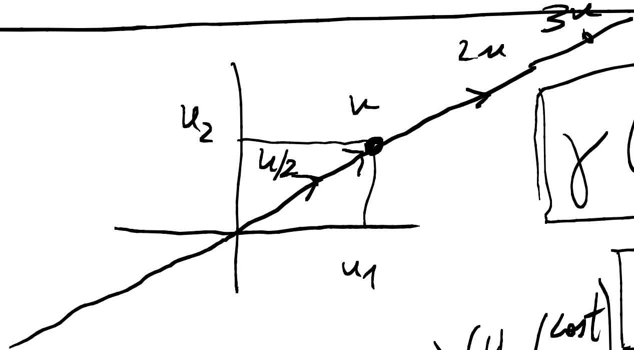
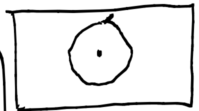
"Equazioni" parametriche

funzioni MATEM.

moti = movimenti FISICA




$$\boxed{\gamma(t) = tu}$$




$\gamma(t)=\begin{pmatrix}\cos t\\ \sin t\end{pmatrix} \longleftarrow t\in[0,2\pi]$

$\gamma : \mathbb{R} \to \mathbb{R}^n$ curva parametrica

$\{ x \in \mathbb{R}^n : \exists t \in \mathbb{R} \quad x = \gamma(t) \}$

$f(t) = t^2 \quad f : \mathbb{R} \to \mathbb{R}$

$\boxed{\operatorname{Im} f = \{ x \geq 0 \}}$

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Eq. param. di una retta per l'origine $\gamma(t) = tu$ $\quad u$ (velocità moto rettil. unif.

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = t \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$\gamma(t)$ ; $u$

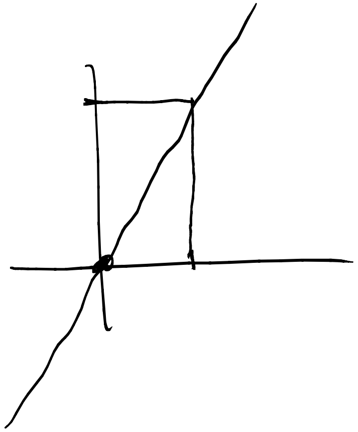
tempo che scorre.

retta per l'origine in direzione di $u$

$$u = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \begin{cases} x = 1 \cdot t = t \\ y = 2t \end{cases} \qquad \gamma(t) = \begin{pmatrix} t \\ 2t \end{pmatrix}$$

$$\begin{cases} x = t \\ y = 2t \end{cases} \Rightarrow t = x \Rightarrow \boxed{y = 2x}$$

eliminando $t$ si ottiene un'eq. cartesiana



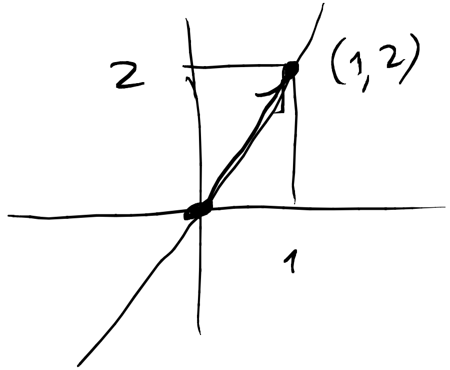
$r \quad 2x - y = 0 \qquad O \in r$

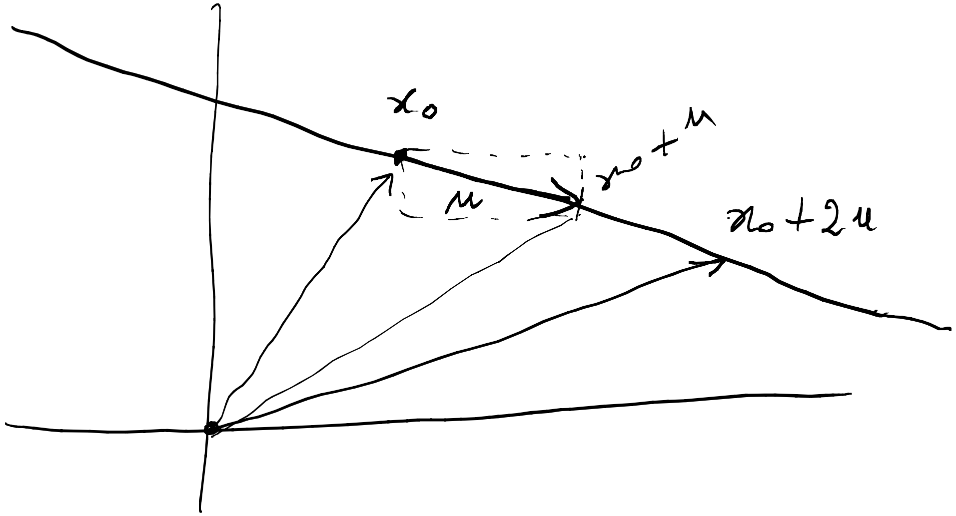
$$y = 2x$$

$x = 1 \Rightarrow y = 2$

$\Downarrow$

$(1,2) \in r$



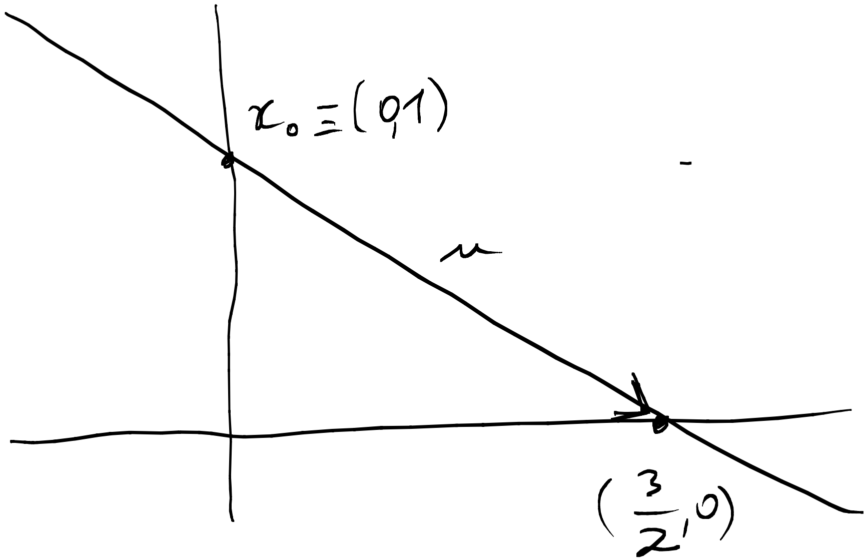



$$\gamma(t) = \begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= x_0 + tu \quad \rightarrow \text{vettoriale}$$

$$x_0 = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{cases} x = a + tu_1 \\ y = b + tu_2 \end{cases} \quad \text{scalare}$$




$$u = \left(\tfrac{3}{2}, 0\right) - (0,1) = \left(\tfrac{3}{2}, -1\right)$$

$$\gamma(t) = (0,1) + t\left(\tfrac{3}{2}, -1\right) \quad \text{vettoriale}$$

$$\begin{cases} x = 0 + \frac{3}{2}t \\ y = 1 - t \end{cases} \quad \text{scalare}$$