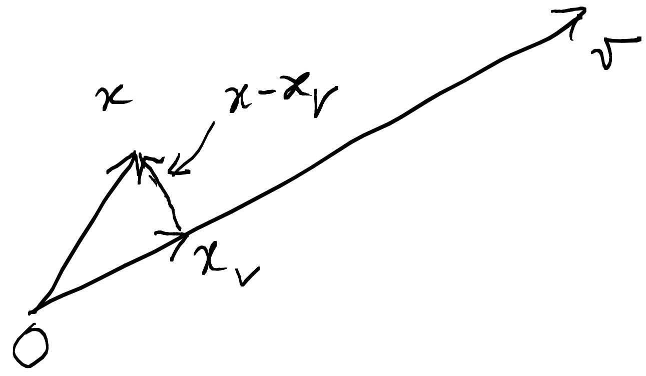
PROPRIETA' DELLA PROIEZIONE

$x \in \mathbb{R}^n$ $\quad v \neq 0 \quad$ $x_v = \underbrace{\boxed{\dfrac{xv}{|v|^2}}}_{\text{scalare}} v$



$$(x - x_v)\, v = 0$$

"resto" della proiezione

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① La proiezione è LINEARE $\qquad P_v(x) = x_v = \dfrac{xv}{|v|^2} v$

a) $P_v(x+y) = P_v(x) + P_v(y)$

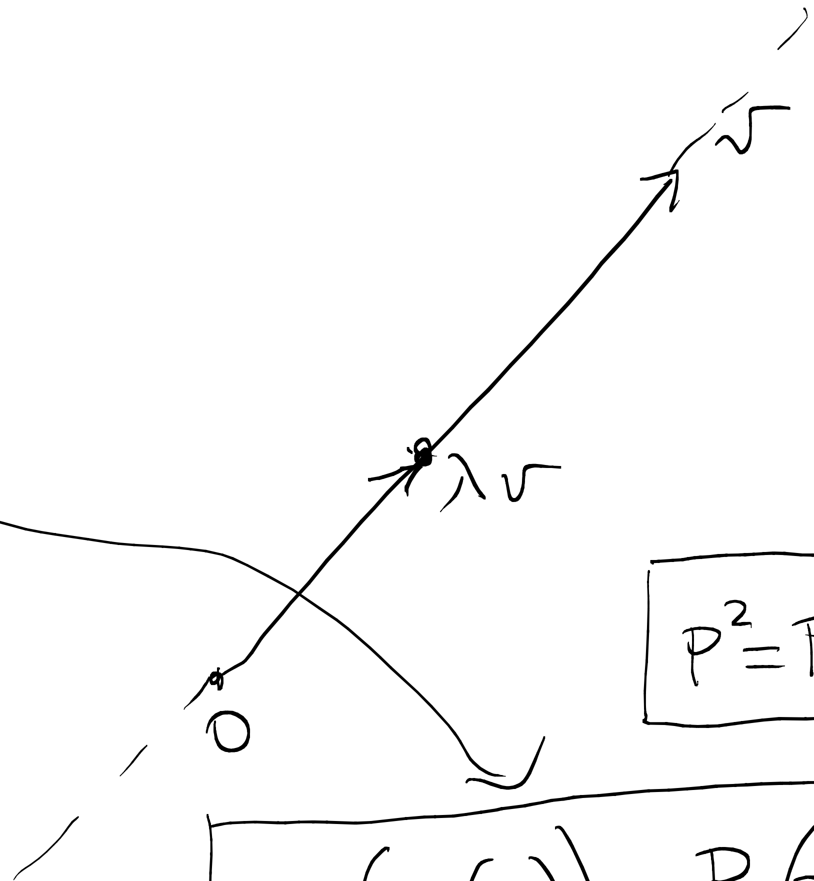
Dim

$$P_v(x+y) = \frac{(x+y)v}{|v|^2} v = \frac{xv + yv}{|v|^2} v = \overbrace{\frac{xv}{|v|^2} v + \frac{yv}{|v|^2} v}^{= P_v(x) + P_v(y)}$$

b) $P_v(\lambda x) = \lambda P_v(x)$

$$\frac{(\lambda x)v}{|v|^2}\, v = \lambda \frac{xv}{|v|^2}\, v$$

IDEMPOTENZA

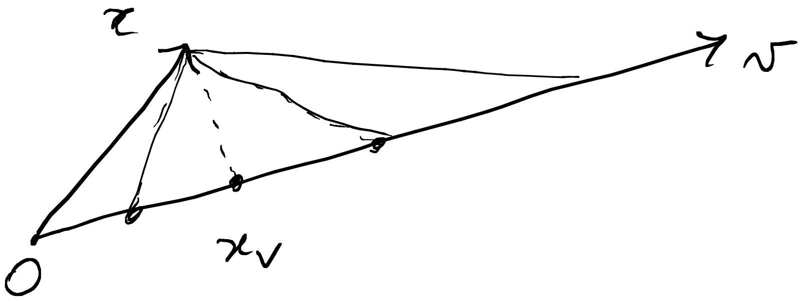


$$\boxed{P^2 = P}$$

<u>Oss</u> $P_v(\lambda v) = \lambda v$

$$= \frac{(\lambda v)v}{|v|^2}\, v = \lambda \cancel{\frac{vv}{|v|^2}}\, v = \lambda v$$

$$\boxed{P_v\big(\underbrace{P_v(x)}_{\alpha v}\big) = \underbrace{P_v(x)}_{\alpha v}}$$



Proprietà di minimo distanza

$\forall v \in \mathbb{R}^n \ \forall \alpha \in \mathbb{R} \qquad |x - \alpha v| \geq |x - x_v|$

$v \neq 0$

$|a+b|^2 = |a|^2 + |b|^2$

se $ab = 0$

$$|x-\alpha v|^2 = |\overbrace{x - x_v}^{a} + \overbrace{x_v - \alpha v}^{b}|^2 \overset{\text{Pitagora}}{=} |x-x_v|^2 + \underbrace{|x_v - \alpha v|^2}_{\geq 0} \geq |x - x_v|^2$$

th. proiezione $\perp v$

è un multiplo di v

sono ortogonali

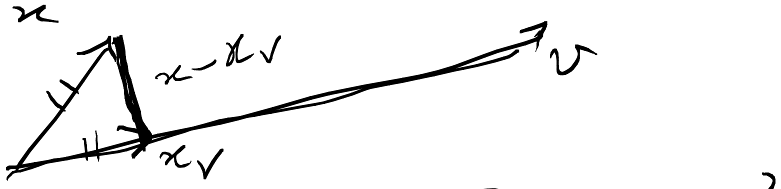
TESI

$\rightarrow \boxed{AL\_2.1}$

Per ogni $\underline{x \in \mathbb{R}^n}$ ed ogni $v \in \mathbb{R}^n$, $v \neq 0$

$$|x_v| \le |x|$$

Proiezione obliqua



DIM.

$$|x|^2 = |\underbrace{x - x_v}_{\perp v} + \underbrace{x_v}_{\text{multiplo di } v}|^2 \overset{\text{Pitagora}}{=} |x-x_v|^2 + |x_v|^2 \ge |x_v|^2$$

$\blacksquare$

Altra dim. della dis. Schwarz.

$$|x| \ge |x_v| = \Big|\underbrace{\frac{xv}{|v|^2}}_{\text{scalare}} v\Big| = \frac{|xv|}{|v|^2}|v| = \frac{|xv|}{|v|}$$

Omogeneità della norma

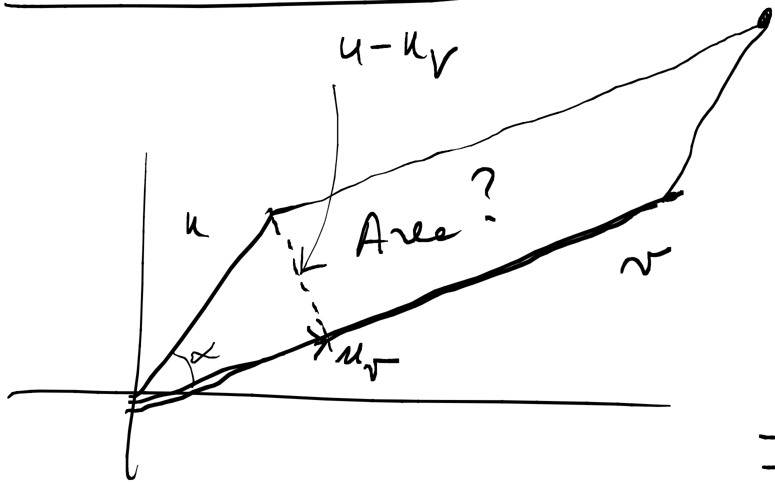
valore assoluto dello scalare xv

norma del vettore v

da cui $|x||v| \geq |xv|$ Schwarz

$v \neq 0$

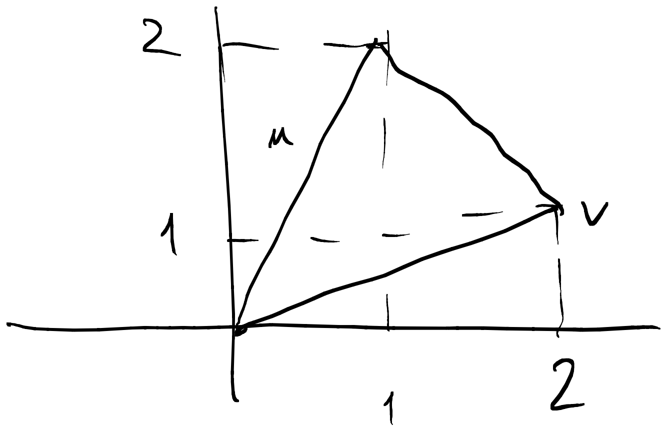
$(x - x_v)v = 0$ Th. proiez. ortogonale



$$\text{Area} = \underset{\text{base}}{|v|} \cdot \underset{\text{altezza}}{|u - u_v|} =$$

$$= |v| \sqrt{|u|^2 - |u_v|^2} =$$

$$= |v| \sqrt{|u|^2 - \frac{|uv|^2}{|v|^2}} = \sqrt{|u|^2|v|^2 - (uv)^2}$$



$u = (1,2) \qquad |u|^2 = 1+4 = 5$

$v = (2,1) \qquad |v|^2 = 4+1 = 5$

$\text{Area (triang.)} = \frac{1}{2}\sqrt{5\cdot 5 - 16} = \frac{3}{2}$

$u\,v = 1\cdot 2 + 2\cdot 1 = 4 \qquad \text{Area del parall.} = 2\,\text{Area triang.}$