Note Title 10/8/2020

1) $xx \geq 0 \quad \forall x \in \mathbb{R}^n$ $\qquad xx = |x|^2 = \sum_1^n x_i^2$

2) $xx = 0 \iff x = 0$

definita positiva

3) $xy = yx$

simmetria

4') $(x+y)z = xz + yz$

$\Longrightarrow (\lambda x)z = \lambda(xz)$

4'') $z(x+y) \overset{3)}{=} (x+y)z = xz + yz \overset{3)}{=}$

$= zx + zy$

$z(\lambda x) \overset{3)}{=} (\lambda x)z = \lambda(xz) \overset{3)}{=} \lambda(zx)$

$e_1 \equiv (1,0,0) \quad e_2 \equiv (0,1,0)$

$(e_1 e_1) e_2 = \underbrace{\big((1,0,0)(1,0,0)\big)}(0,1,0) = 1(0,1,0) = (0,1,0)$

$e_1(e_1 e_2) = (1,0,0)\big((1,0,0)(0,1,0)\big) = (1,0,0)\cdot 0 =$

$= (0,0,0)$

$$\frac{2}{\frac{2}{2}} \overset{?}{=}$$

$$xx = \sum_1^n x_i^2 = |x|^2$$

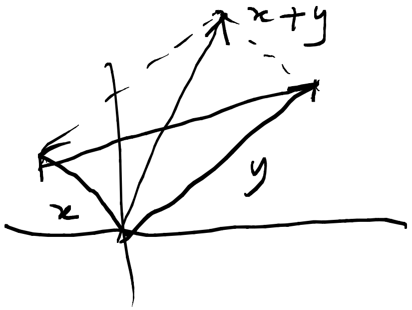
$$(1,0,0)(1,0,0) = |(1,0,0)|^2 = 1$$

$$xy = \sum_1^n x_i y_i$$

$x = (1,0,0)$

$y = (1,0,0)$

$xy = 1 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 = 1$



Hp $\boxed{xy = 0} \equiv x \perp y$ x è ortogonale a y

$$|x+y|^2 = \underbrace{(x+y)}_{z}(x+y) \overset{4')}{=} (x+y)x + (x+y)y \overset{4'')}{=}$$

$$= xx + yx + xy + yy \overset{3)}{=} |x|^2 + |y|^2 + \underbrace{2xy}_{=0}$$

Perché $xy = 0 \Rightarrow |x+y|^2 = |x|^2 + |y|^2$

PITAGORA E' VERO!!!

DISUGUAGLIANZA DI $\boxed{\text{SCHWARZ}}$

$$\forall x, y \in \mathbb{R}^n \qquad |xy| \le |x||y|$$

valore assoluto

norma in $\mathbb{R}^n$

$$|(1,0)\cdot(0,1)| = 0$$

$$|(1,0)| = 1 \qquad |(0,1)| = 1$$

$$|(1,0)(1,0)| = 1$$

$$|(1,0)||(1,0)| = 1$$

$$\boxed{0 \le |x+\lambda y|^2} \qquad \lambda \in \mathbb{R} \quad x, y \in \mathbb{R}^n$$

$$|x+\lambda y|^2 = (x+\lambda y)(x+\lambda y) = |x|^2 + |\lambda y|^2 + x(\lambda y) + (\lambda y)x =$$

$$= \lambda^2 \underbrace{|y|^2}_{a} + \underbrace{2\lambda xy}_{b} + \underbrace{|x|^2}_{c} \ge 0$$

x e y sono fissati

λ è un numero reale arbitrario

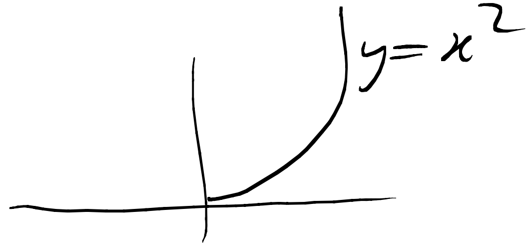
$\boxed{\text{trinomio di II grado in } \lambda \text{ sempre } \ge 0} \iff \Delta \le 0$

$\Delta \quad b^2 - 4ac$

$\Delta/4 = \left(\frac{b}{2}\right)^2 - ac$

$\boxed{\text{AL\_2.1}}$

$$\frac{\Delta}{4} = (xy)^2 - |x|^2|y|^2 \le 0$$



$$(xy)^2 \le |x|^2|y|^2$$

Passando alla radice

$$|xy| \le |x||y| \quad \text{(la tesi!)}$$

$$xy = |x||y|\cos\widehat{xy}$$

$$|xy| = |x||y|\underbrace{|\cos\widehat{xy}|}_{\le 1}$$

$ax^2+bx+c=0$ ha $\underline{\underline{2}}$ radici x_1, x_2

$\Delta > 0$

$a>0$

$a(x-x_1)(x-x_2) > 0$ fuori da $[x_1, x_2]$

< 0 dentro $]x_1, x_2[$

$$\underset{\ge 0}{|x+y|} \le \underset{\ge 0}{|x|+|y|} \iff \underbrace{|x+y|}_{\text{vettori}}^2 \le \Big(\underbrace{|x|+|y|}_{\text{scalari}}\Big)^2 = |x|^2+|y|^2+2|x||y|$$

$$\Leftrightarrow \quad \cancel{|x|^2} + \cancel{|y|^2} + \cancel{2}\underbrace{xy} \leq \cancel{|x|^2} + \cancel{|y|^2} + \cancel{2}|x||y|$$

vettori

prodotto scalare

$$\Updownarrow$$

$$\boxed{xy \leq |x||y|}$$ Vero perché

Schwarz

$$xy \leq |xy| \leq |x||y| \implies$$ $\mathbb{R}^n$ è uno spazio normato colla norma $|x| = \sqrt{xx}$

valore assoluto

$a \leq |a| \ \forall a \in \mathbb{R}$

$$|xy| = |x||y|$$

$xy = |x||y| \cos \widehat{xy}$

$\Longrightarrow$ $|\cos \widehat{xy}| = 1$

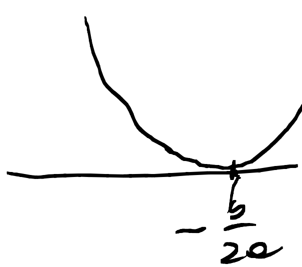
se uno dei vettori è nullo

oppure se $\longrightarrow$ x è multiplo di y

$$|xy| = |x||y| \iff \frac{\Delta}{4} = 0$$

Cosa accade al trinomio $|x+\lambda y|^2$ se il suo discriminante è nullo?

Il trinomio ha uno zero (unico)



$ax^2+bx+c=0$

$x = -\frac{b}{2a}$ se $\Delta = 0$ e solo se

$\Updownarrow$

$|x+\lambda y|^2 = 0$ per un opportuno λ

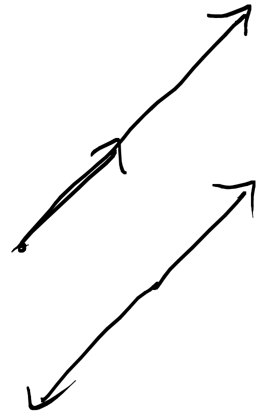
$\Downarrow$

$|x+\lambda y| = 0$

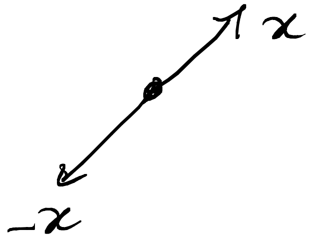
$\Downarrow$

$x+\lambda y = 0$ e quindi x è multiplo di y

Vale l'uguaglianza



$$\boxed{\text{Quando accade } |x+y| = |x|+|y| \text{ ?}}$$



$x \quad y = -x \Rightarrow |x+y| = |x-x| = 0$

$\neq 0 \quad \neq 0$

$|x|+|y| = |x|+|-x| = 2|x| \neq 0$

In realtà $|x+y| \le |x|+|y|$

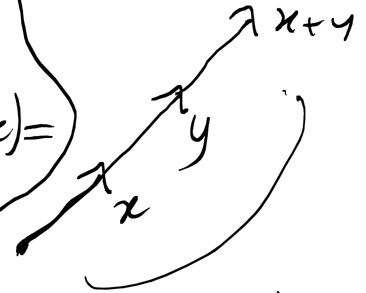
$\Updownarrow$

$$\boxed{xy \le |x||y|}$$

$x(-x) = -(xx) =$

$= -|x|^2$

$x(-x) \neq |x|^2$



$x = \frac{1}{2} y$

$|x+y| = |x|+|y| \iff xy = |x||y| \nearrow xy \ge 0$

$\Downarrow$

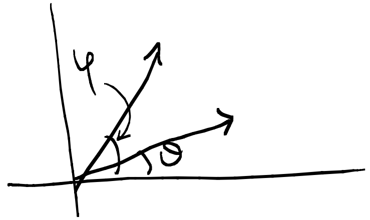
$|xy| = |x||y| \Rightarrow$

x e y sono allineati

Quando $xy \ge 0$ e $x = \lambda y$ $\quad xy = (\lambda y)y = \lambda |y|^2 \Rightarrow \lambda > 0$

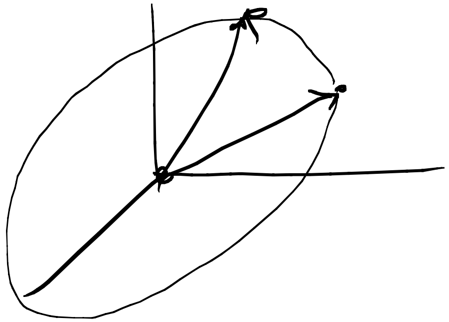
Perché $uv = |u||v|\underbrace{\cos \widehat{uv}}$?

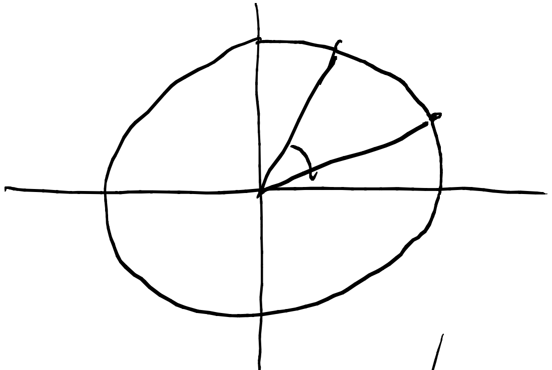
$|u| = |v| = 1$ <u>Versori</u>

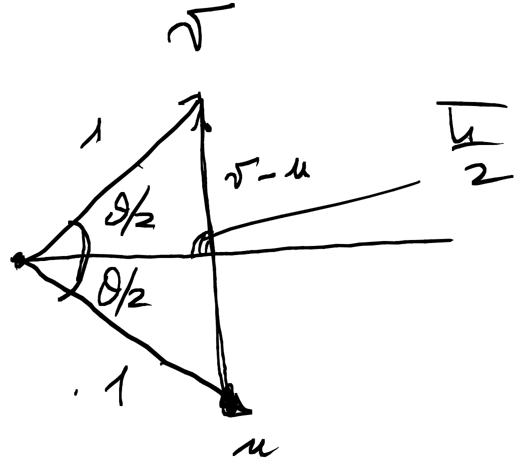


$uv = \cos(u - v)$

$$\cos(\varphi - \theta) = \cos\varphi \cos\theta + \sin\varphi \sin\theta = \underline{(\cos\varphi, \sin\varphi)(\cos\theta, \sin\theta)}$$







la misura del segmento $v - u$ è $|v - u|$

$$\frac{|v-u|}{2} = \underset{1}{|v|} \sin\frac{\theta}{2}$$

Passando ai quadrati

$$|v-u|^2 = 4\sin^2\frac{\theta}{2}$$

$$\underset{\substack{\| \\ 1}}{|v|^2} + \underset{\substack{\| \\ 1}}{|u|^2} - 2uv = 4\sin^2\frac{\theta}{2}$$

$$-2uv = -2 + 4\sin^2\frac{\theta}{2}$$

$$uv = \underbrace{1 - 2\sin^2\frac{\theta}{2}} = \cos\theta$$

$$\underline{\cos\theta} = \cos\left(2\left(\frac{\theta}{2}\right)\right) =$$

$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} =$$

$$= \underline{1 - 2\sin^2\frac{\theta}{2}}$$

In generale si pone $\forall u,v \in \mathbb{R}^n$ $|u|=|v|=1$

$$\cos\widehat{uv} = uv$$

$u, v \in \mathbb{R}^n$ e siano $\hat{u} = \frac{u}{|u|}$ e $\hat{v} = \frac{u}{|v|}$ multipli positivi di u, v

$u, v \neq 0$

$\cos(\alpha) = \cos(-\alpha)$

$$\boxed{\cos \widehat{uv} = \cos \widehat{\hat{u}\hat{v}} = \frac{u}{|u|}\,\frac{v}{|v|} = \boxed{\frac{uv}{|u||v|}}}$$

DEFINIZ.

$\Updownarrow$ u e $v \neq 0$ $\quad \widehat{\hat{u}\hat{v}}$

$$uv = |u||v| \cos \widehat{uv}$$

$$\cos \widehat{(1,1,0)(0,1,1)} = \frac{1\cdot 0 + 1\cdot 1 + 0\cdot 1}{\sqrt{2}\,\sqrt{2}} = \frac{1}{2}$$

$$\cos \widehat{\underset{u}{(1,2,2,-3)}\underset{v}{(0,1,3,-2)}} = \frac{\overbrace{1\cdot 0 + 2\cdot 1 + 2\cdot 3 + (-3)(-2)}^{uv}}{\sqrt{1+4+4+9}\,\sqrt{1+9+4}} =$$

Calcolo del coseno