NORMA, PRODOTTO SCALARE, PROIEZIONE

Note Title 10/2/2020

$\mathbb{R}^n \qquad 0 \equiv (0, 0, \ldots, 0) \qquad -x \equiv (-x_1, -x_2, \ldots, -x_n)$

$x + y \equiv (\ldots, x_i + y_i, \ldots) \qquad \underline{\lambda x \equiv (\lambda x_1, \lambda x_2, \ldots, \lambda x_n)}$

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1) $\underset{\text{scalare}}{0}\, x = \underset{\text{vettore}}{0} \quad \underline{\forall x \in \mathbb{R}^n}$

$\underset{\text{scalare}}{0}\, x = (0x_1, 0x_2, 0x_3 \ldots 0x_n) = $

$= (0, 0, \ldots, 0) = \underset{\text{vettore}}{0}$

2) $\boxed{\lambda \underset{\text{vettore}}{0}} = \underset{\text{vettore}}{0} \quad \forall \lambda \in \mathbb{R} \quad 0 \in \mathbb{R}^n$

$\underline{\lambda 0} \equiv (\overbrace{\lambda 0, \lambda 0, \ldots \lambda 0}^{\text{scalari}})$

$\underline{0 \quad\; 0 \quad\; \ldots \quad 0}$

3) $\lambda x = 0 \Longrightarrow \underline{\lambda = 0}$ oppure $\underline{x = 0} \qquad \lambda \in \mathbb{R} \quad x \in \mathbb{R}^n$

$$(0,0,\dots,0) = (\lambda x_1, \lambda x_2, \dots, \lambda x_n)$$

$$\Downarrow$$

$$\boxed{\lambda x_1 = 0, \ \lambda x_2 = 0, \dots, \lambda x_n = 0}$$

$\Downarrow$ legge di annullamento del prodotto in $\mathbb{R}$

$$\underline{\lambda = 0} \quad \text{o} \quad (\text{se } \lambda \neq 0) \Rightarrow x_1 = x_2 = \dots = x_n = 0$$

scalare

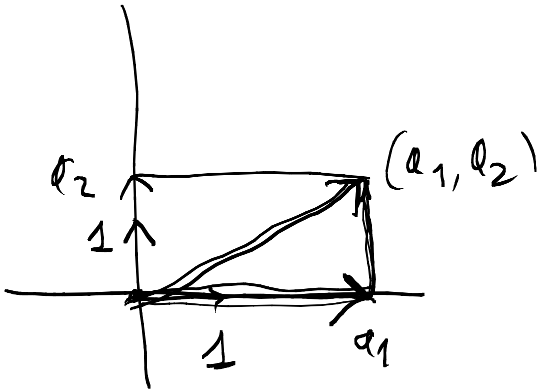
$$\underline{(x_1, x_2 \dots x_n) = 0}$$

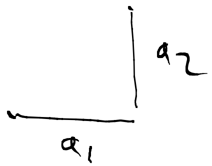
vettore

$$a \equiv (a_1, a_2) = \underline{(a_1, 0)} + \underline{(0, a_2)} = a_1 \underline{(1, 0)} + a_2 \underline{(0, 1)}$$

spostam. verso destra

verso l'alto





$$a \equiv (a_1, a_2, \dots, a_n) =$$

$$= a_1 \underbrace{(1, 0, \dots, 0)}_{e_1} + a_2 \underbrace{(0, 1, 0 \dots 0)}_{e_2} + \dots$$

$$\dots + a_n \underbrace{(0, \dots, 0, 1)}_{e_n}$$

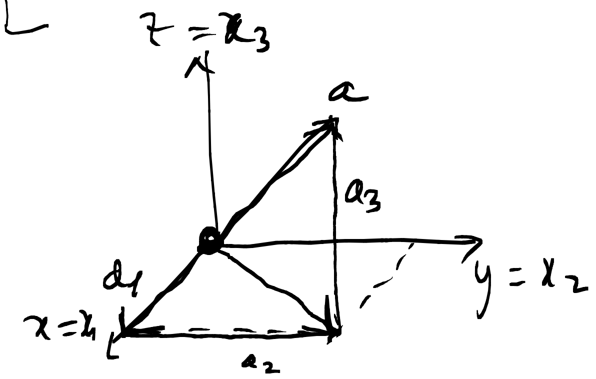
BASE CANONICA

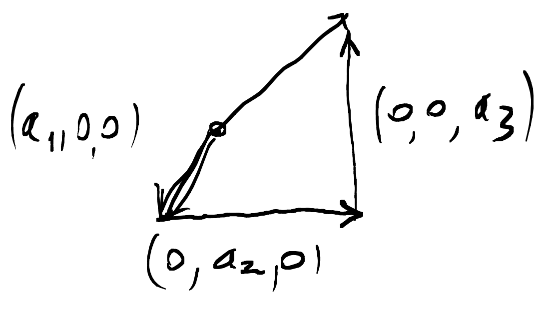
$$\left[\begin{array}{l} e_1 \equiv (1, 0, \dots, 0) \\ e_2 \equiv (0, 1, 0 \dots 0) \\ \vdots \\ e_n \equiv (0, \dots, 0, 1) \end{array}\right.$$

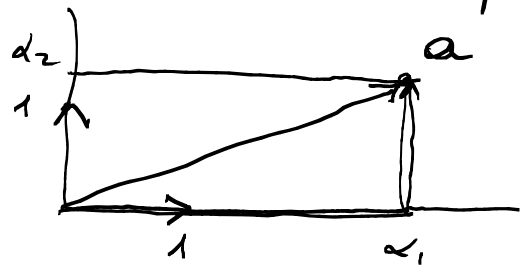
$$\boxed{\forall a \in \mathbb{R}^n \ \exists \alpha_i \in \mathbb{R} : \ ? \quad a = \sum_1^n \alpha_i e_i}$$

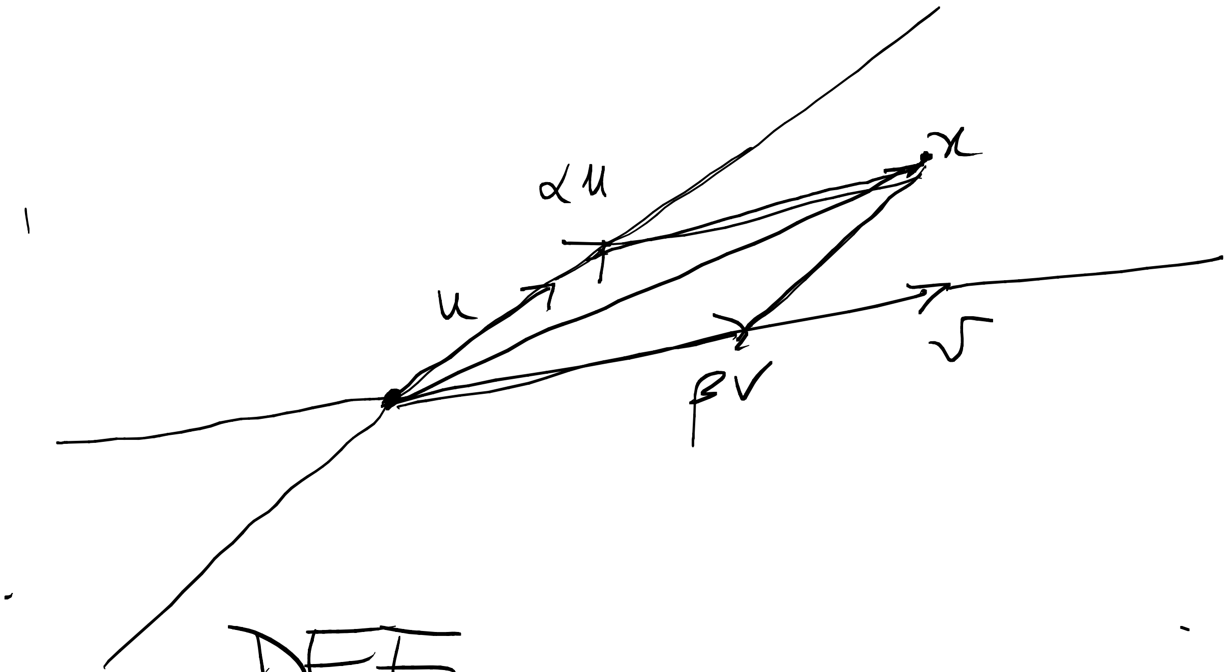
vero se si pone $\alpha_i = a_i$

componenti di a









Dati x, $\underline{\underline{u, v}}$

$\exists\ \alpha, \beta$:

$x = \alpha u + \beta v$

stessa retta di u — stessa retta di v

<u>**DEF**</u>

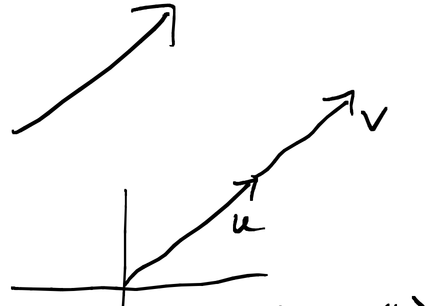
Si dice combinazione lineare di $u_1 \dots u_k \in \mathbb{R}^n$ ogni somma di loro multipli. L'insieme delle combinazioni lineari di $u_1 \dots u_k$ si chiama spazio generato (o span) da $u_1 \dots u_k$, che sono detti generatori dello spazio. Lo spazio si denota con $\langle u_1, \dots, u_k \rangle$.

In simboli $x \in \langle u_1, \dots, u_n \rangle$ vuol dire che

$$\to \exists\, \alpha_1, \dots, \alpha_n \in \mathbb{R} : \quad x = \sum_{1}^{n} \alpha_i u_i$$

$$\langle u_1, \dots, u_n \rangle = \left\{ \sum_{1}^{n} \alpha_i u_i \ : \ \alpha_i \in \mathbb{R} \right\}$$

$$\mathbb{R}^2 = \langle e_1, e_2 \rangle$$

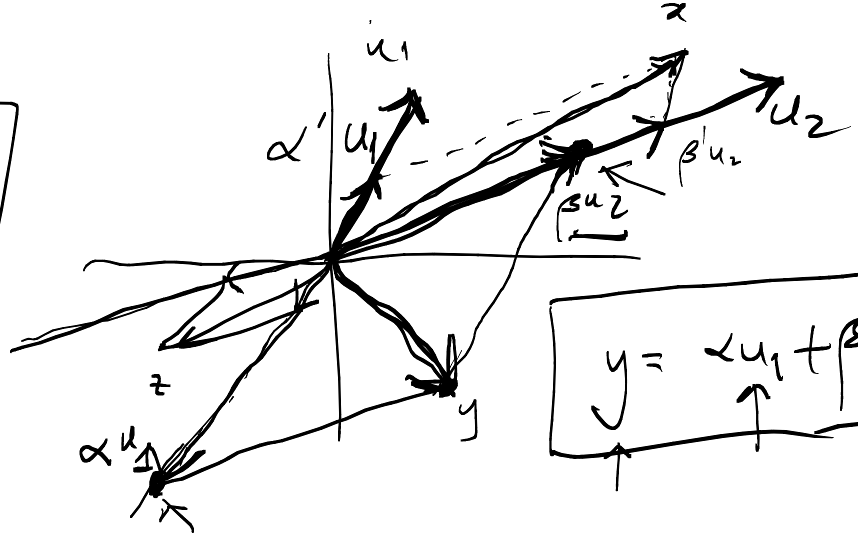


$$\langle u, v \rangle = \langle u \rangle$$

$$\alpha u + \beta u = (\alpha + \beta) u$$

$$\langle u_1, u_2 \rangle = \mathbb{R}^2$$

$$\forall x \in \mathbb{R}^2 \ \exists\, \alpha, \beta : \ x = \alpha u_1 + \beta u_2$$



$$y = \alpha u_1 + \beta u_2$$

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

coeff. $\quad a\,\alpha = x$

incognita $\quad$ termine noto

$$x \in \langle a, b \rangle \, ?$$

$$\Updownarrow$$

$$\exists\, \alpha, \beta \in \mathbb{R} : \quad \underbrace{\underset{\in\mathbb{R}^2}{x} = \overset{\in\mathbb{R}}{\alpha}\, \underset{\in\mathbb{R}^2}{a} + \overset{\in\mathbb{R}}{\beta}\, \underset{\in\mathbb{R}^2}{b}}\, ?$$

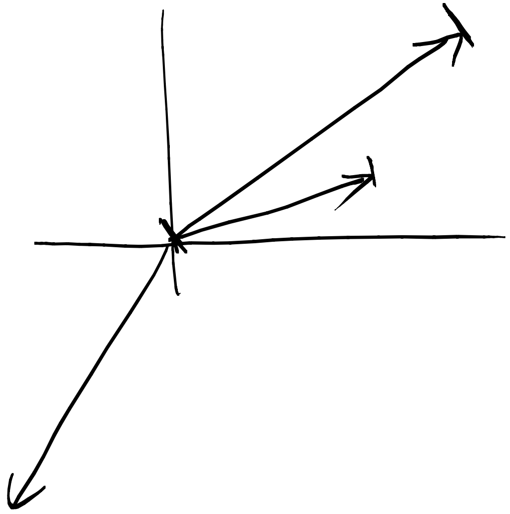
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \alpha \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \beta \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \alpha a_1 \\ \alpha a_2 \end{pmatrix} + \begin{pmatrix} \beta b_1 \\ \beta b_2 \end{pmatrix} = \begin{pmatrix} \alpha a_1 + \beta b_1 \\ \alpha a_2 + \beta b_2 \end{pmatrix}.$$

$$\boxed{\begin{cases} \alpha a_1 + \beta b_1 = x_1 \\ \alpha a_2 + \beta b_2 = x_2 \end{cases}}$$

sistema di 2 eq. in 2 incognite α, β

$\boxed{\text{AL\_11}}$

MODULO ≡ NORMA ≡ LUNGHEZZA



$\forall x \in \mathbb{R}^n$ la NORMA DI x

(simbolo "commerciale" è $|x|$, $\|x\|$)

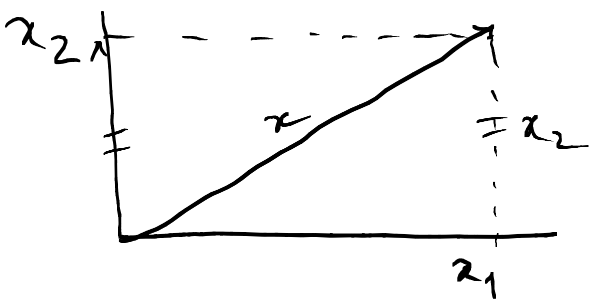
si definisce ponendo

$$|x| = \left(x_1^2 + x_2^2 + \cdots + x_n^2\right)^{1/2} = \left(\sum_1^n x_i^2\right)^{1/2}$$

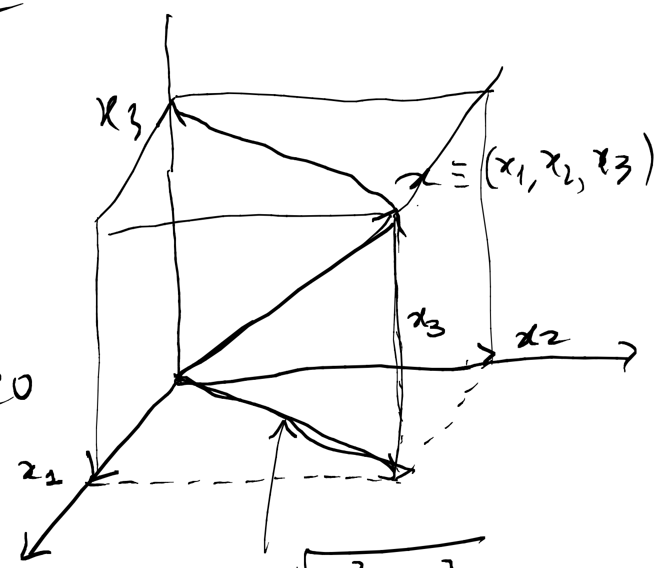
$$\underset{\in \mathbb{R}^2}{x = (1,2)} \qquad \|x\| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\underset{\in \mathbb{R}^4}{|(1,-1,3,0)|} = \sqrt{1^2 + (-1)^2 + 3^2 + 0^2} = \sqrt{11}$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2$$



$$|x| = \sqrt{x_1^2 + x_2^2}$$



$$\sqrt{x_1^2 + x_2^2}$$

① $\forall x \in \mathbb{R}^n \quad |x| \geq 0$

$\sqrt{\sum_1^n x_n^2}$ è definita e ≥ 0

② $|x| = 0 \Leftrightarrow x = 0$

$$|x| = \left(\sum_1^n x_1^2 \right)^{1/2}$$

$$\geq 0$$

$$\Downarrow$$

$$\left(\sum_1^n x_i^2 \right)^{1/2} = 0$$

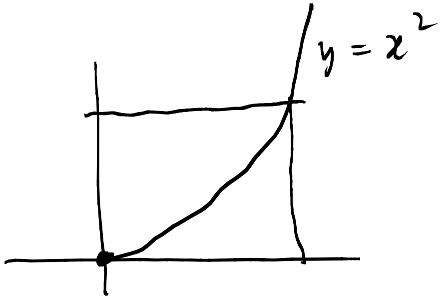
$$\Updownarrow$$

$$\sum_1^n x_i^2 = 0$$

$$\|$$

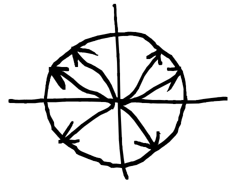
$$\underset{\geq 0}{x_1^2} + \underset{\geq 0}{x_2^2} + \dots + \underset{\geq 0}{x_n^2} = 0 \Rightarrow x_i^2 = 0 \quad \forall i = 1..n$$

$$\Longleftrightarrow |x_i| = 0 \Rightarrow x_i = 0 \; \forall i$$



$$|x| = \sqrt{\left(\sqrt{x_1^2 + x_2^2}\right)^2 + x_3^2} =$$

$$= \sqrt{x_1^2 + x_2^2 + x_3^2}$$

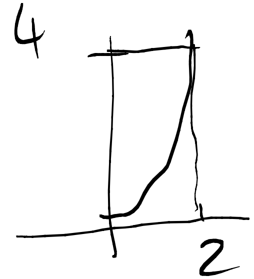


③ $|\lambda x| = |\lambda||x| \quad \forall \lambda \in \mathbb{R}\ \forall x \in \mathbb{R}^n$

$$|\lambda x| = \sqrt{(\lambda x_1)^2 + (\lambda x_2)^2 + \dots + (\lambda x_n)^2} = \sqrt{\lambda^2 (x_1^2 + x_2^2 + \dots + x_n^2)} =$$

$$= \sqrt{\lambda^2}\ \sqrt{\sum_1^n x_i^2} = |\lambda||x|$$

$$\boxed{\sqrt{\lambda^2} = |\lambda|} \qquad \sqrt{(-2)^2} = \sqrt{4} = 2$$



④ $|x+y| \le |x| + |y| \quad \forall x, y \in \mathbb{R}^n$

$\mathbb{R}^1 \simeq \mathbb{R}$

Uno spazio vettoriale si dice NORMATO se su di esso è definita una norma

Se $|u| = 1$ u si chiama anche versore (VERSORE)

$u \neq 0$

$$\boxed{\underbrace{\hat{u}}_{\text{VERSORE di } u} = \frac{1}{\underbrace{|u|}_{\text{scalare}}} \underbrace{u}_{\text{vettore}}} = \frac{u}{|u|}$$

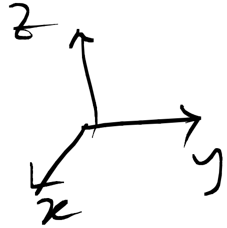
$$|\hat{u}| = \Big|\underbrace{\frac{1}{|u|}}_{\lambda} u\Big| = \Big|\frac{1}{|u|}\Big| |u| = \frac{1}{|u|} |u| = 1$$

SCALARI

$u = (1,1,1)$ $\quad \hat{u} = \frac{1}{\sqrt{3}}(1,1,1) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

$|u| = \sqrt{1+1+1} = \sqrt{3}$

FISICA



$\hat{x} = (1,0,0)$ $(= e_1$ Mat$)$

$\hat{y} = (0,1,0)$ $= e_2$ Mat

$\hat{z} = (0,0,1)$ $(e_3$ Mat$)$