$|xy| \leq |x||y|$ Schwarz $\forall x, y \in \mathbb{R}^n$

1) $|x+\lambda y|^2 \geq 0$

$|(x+\lambda y)|^2 = (x+\lambda y)(x+\lambda y) = |x|^2 + \lambda^2 |y|^2 + 2\lambda xy$

$\boxed{\forall x, y}$

$\underbrace{|y|^2 \lambda^2 + 2xy\,\lambda + |x|^2}$ ≥ 0 trinomio di II grado in λ

$\uparrow$

$\frac{\Delta}{4} \leq 0$

$(xy)^2 - |x|^2 |y|^2 \leq 0$

$(xy)^2 \leq |x|^2 |y|^2$

$\frac{\Delta}{4} = (xy)^2 - |x|^2 |y|^2$

passando alla radice

$|xy| \leq |x||y|$

$\boxed{AL\_2.1}$

$\forall v \neq 0 \qquad |u_v| \leq |u| \quad \forall u \in \mathbb{R}^n$

$$|u|^2 = |\underbrace{u - u_v}_{\text{perpendic. a } v} + \underbrace{u_v}_{\substack{\text{parallelo a } v \\ \text{(un multiplo)}}}|^2 \quad \text{P.tagora} \quad \underbrace{|u-u_v|^2}_{\geq 0} + |u_v|^2 \geq |u_v|^2$$

(Th. proiez.)

sono ortogonali

ne discende Schwarz.

$$|u_v| = \left| \frac{uv}{|v|^2} v \right| = \frac{|uv|}{|v|^2} \cdot |v| = \frac{|uv|}{|v|}$$

$$\frac{|uv|}{|v|} \leq |u| \implies \text{Schwarz.}$$

$X = \langle u_1, u_2, \dots, u_n \rangle \quad Y = \langle v_1, \dots, v_m \rangle \quad Z = \langle w_1 \dots w_p \rangle$

$$\boxed{X + Y + Z \underset{?}{=} X \oplus Y \oplus Z \quad ?}$$

$x \in X \quad y \in Y \quad z \in Z \qquad x + y + z = 0 \Rightarrow x = y = z = 0$

$\exists \alpha_i : x = \sum \alpha_i u_i \quad \exists \beta_j : y = \sum \beta_j v_j \quad \exists \gamma_h : z = \sum \gamma_h w_h$

$$\sum \alpha_i u_i + \sum \beta_j v_j + \sum \gamma_h w_h = 0 \qquad (*)$$

La somma è diretta se per ogni soluzione

$(\overline{\alpha}_1, \dots, \overline{\alpha}_n, \overline{\beta}_1, \dots, \overline{\beta}_m, \overline{\gamma}_1, \dots, \overline{\gamma}_p)$ di $(*)$

accade $\sum_1^n \overline{\alpha}_i u_i = 0 \quad \sum_1^m \overline{\beta}_j v_j = 0$ e $\sum_1^p \overline{\gamma}_h w_h = 0$

$A(u) = u'$ $\qquad \langle \underset{e_1}{\sin t}, \underset{e_2}{\cos t} \rangle \qquad \{\sin t, \cos t\}$

$\langle \underset{e'_1}{\sin t}, \underset{e'_2}{\cos t} \rangle$

$$A(e_1) = A(\sin t) = (\sin t)' = \cos t = 0 \sin t + 1 \cos t$$

$$A(e_2) = A(\cos t) = (\cos t)' = -\sin t = -1 \sin t + 0 \cos t$$

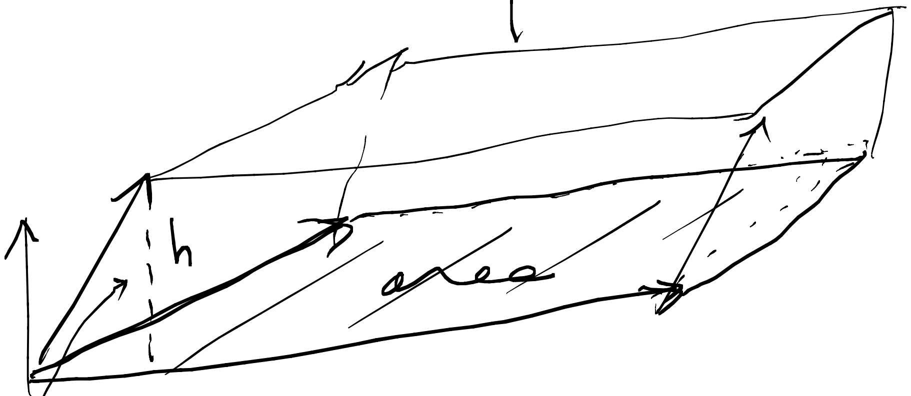
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$\boxed{A(u) = u' \quad \{\sinh t, \cosh t\}}$

Diagonalizzabilità in pratica (II) AN_7.9

u, v, w

$$\text{vol} = \left| \det \begin{pmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{pmatrix} \right|$$



$$\text{Area} = \sqrt{|u|^2 |v|^2 - (u \cdot v)^2}$$

parallelogramma di lati

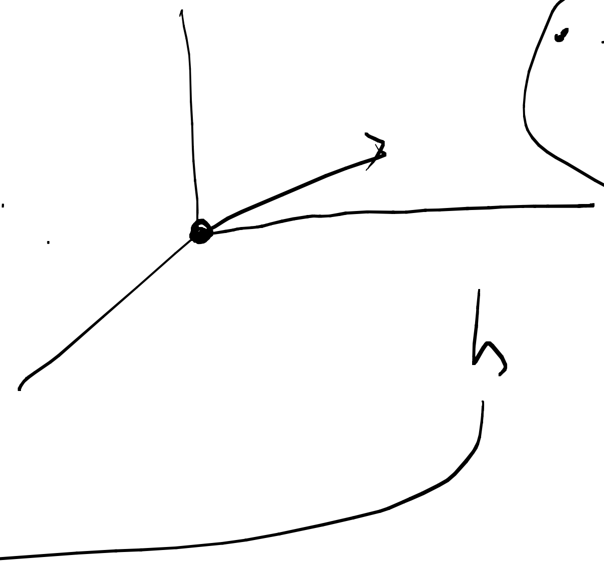
Area $\big((1,0,0,0), (1,0,1,0)\big)$

- la norma della proiezione di $(1,1,1,1) \perp \langle (1,0,0,0), (1,0,1,0) \rangle$

$(1,0,0,0)$

$(1,0,1,0)$

$(1,1,1,1)$



h