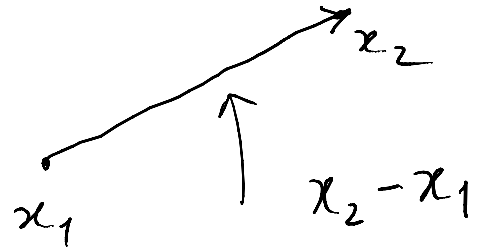
Note Title 9/9/2020

$x_1 \quad x_2 \in \mathbb{R}^n$



segmento di estremi x_2 e x_1

$$\left\{ \begin{array}{l} x_1 + \lambda(x_2 - x_1) \qquad \lambda \in [0,1] \\ (1-\lambda)x_1 + \lambda x_2 \end{array} \right.$$

Ω è stellato se $\exists\, x_0 \in \Omega : \forall x_1 \in \Omega \ \ \overline{x_0 x_1} \subseteq \Omega$.

ALTERN.:
$(1-\lambda)x_0 + \lambda x_1 \in \Omega$
$\rightarrow x_0 + \lambda(x_1 - x_0) \in \Omega$

$\gamma(t) \quad t \in [0,1]$, continua

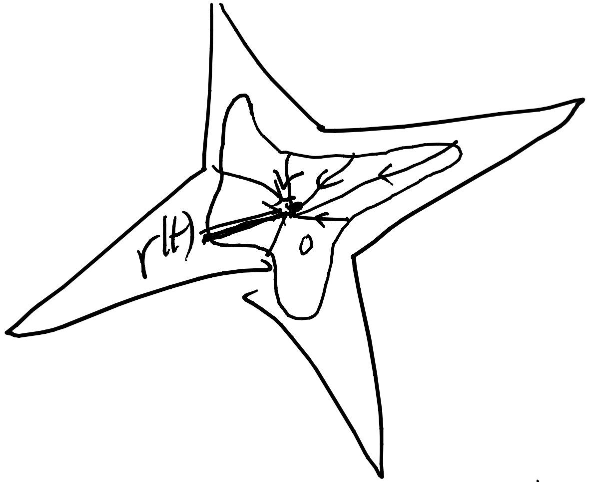
$\gamma(t) \in \Omega \ \ \forall t \in (0,1) \quad \gamma(0) = \gamma(1)$

$\underline{x_0 = 0}$ $\qquad h(\lambda,t) = \underline{(1-\lambda)}\,\underline{\gamma(t)}$

— $\underline{h \text{ è continua}}$ su $[0,1]\times[0,1]$

— $\underline{h(\lambda,t)\in\Omega}$ $\forall\, \lambda, t \in [0,1]$

$(1-\lambda)\gamma(t)$ è un punto del segmento di estremi 0 e $\gamma(t)$ e quindi appartiene ad Ω (che è stella rispetto a 0).



— $\lambda = 0$ $\quad h(\lambda,t) = \gamma(t)$ ←

$\lambda = 1$ $\quad h(\lambda,t) = 0$ ←

$$\boxed{h(\lambda,t) = \underbrace{x_0 + (1-\lambda)(\gamma(t)-x_0)}_{\text{è un punto del segm. di estremi } x_0 \text{ e } \gamma(t)}}$$

$\phi : \Delta \to \mathbb{R}^3$ — $\phi_u \times \phi_v \neq 0$ su Δ

— $\phi \in C^1$

— ϕ sia iniettiva su $\mathring{\Delta}$

Area di ϕ $\equiv \int_\Delta |\phi_u(u,v) \times \phi_v(u,v)|\, du\, dv$

l'area di una superficie parametrica.

$f : \Omega \to \mathbb{R}$ $\quad \Omega \subseteq \mathbb{R}^3$ $\quad \phi : \Delta \to \Omega$ sup. regolare

$$\int_\phi f \, d\sigma \equiv \int_\Delta f(\phi(u,v)) \underbrace{|\phi_u(u,v) \times \phi_v(u,v)|\, du\, dv}_{dS}$$

Integrale superficiale

$$\phi(u,v) = \begin{pmatrix} u\cos v \\ u\sin v \\ v \end{pmatrix}$$

$\underline{\left(0, 0, \frac{\pi}{4}\right)} \in$ codominio di ϕ

$$\nu(u,v) = \phi_u(u,v) \times \phi_v(u,v)$$

$$\phi_u = \begin{pmatrix} \cos v \\ \sin v \\ 0 \end{pmatrix}_{\substack{u=0 \\ v=\pi/4}} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\phi_v = \begin{pmatrix} -u\sin v \\ u\cos v \\ 1 \end{pmatrix}_{\substack{u=0 \\ v=\frac{\pi}{4}}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} u_0\cos v_0 = 0 \\ u_0\sin v_0 = 0 \\ v_0 = \frac{\pi}{4} \end{cases}$$

$\exists (u_0, v_0) \in \Delta$:

$\phi(u_0, v_0) = \left(0, 0, \frac{\pi}{4}\right)$?

$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} \neq 0 \Rightarrow$

l'unica soluzione è $\underbrace{\left(0, \frac{\pi}{4}\right)}_{(u_0, v_0)} =$

$$\nu\left(0, \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \,/\!/\, (1, -1, 0)$$

Piano per $\left(0,0,\frac{\pi}{4}\right)$ normale a $(1,-1,0)$

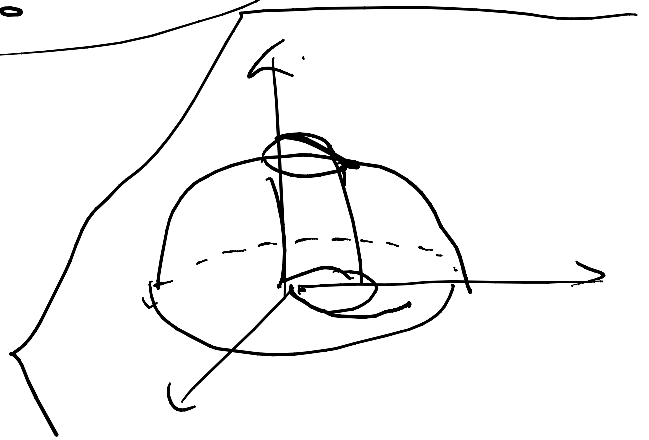
$$1(x-0)-1(y-0)+0\left(z-\frac{\pi}{4}\right)=0$$

$$x-y=0$$

$z=0$ $\qquad x^2+y^2=4y$ $\qquad \underbrace{x^2+y^2+z^2=16}$

$$0 \le z \le \underbrace{\sqrt{16-x^2-y^2}}_{\ge 0}$$

$$\int_{x^2+y^2-4y\le 0} \sqrt{16-x^2-y^2}\,dx\,dy =$$



$$= \int_0^{\pi} d\theta \int_0^{4\sin\theta} d\rho\, \underset{\text{jacobiano}}{\rho} \sqrt{16-\rho^2} =$$

$x^2+y^2-4y \le 0$

$\rho^2 - 4\rho\sin\theta \le 0$

$\rho < 4\sin\theta \Rightarrow \sin\theta > 0$

$\theta \in [0, \pi]$

$$= -\frac{1}{2} \int_0^{\pi} d\theta \underbrace{\int_0^{4\sin\theta} d\rho\, (-2\rho) \sqrt{16-\rho^2}}_{} =$$

$$= \int_{16}^{16\cos^2\theta} \sqrt{t}\, dt = \frac{2}{3} t^{3/2} \Big|_{16}^{16\cos^2\theta} = \frac{2}{3}\left[(16\cos^2\theta)^{\frac{3}{2}} - 16^{\frac{3}{2}} \right] =$$

$16-\rho^2 = t$

$-2\rho\, d\rho = dt$

$$= \frac{2}{3}\left[16\cos^2\theta\sqrt{16\cos^2\theta} - 64\right]$$

$$4\cos^2\theta\,|\cos\theta|$$

$$\sqrt{\cos^2\theta} = |\cos\theta|$$

$$\iint_{x^2+y^2-2y\le 0} \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy = \int_0^{\pi} d\theta \int_0^{2\sin\theta} \underbrace{\rho\,\frac{1}{\rho}}_{1}\,d\rho$$

$(0,1)$

$$\rho^2 - 2\rho\sin\theta \le 0$$

$$0 < \rho \le 2\sin\theta$$

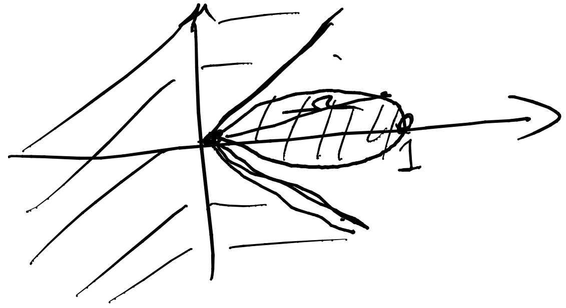
$$\sin\theta \ge 0$$

$$\theta \in [0,\pi]$$

$$\int_0^{\pi} d\theta\, 2\sin\theta$$

$\rho = \sqrt{\cos 2\theta}$ ≥ 0

$x \geq 0$ $\longleftarrow$ $\cos 2\theta \geq 0$



$2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$x = \rho \cos\theta$

$y = \rho \sin\theta$

$$\int_\Omega 1 \, dx\,dy = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta \int_0^{\sqrt{\cos 2\theta}} \rho \, d\rho =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\theta \ \frac{1}{2}\rho^2 \Big|_0^{\sqrt{\cos 2\theta}}$$

es.

$$\phi(u,v) = \begin{pmatrix} e^u + e^v \\ e^u - e^v \\ uv \end{pmatrix} \qquad \begin{pmatrix} 2e \\ 0 \\ 1 \end{pmatrix}$$

$\exists$ unico (u_0, v_0) :

$$\phi(u_0, v_0) = \begin{pmatrix} 2e \\ 0 \\ 1 \end{pmatrix}$$

$u_0 = v_0 = 1$

$$\phi(u_0, v_0) = \begin{pmatrix} 2e \\ 0 \\ 1 \end{pmatrix}$$

$u=1, v=1 \Rightarrow e^u + e^v = 2e$ ok

$u=-1, v=-1 \Rightarrow e^{-1} + e^{-1} = 2e^{-1}$ NO

$$\begin{cases} e^u + e^v = 2e \\ e^u - e^v = 0 \Rightarrow e^u = e^v \Rightarrow u = v \\ uv = 1 \end{cases}$$

$u^2 = 1$ $\quad (1, 1)$?

$v = u = \pm 1$ $\quad (-1, -1)$?

$$\phi_u = \begin{pmatrix} e^u \\ e^u \\ v \end{pmatrix}_{(1,1)} = \begin{pmatrix} e \\ e \\ 1 \end{pmatrix}$$

$$\phi_v = \begin{pmatrix} e^v \\ -e^v \\ u \end{pmatrix}_{(1,1)} = \begin{pmatrix} e \\ -e \\ 1 \end{pmatrix}$$

$$v(1,1) = \begin{pmatrix} e \\ e \\ 1 \end{pmatrix} \times \begin{pmatrix} e \\ -e \\ 1 \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 2e \\ -0 \\ -2e^2 \end{pmatrix}}} \ // \begin{pmatrix} 1 \\ 0 \\ -e \end{pmatrix}$$

$$1(x-2e) + 0(y-0) - e(z-1) = 0$$

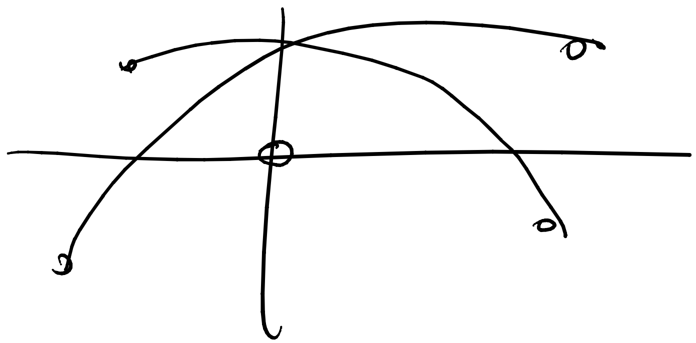
$$x - ez - 2e + e = 0$$

$$\boxed{x - ez = e}$$

$$(4x^2+3y^2)^{-1/2}\,(4x,3y)$$

$\operatorname{dom} A = \mathbb{R}^2 \setminus \{(0,0)\}$

connesso



$$\frac{1}{\sqrt{4x^2+3y^2}}$$

$$f_x = \frac{4x}{\sqrt{4x^2+3y^2}}$$

$$\boxed{\sqrt{4x^2+3y^2} + C}$$

$$f_y = \frac{3y}{\sqrt{4x^2+3y^2}}$$

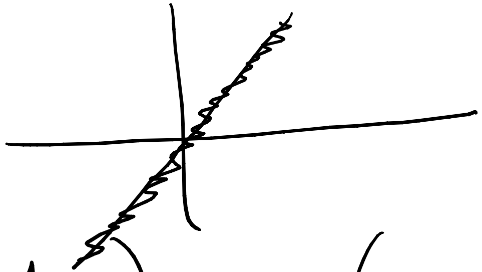
integrando in dx

$$f = \int \frac{4x\,dx}{\sqrt{4x^2+3y^2}} + c(y)$$

derivando in y

$$\frac{3y}{\sqrt{\ldots}} = (f)_y$$

$$\frac{1}{x-y}\left[dx - dy\right] \qquad \text{dom} = \{y \neq x\}$$



unione di due convessi (semplic. connessi)

$$\left(\frac{1}{x-y}\right)_y \overset{?}{=} \left(-\frac{1}{x-y}\right)_x$$

(condiz. rotore)

se sì, la forma è integrabile in ogni semipiano $y > x$ e $y < x$

$$\lim_{(0,0)} \frac{x^3 y}{x^4 + y^2}$$

$y = k x^2$

$x \to 0$

$y \to 0$

$$f(x, kx^2) = \frac{k x^4}{x^4 + k^2 x^4} = \frac{k}{1+k^2} \qquad x \neq 0$$

$\to$ varia al variare di k

$k = 0 \quad f(x, 0) = \underline{0}$

$k = 1 \quad f(x, x^2) = \dfrac{x^4}{2x^4} = \underline{\dfrac{1}{2}}$

Se x_0 è di accum. per Ω ed f è continua in x_0

$$\boxed{\forall \varepsilon>0 \ \exists \delta>0 : \forall x\in\Omega \quad |x-x_0|<\delta \qquad |f(x)-f(x_0)|<\varepsilon}$$

limite

$$\Updownarrow$$

$$\lim_{x\to x_0} f(x) = f(x_0) = L$$

$w = x - x_0$

$$\boxed{\lim_{w\to 0} f(x_0+w) = f(x_0)}$$

$$g(y) = \begin{cases} 1 & y \neq 0 \\ 0 & y = 0 \end{cases}$$

$$\lim_{y \to 0} g(y) = 1$$

$$f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\lim_{x \to 0} x \sin \frac{1}{x} = 0$$

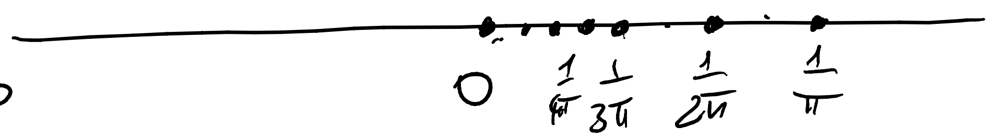
$$g(f(x)) = \begin{cases} 0 & \text{se } x \sin \frac{1}{x} = 0 \iff \begin{matrix} x = 0 \\ \frac{1}{x} = k\pi \end{matrix} \\ 1 & \text{altrimenti} \end{cases}$$

$$\boxed{g\left(f\left(\tfrac{1}{n}\right)\right) \to 1 \qquad g\left(f\left(\tfrac{1}{k\pi}\right)\right) \to 0}$$

$$a_k = \frac{1}{k\pi} :$$

$$g\left(f\left(\tfrac{1}{k\pi}\right)\right) \equiv 0$$

$$\frac{1}{n} \to 0 \qquad \frac{1}{k\pi} \to 0$$



$\lim_{x\to x_0} f(x) = L \qquad \lim_{y\to L} g(y) = M$

$\lim_{x\to x_0} g(f(x))$ NON esiste se

$$\begin{cases} g \text{ è definita in } L, \text{ ma } g(L) \neq M \\ \forall n > 0 \quad \exists x_n : |x_n - x_0| < \frac{1}{n} \ : \ f(x_n) = L \end{cases}$$

$g(f(x_n)) = g(L)$ costante in n e quindi

il limite di $g(f(x_n))$ è $g(L) \neq M$

$\lim g(f(x))$ sull'insieme $f \neq L$ il limite è M.