Note Title 7/24/2020

$$\left| \int_a^b \sigma(t)\,dt \right|_{\mathbb{R}^n} \le$$

$\sigma : [a,b] \to \mathbb{R}^n$

$a < b$

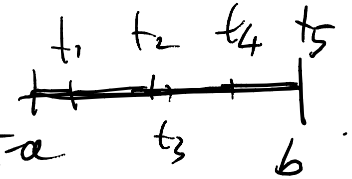
$$\le \int_a^b |\sigma(t)|\,dt$$

$\sigma_i \in \mathbb{R}^n$

$$\left| \sum_1^n \sigma_i \right|_{\mathbb{R}^n} \le \sum_1^n |\sigma_i|$$

$$\gamma(t_{i+1}) - \gamma(t_i) = \int_{t_i}^{t_{i+1}} \dot\gamma(t)\,dt$$

$$\underbrace{\left| \qquad\qquad \right|} = \left| \int \qquad\qquad \right| \le \underbrace{\int_{t_i}^{t_{i+1}} |\dot\gamma(t)|\,dt}$$

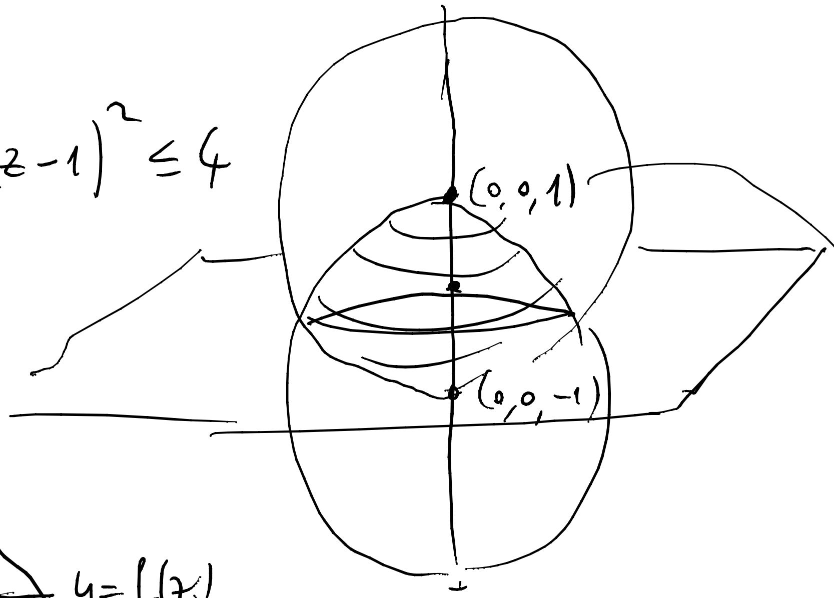


Sommando $\Lambda(\pi) \le \int_a^b |\dot\gamma(t)|\,dt$

$$\{x^2+y^2+z^2-2z \le 3\} \cap \{x^2+y^2+z^2+2z \le 3\}$$

SOLIDO DI ROTAZIONE (attorno a z)

$x^2+y^2+(z-1)^2 \le 4$

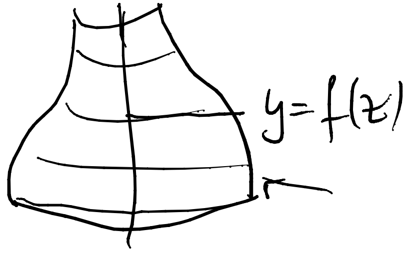


$x=0$

$$y^2+z^2-2z \le 3$$

$$y^2 \le 3+2z-z^2$$

$$y = \sqrt{3+2z-z^2}$$



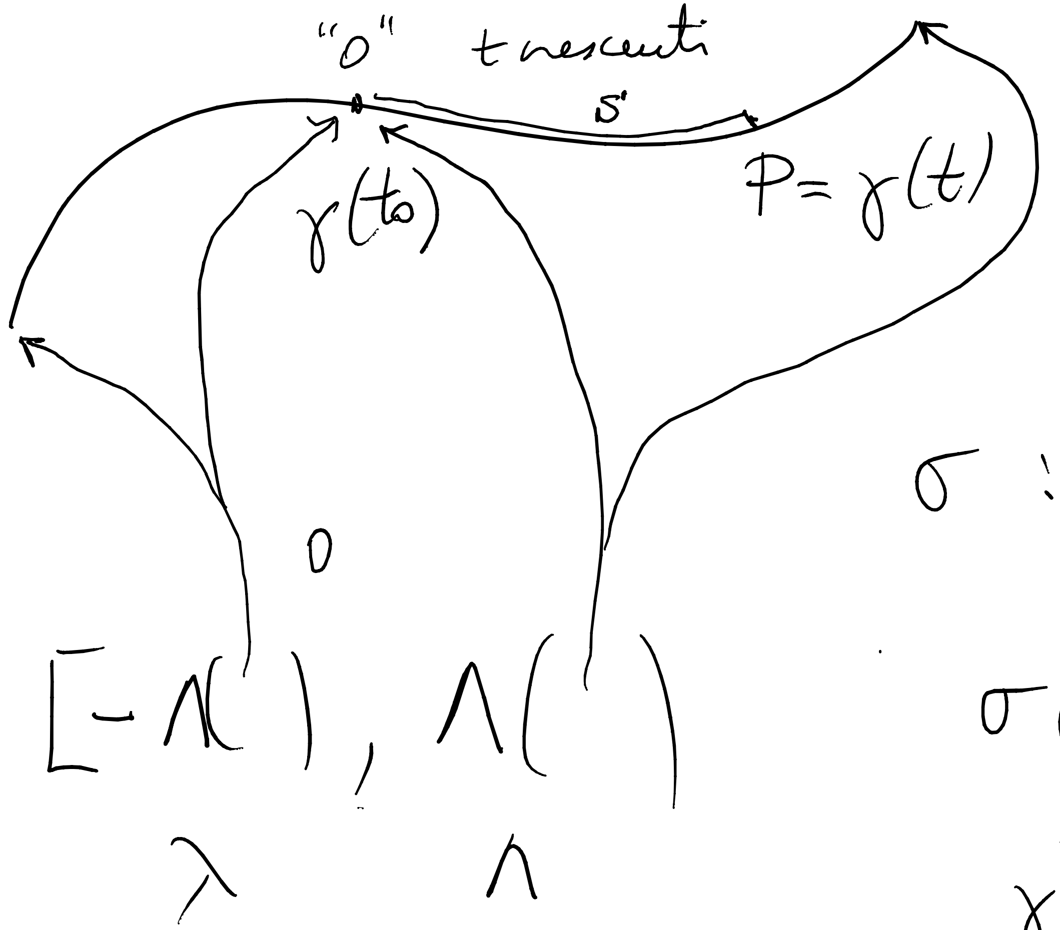
$$\text{vol.} = \pi \int_{z_1}^{z_2} f^2(z)\,dz$$

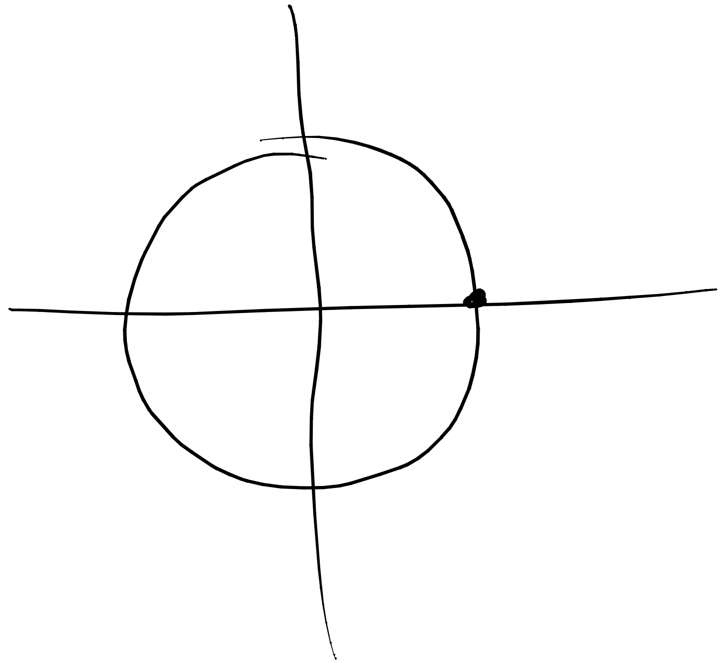
2 volte vol. per $z>0$

$$\boxed{2\pi \int_0^1 3+2z-z^2\,dz}$$

$\gamma(t) \qquad \gamma : [a,b] \to \mathbb{R}^n$

rettificabile



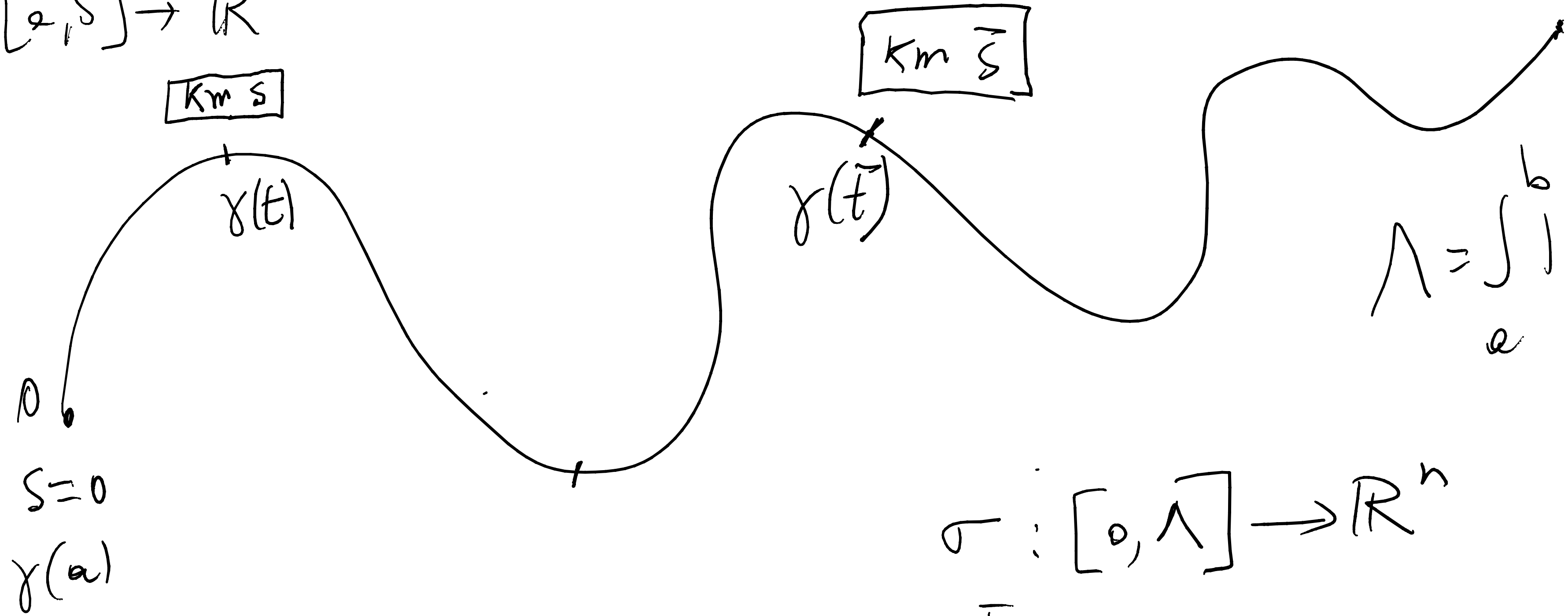


$\sigma : [\lambda, \Lambda] \to \mathbb{R}^n$

$\sigma(s)$ il punto tale che

$\overset{\shortparallel}{\gamma(t)} : \boxed{\int_{t_0}^{t} |\gamma'(t)|\,dt = S(t)}$

$\gamma : [a,b] \to \mathbb{R}^n$



$$\Lambda = \int_a^b |\dot{\gamma}(t)|\,dt$$

$$s(t) = \int_a^t |\dot{\gamma}(\tau)|\,d\tau$$

$\sigma : [0,\Lambda] \longrightarrow \mathbb{R}^n$

$$\int_a^{\bar{t}} |\dot{\gamma}(t)|\,dt = \bar{s}(\bar{t})$$

è invertibile se γ è regolare

$\dot{\gamma} \neq 0$

$|\dot{\gamma}| > 0$

$\sigma(t) = \gamma(t(s))$

$$x^3+y^3=3 \qquad f(x,y)=x^3+y^3$$

studiare l'insieme di livello 3 $= \{(x,y)\in\mathbb{R}^2 : f(x,y)=3\}$

$f_x = 3x^2$

$f_y = 3y^2$

$\nabla f = \begin{pmatrix}0\\0\end{pmatrix}$ se $x=y=0$

L'unico punto critico è $(0,0)$

$(0,0)$ appartiene all'insieme da studiare?

No, perché $\boxed{f(0,0)}=0$ e non 3.

$f_x=0$ se $x=0$ ←

$f_y=0$ se $y=0$

$f : \Omega \to \mathbb{R} \qquad \Omega \subseteq \mathbb{R}^n$

$$\operatorname{graph} f = \{ (x, z) : x \in \underbrace{\Omega}_{\text{dom} f} \; z \in \underbrace{\mathbb{R}}_{\text{codom} f} : z = f(x) \}$$

Il piano tangente a graph f in (x_0, z_0) $\left[\left(x_0, \underset{z_0}{\overset{}{f(x_0)}} \right) \right]$

è

$$\boxed{z = f(x_0) + \underbrace{\nabla f(x_0)(x - x_0)}_{df(x_0, x - x_0)}}$$

Calcolo differenz.

$n = 2$ scritto in piano

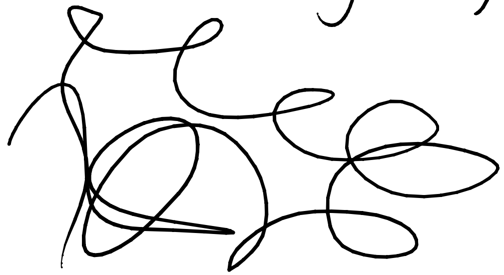
$$\boxed{z = \underbrace{f(x_0, y_0)}_{\in \mathbb{R}^2} + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

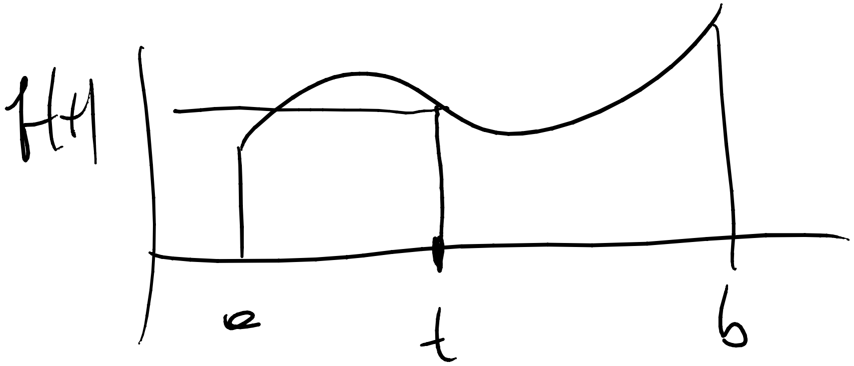
$$\phi : \Delta \to \Sigma \subseteq \mathbb{R}^3$$

$$f : \Sigma \to \mathbb{R}$$

$$\int_\phi f = \int_\Delta f(\phi(u,v))\,|\phi_u(u,v) \times \phi_v(u,v)|\ dudv$$

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$





$$\text{graph. } f = \{(x,y) \in \mathbb{R}^2 : x \in \text{dom} f,\ y \in \text{cod} f,\ y = f(x)\}$$

$$\gamma : \text{dom} f \to \mathbb{R}^2$$

$$\gamma(t) = \begin{pmatrix} t \\ f(t) \end{pmatrix} \quad t \in \text{dom} f$$

t varia in $[a,b]$ $\quad (t, f(t))$

descrive il grafico di f su $[a,b]$

$f : \Omega \to \mathbb{R}$ graph. $\{(x,y,z) \in \mathbb{R}^3 : (x,y) \in \Omega \ \underline{\underline{z = f(x,y)}}\}$

$\phi : \underline{\underline{\Omega}} \to \mathbb{R}^3$ $\phi(u,v) = \begin{pmatrix} u \\ v \\ f(u,v) \end{pmatrix}$

al variare di (u,v) in Ω

$\phi(u,v)$ descrive il grafico di f

$$x = f(u,v) \qquad u = \tilde{f}(x,y) \qquad \to \begin{pmatrix} x \\ y \\ h(\tilde{f}(x,y), \tilde{g}(x,y)) \end{pmatrix}$$

$$y = g(u,v) \qquad v = \tilde{g}(x,y) \qquad \to$$

$$z = h(u,v) \qquad z = h(\underbrace{\tilde{f}(x,y)}_{u}, \underbrace{\tilde{g}(x,y)}_{v})$$

$$\phi_u = \begin{pmatrix} 1 \\ 0 \\ f_u(u,v) \end{pmatrix} \qquad \phi_v = \begin{pmatrix} 0 \\ 1 \\ f_v(u,v) \end{pmatrix}$$

$$\nu = \phi_u \times \phi_v$$

$$f(x,y,z) = xy \qquad \phi(u,v) = \begin{pmatrix} u \\ v \\ u^2 v^2 \end{pmatrix}$$

$x = u$
$y = v$
$z = u^2 v^2$

$g(u,v)$

$$\int_\phi f = \iint_{\underbrace{[0,1]\times[0,1]}_{\Delta}} \overbrace{u v}^{f(\phi(u,v))} \cdot \sqrt{1 + \underbrace{(2uv^2)^2}_{g_u^2} + \underbrace{(2u^2v)^2}_{g_v^2}}\, du\, dv$$

$$\Delta = \underbrace{[0,1]}_{u} \times \underbrace{[0,1]}_{v}$$

$$a(x,y)\,dx + b(x,y)\,dy = \alpha\big((x,y),(dx,dy)\big)$$

$$\underbrace{\begin{pmatrix} a(x,y) \\ b(x,y) \end{pmatrix}}_{A(x,y)} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$A(x,y)$ campo vettoriale ad α

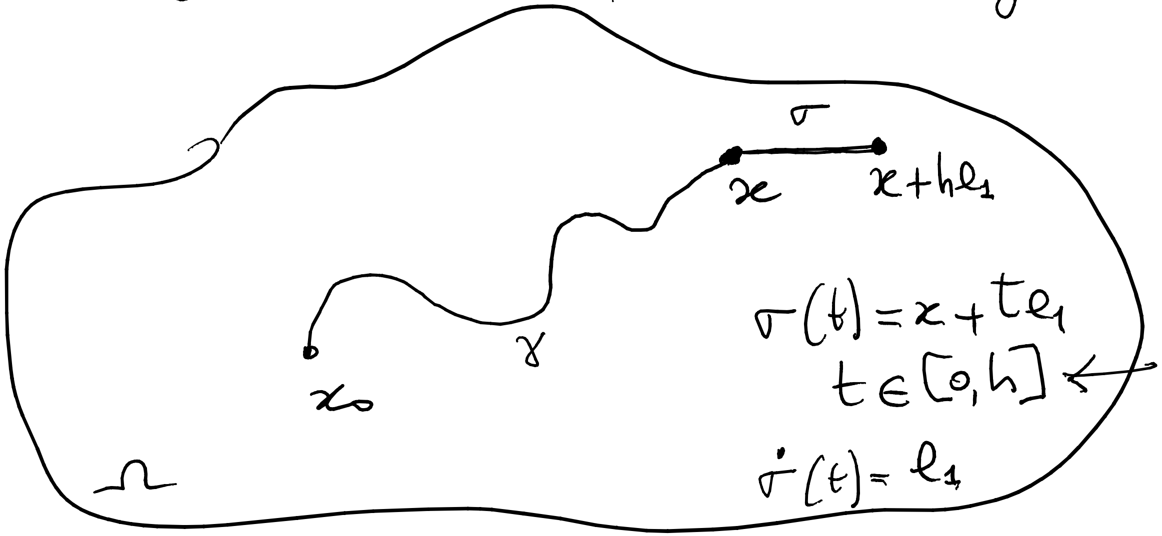
$$\int_\gamma \alpha = \int_\gamma A = \int_a^b A(\gamma(t))\,\dot\gamma(t)\,dt$$

$\gamma : [a,b] \to \operatorname{dom} \alpha$

$$-\frac{y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy$$

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad t \in [0, 2\pi]$$

C.S. $A \in C^0(\Omega)$ sia integrabile è che $\int A$ non dipenda dal cammino, ma solo dagli estremi γ (Ω connesso)



$$f(x) = \int_{\gamma(x_0 \to x)} A$$

$A \in C^0(\Omega)$ e $\gamma \in C^1[0,1]$

$$= \int_0^1 \underbrace{\underbrace{A(\gamma(t))}_{C^0[0,1]} \cdot \underbrace{\dot\gamma(t)}_{C^0[0,1]}}_{C^0} \, dt$$

$$f_{x_1}(x) = A_1(x) \quad (\forall x \in \Omega) \; ?$$

$$\parallel$$

$$\lim_{h \to 0} \frac{f(x+he_1) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h} \int_0^h \underbrace{A(x+te_1) \cdot e_1}_{A_1(x+te_1)} \, dt = \lim_{h \to 0} A_1(x + \xi e_1)$$

th. media integrale

$\xi \in [0,h]$

$$h \to 0 \Rightarrow \xi \to 0 \ (\text{th. confronto}) \Longrightarrow x + \xi e_1 \to x$$

$$\Big\downarrow \ \text{continuità d'}A$$

$$A_1(x + \xi e_1) \to \underline{A_1(x)}$$