Note Title 7/24/2020

$A_1, \dots, A_n$ sono indipendenti

allora $A_1, \dots, A_n, B$ sono indip se e solo se $B \notin \langle A_1 \dots A_n \rangle$

$B \in \langle A_1, \dots, A_n \rangle$ $\quad \exists \alpha_i : B = \sum_1^n \alpha_i A_i$

$\alpha_1 A_1 + \dots + \alpha_n A_n - \underset{\neq 0}{B} = 0$

se $A_1 \dots A_n, B$ sono indip allora

$\forall \alpha_1 \dots \alpha_n \beta \quad \sum \alpha_i A_i + \beta B = 0 \implies \alpha_i = \beta = 0 \ \forall i$

$$\boxed{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}}$$

$\boxed{AL\_7.1}$

$X$ spazio vettoriale

$\alpha : X \times X \to \mathbb{R}$

è bilineare se e solo se

$$x \longrightarrow \alpha(x,y) \quad \text{è lineare } \forall x \in X$$

e

$$y \longrightarrow \alpha(x,y) \quad \text{è lineare } \forall y \in X$$

$$\boxed{\begin{aligned} (u+v)w &= uw + vw \\ (\lambda u)v &= \lambda(uv) \end{aligned}}$$

$$\langle A_1, A_2, \dots, A_n \rangle = \langle A_2, \dots, A_n \rangle \quad \Downarrow \quad \Uparrow$$

se e solo se $A_1 \in \langle A_2, \dots, A_n \rangle$

DIM

$$\underline{\langle A_1, A_2 \dots A_n \rangle} = \underline{\langle A_2, \dots, A_n \rangle}$$

$$\Downarrow$$

$$A_1 \in \langle A_2, \dots, A_n \rangle$$

$$=$$

CS

$$A_1 \in \langle A_2, \dots, A_n \rangle \overset{?}{\Longrightarrow} \langle A_1, A_2 \dots A_n \rangle = \langle A_2, \dots, A_n \rangle$$

$\exists \alpha_i :$ $A_1 = \sum_2^n \alpha_i A_i$

$B \in \langle A_1, A_2, \dots, A_n \rangle$

$$\exists \beta, \gamma_2 \dots \gamma_n : \ B = \beta A_1 + \sum_2^n \gamma_i A_i = \boxed{\underbrace{\sum_2^n \beta \alpha_i A_i}_{\beta A_1} + \sum_2^n \gamma_i A_i}$$

$\underline{0}, A_1, \dots, A_n$

$\underline{1} \cdot \underline{0} + 0A_1 + 0A_2 + \dots + 0A_n = 0$

$\neq 0$

L'algoritmo di Gauss

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**A1.2**

$\dim X = 1 \qquad x_0 \neq 0$ (base per $X$)

$\lambda \longleftrightarrow \lambda x_0$ al variare di $\lambda$ è tutto $X$

$\langle A_1 \dots A_n \rangle \cap \langle B_1 \dots B_m \rangle$

$$w = \sum_{1}^{n} \alpha_i A_i = \sum_{1}^{m} \beta_j B_j \qquad \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \\ \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}$$

incognite

$w \in \langle A_1, \dots, A_n \rangle \quad \exists\, \alpha_1 \dots \alpha_n : \quad w = \sum_1^n \alpha_i A_i$

$w \in \langle B_1 \dots B_m \rangle \quad \exists\, \beta_1 \dots \beta_m : \quad w = \sum_1^m \beta_j B_j$

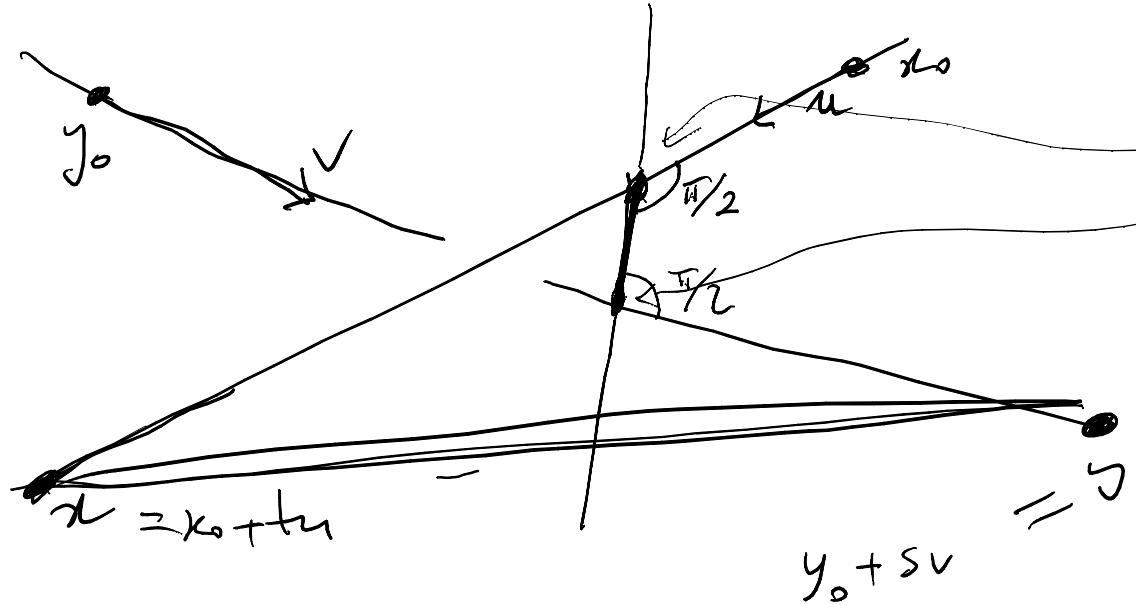
$w \in \langle A_1 \dots A_n \rangle \cap \langle B_1 \dots B_m \rangle$

$$\boxed{\sum_1^n \alpha_i \underset{\uparrow \text{noti}}{A_i} - \sum_1^m \beta_j \underset{\uparrow \text{noti}}{B_j} = 0} \quad *$$

soluzioni $(\bar{\alpha}_1 \dots \bar{\alpha}_n, \bar{\beta}_1 \dots \bar{\beta}_m) \Rightarrow$

$\sum_1^n \bar{\alpha}_i A_i$ è nell'intersezione e anche $\sum_1^m \bar{\beta}_j B_j$ lo è

L'algoritmo di Gauss e i sottospazi AL_1.2?



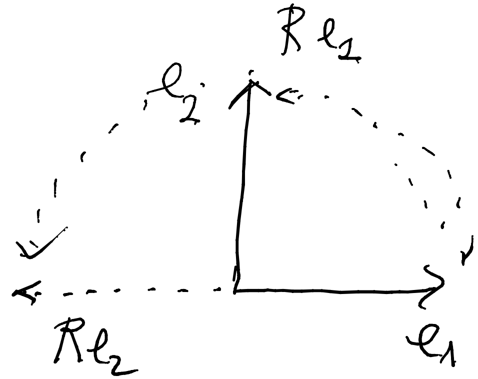

$$\left[(x_0+tu)-(y_0+sv)\right]$$

$$\begin{cases} [\quad]\, u = 0 \\ [\quad]\, v = 0 \end{cases}$$

t e s

$$\boxed{\left|(x-y)_{u\times v}\right|} \leftarrow \text{Minima distanza}$$

$u\times v$ perpendicolare a $u$ e $v$.




$$Re_1 = 1\cdot e_2 + 0\cdot e_1$$

$$Re_2 = -e_1 + 0\, e_2$$

Matrice associata ad $R$ $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$