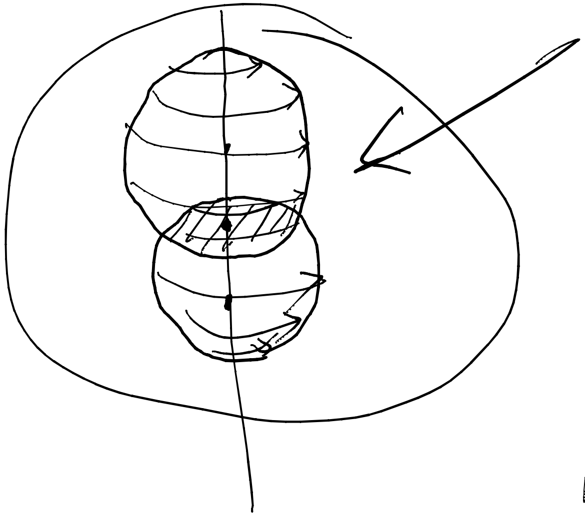
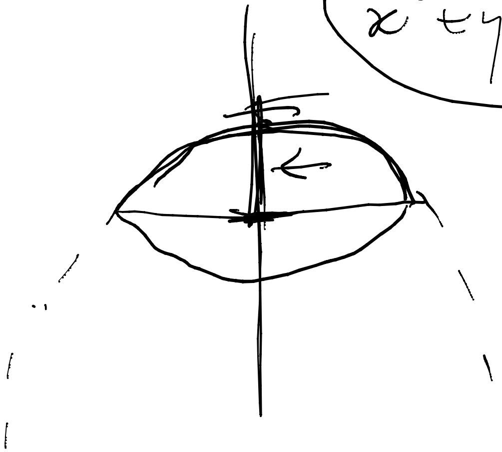
Note Title 7/20/2020





$x^2+y^2+z^2-2z \le 3$

$+2z$

$x=0$

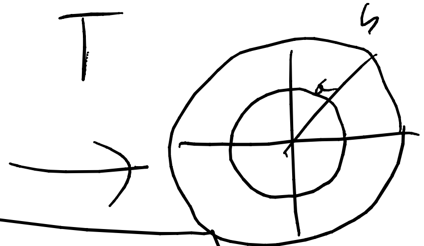
[illegible]

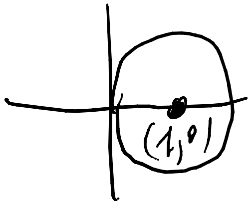
$y=\sqrt{3+\underbrace{2z-z^2}_{f(z)}}$

formule solidi di rotazione

$$\underline{Vol} = \pi\int_{z_1}^{z_2} f^2(z)\,dz$$

$$\int_T \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy$$





$x^2+y^2-2x \le 0$

$a^2 < x^2+y^2 < b^2$

$\theta = [0,2\pi]$

$\rho\in[a,b]$

$\rho^2-2\rho\cos\theta \le 0$

$\rho < 2\cos\theta \Rightarrow \underline{\cos\theta > 0}$

$\theta\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \qquad A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

$$\text{rot}\, A = \text{"}\nabla \times A\text{"} = \boxed{\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \times \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}} =$$

$$= \begin{pmatrix} \partial_y A_3 - \partial_z A_2 \\ -(\partial_x A_3 - \partial_z A_1) \\ \partial_x A_2 - \partial_y A_1 \end{pmatrix}$$

$$\boxed{\text{rot}\, A = 0}$$

$$\partial_{x_i}(A_j) = \partial_{x_j}(A_i)$$

$$\boxed{\int_{\phi} \overset{\nabla \times A}{\text{rot}\, A}\, d\sigma = \int_{\partial\phi^+} A\, dl}$$

$\rho = \sqrt{\cos 2\theta}$

$\boxed{x \geq 0}$

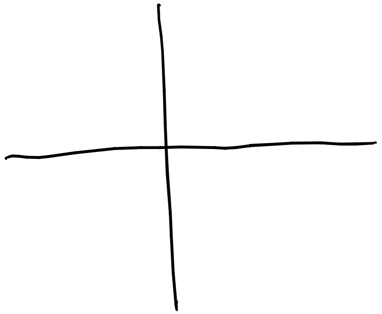
$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

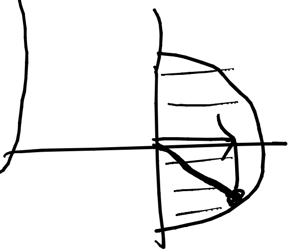
$\sqrt{\cos 2\theta}$

$\cos(2\theta) \geq 0$

$2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\boxed{\theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]}$





$\rho = f(\theta)$

$\begin{pmatrix} f(\theta) \\ \theta \end{pmatrix}$

e poi
verticale
di

$$\Lambda = \int_{\theta_1}^{\theta_2} \sqrt{\dot{f}^2(\theta) + f^2(\theta)}$$

$\gamma(t) = (\rho(t), \theta(t))$

$t \in [a, b)$

$x(t) = \rho(t) \cos \theta(t)$

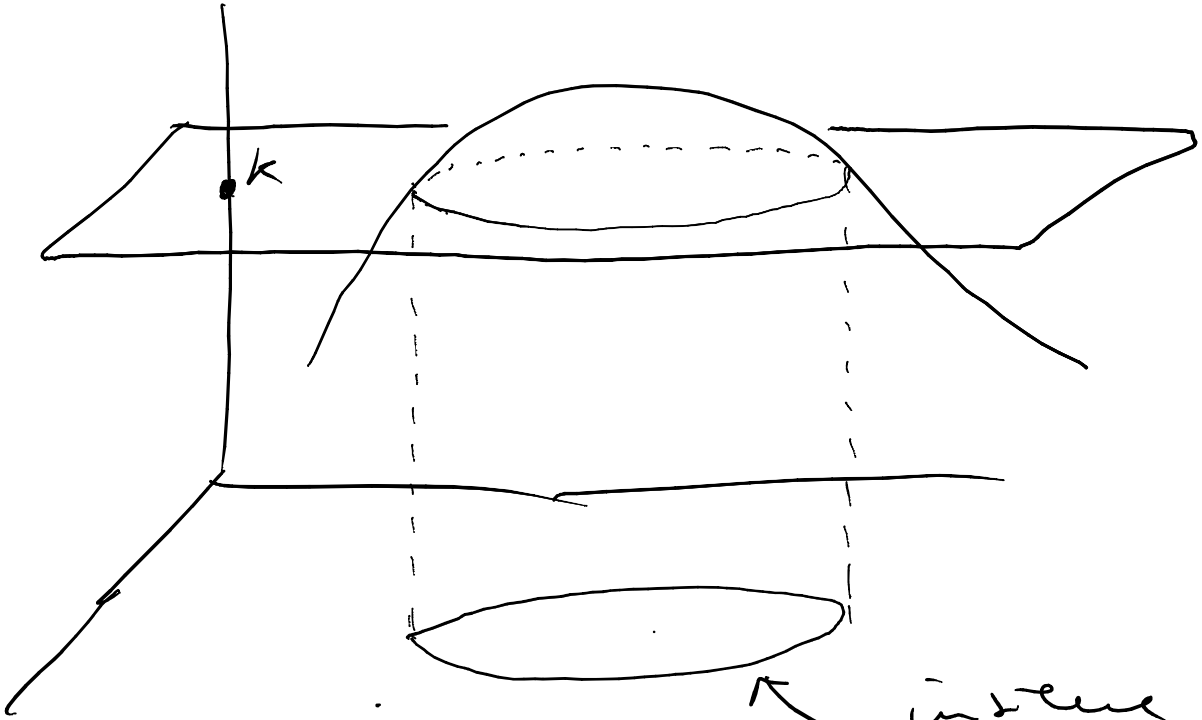
$y(t) = \rho(t) \sin \theta(t)$

$$\boxed{\Lambda(\gamma) = \int_a^b \sqrt{\dot{\rho}^2(t) + \rho^2(t)\dot{\theta}^2(t)}\, dt}$$

$\int |\dot{\gamma}(t)|\, dt$

$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

$f : \Omega \to \mathbb{R}$



insieme di livello k

$$\boxed{f^{-1}(k) = \{ x \in \operatorname{dom} f \, : \, f(x) = k \}}$$

① $f : \Omega \to \mathbb{R}$, $\Omega \subseteq \mathbb{R}^n$ $\qquad x_0 \qquad$ $\operatorname{graph} f = \{(x,z) : x \in \Omega,\ z \in \mathbb{R} : z = f(x)\}$

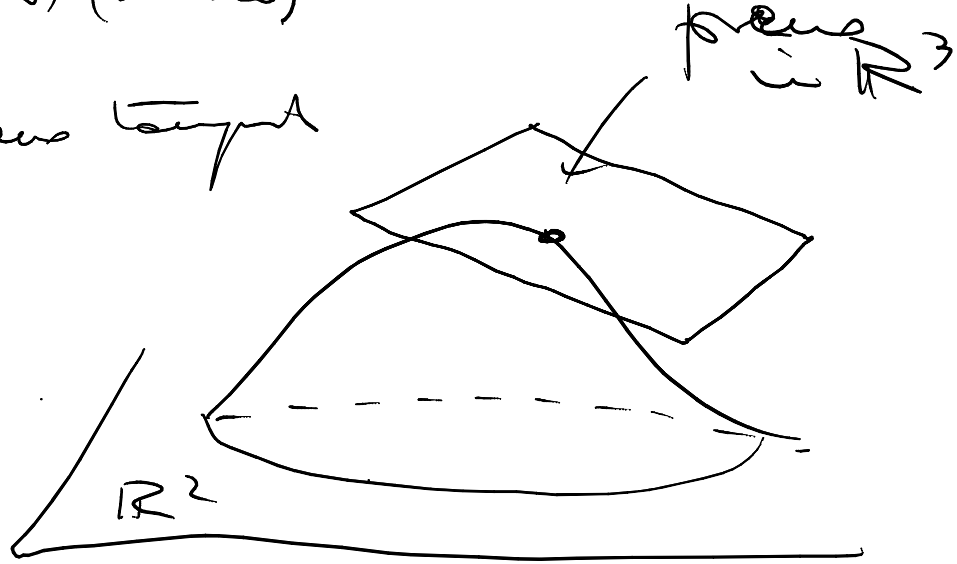
Piano tangente al graph f in (x_0, z_0) è

$$z = f(x_0) + \nabla f(x_0)(x - x_0)$$

equazione implicita del piano tangente

$$\{(x,z) \in \mathbb{R}^n \times \mathbb{R} : \quad \}$$

$$\boxed{\sum_{1}^{n} f_{x_i}(x_0)\,(x_i - (x_0)_i) - z + f(x_0) = 0}$$



$(-\nabla f(x_0), 1)$ vettore normale orientato in alto

$$\phi : \Delta \to \mathbb{R}^3 \qquad \Delta \subseteq \mathbb{R}^2$$

$$\phi(x,y) = \begin{pmatrix} x \\ y \\ f(x,y) \end{pmatrix}$$

$$z = f(x,y)$$

$$\nu(u,v) = \phi_u \times \phi_v = \qquad \forall (u,v) \in \Delta$$

$$= \begin{pmatrix} \phi_u^1 \\ \phi_u^2 \\ \phi_u^3 \end{pmatrix} \times \begin{pmatrix} \phi_v^1 \\ \phi_v^2 \\ \phi_v^3 \end{pmatrix}$$

vettore normale al sostegno nel punto $\phi(u,v)$

$$\phi(u_0,v_0) = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} \qquad \nu(u_0,v_0) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\boxed{a(x-x_0) + b(y-y_0) + c(z-z_0) = 0}$$

$\nu \neq 0$ se ϕ è regolare in (u_0,v_0)

$\underbrace{\phi_u(u_0,v_0)}_{}$ e $\underbrace{\phi_v(u_0,v_0)}_{}$ sono vettori tangenti al

sostegno

se ϕ è regolare sono indipendenti e

le loro combinazioni formano lo spazio tangente

(lo spazio dei vettori tangenti alla superficie)

$$\psi(\alpha,\beta) = \overbrace{\underbrace{\phi(u_0,v_0)}_{\text{punto di tangenza del sostegno}} + \alpha\,\phi_u(u_0,v_0) + \beta\,\phi_v(u_0,v_0)}$$

$(\alpha,\beta)\in\mathbb{R}^2$

$\phi : \Delta \to \mathbb{R}^3 \quad \Delta \subseteq \mathbb{R}^2$

$$\phi(u,v) = \begin{pmatrix} \phi_1(u,v) \\ \phi_2(u,v) \\ \phi_3(u,v) \end{pmatrix}$$

1) $\boxed{C^1(\Delta)}$

2) $\boxed{\nu(u,v) = \phi_u(u,v) \underset{\uparrow \text{ vettore}}{\times} \phi_v(u,v)}$

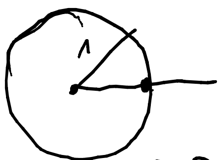
$\nu(u,v)$ vettore normale standard in $\phi(u,v)$

$\boxed{\nu \neq 0 \quad \forall (u,v) \in \mathring{\Delta}}$

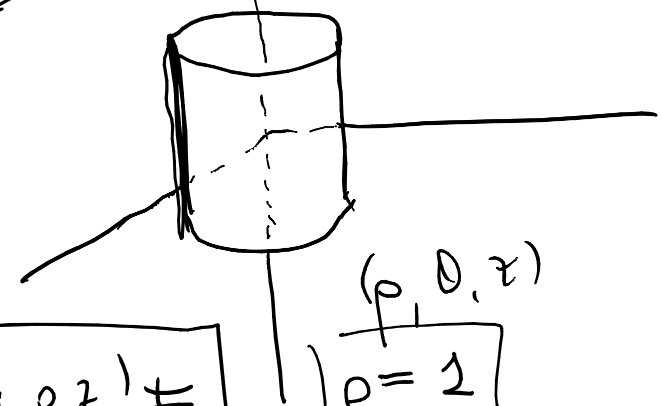
3) ϕ è iniettiva su $\mathring{\Delta}$ (l'insieme dei punti interni a Δ)

$\Updownarrow$

ϕ è Regolare



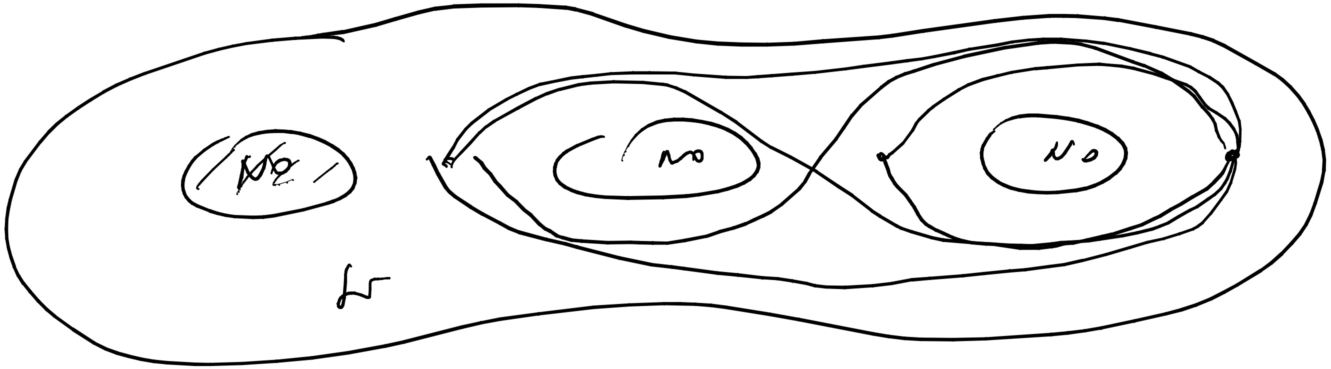
$\begin{matrix} \rho & \theta \\ (1, & 0) \end{matrix} \qquad \begin{matrix} \rho & \theta \\ (1 & 2\pi) \end{matrix}$



(ρ, θ, z)

$\boxed{\rho = 1}$

$\boxed{\begin{matrix} (1, 0, z) \equiv \\ (1, 2\pi, z) \end{matrix}}$



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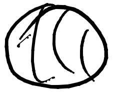
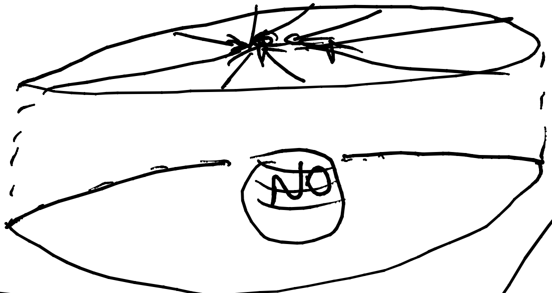
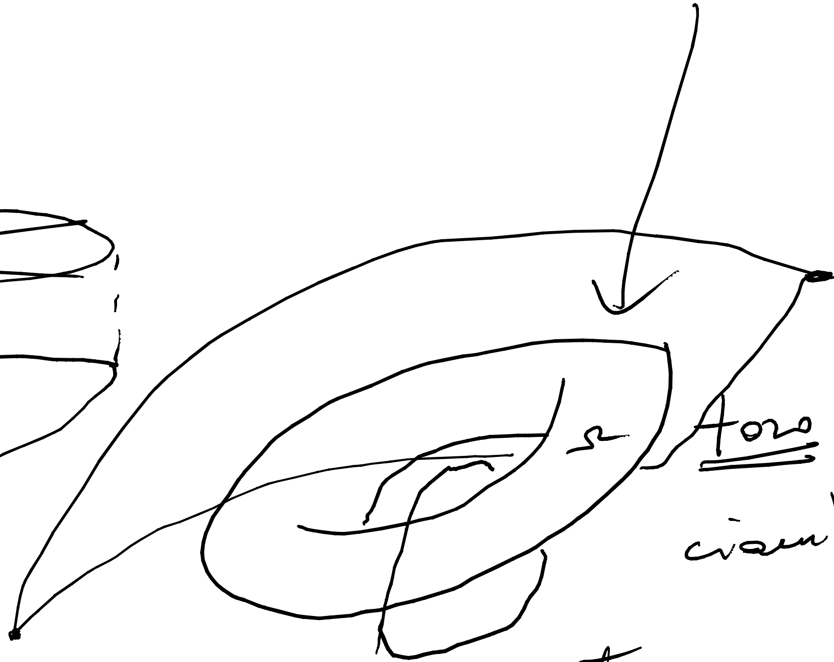






Toro
ciambella

il complementare
è non sempl. connesso

CAMPI E FORME II

$\alpha(x, w)$ $\quad \boxed{w \to \alpha(x,w) \text{ è lineare in } w \ \forall x}$

$\alpha : \Omega \times \mathbb{R}^n \to \mathbb{R} \qquad \Omega \subseteq \mathbb{R}^n$

$$\boxed{\alpha(x, w) = \underbrace{A(x)}_{\text{vettore che rappresenta } w \to \alpha(x,w)} \cdot w}$$

prodotto scalare

vettore che rappresenta $w \to \alpha(x,w)$

al variare di x è il campo associato ad $\alpha(x,w)$

$$\underbrace{df(x,w)}_{\text{forme}} = \underbrace{\nabla f(x_0)}_{\text{campo}} w$$

forme $\longleftrightarrow$ campo associati

$A(x)$ è lineare a valori scalari

$A(x) = A(x_i e_i) =$ (base canonica)

$= x_i A(e_i) = a_i x$ (scalare)

$A(e_i) = a_i$

$$a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$$\lim_{(0,0)} \frac{x^3 - x^2 y}{\sqrt{x^2+y^2}} \quad \begin{array}{l} \to \text{3-omogenea} \\ \to \text{1-omogenea} \end{array}$$

f

Il rapporto è 2-omog. grado >0

il denominatore non si annulla mai sulla sfera unitaria

$\Downarrow$

la f è limitata su $\{x^2+y^2=1\}$ per Weierstrass.

Per il th. f. omogenea il limite è $\underline{0}$.

$$\frac{x^3 - x^2 y + xy}{\sqrt{x^2+y^2}}$$

$$\underbrace{\frac{x^3 - x^2 y}{\sqrt{}}}_{\text{2-omog.}} \to 0 + \underbrace{\frac{xy}{\sqrt{x^2+y^2}}}_{\text{1-omog.}} \to 0$$

X insieme arbitrario

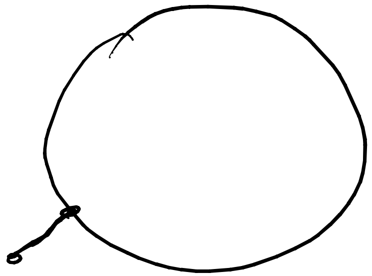
$d : X \times X \to \mathbb{R}$

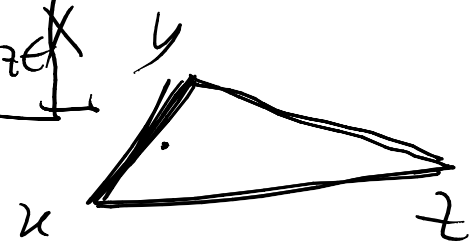
1) $d(x,y) \geq 0 \quad \forall x,y \in X$

2) $d(x,y) = 0 \iff x = y \quad \forall x,y \in X$

3) $d(x,y) = d(y,x) \quad \forall x,y \in X$

4) $d(x,y) \leq d(x,z) + d(z,y) \quad \forall x,y,z \in X$





~~X si dice METRICO se su di esso è definita una distanza d~~

(X,d) è uno spazio <u>metrico</u> se d verifica gli assiomi 1)..4)

$\Lambda(\text{graph } f) =$ $f(x) = x + x^2$ $x \in [0,1]$

$$= \int_0^1 \sqrt{1 + (1+2x)^2}\, dx =$$

$1 + 2x = u$

$$\boxed{\Lambda(\gamma) = \int_a^b |\dot\gamma(t)|\, dt}$$

$$\gamma(t) = \begin{pmatrix} t \\ t + t^2 \end{pmatrix}$$

$f(t)$ $t \in \underbrace{[0,1]}_{\text{dom} f}$

$$= \int \sqrt{1 + u^2}$$

$u = \sinh v$

$$\Lambda(\text{graph } f) = \int_a^b \sqrt{1 + f'(t)^2}$$

$$\int_\gamma \alpha\, dx \qquad \gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \quad t \in [0, 2\pi]$$

$$\alpha(x,y)\, dx \qquad \gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$\boxed{\int_0^{2\pi} \alpha(\cos t, \sin t)\,(-\sin t)\, dt}$$

$\underline{x = \cos t}$ $\quad dx = -\sin t\, dt$

$\underline{y = \sin t}$ $\quad dy = \cos t\, dt$

$$A \begin{pmatrix} A_1(x,y) \\ A_2(x,y) \end{pmatrix} \qquad \int_\gamma A = \int_a^b A(\gamma(t)) \cdot \dot\gamma(t)\, dt$$

$$\begin{pmatrix} A_1(\gamma_1(t), \gamma_2(t)) \\ A_2(\gamma_1(t), \gamma_2(t)) \end{pmatrix} \cdot \begin{pmatrix} \dot\gamma_1(t) \\ \dot\gamma_2(t) \end{pmatrix}$$

$$= \int_a^b \left[A_1\left(\gamma_1(t), \gamma_2(t)\right) \dot{\gamma}_1(t) + A_2(\quad) \dot{\gamma}_2(t) \right] dt$$

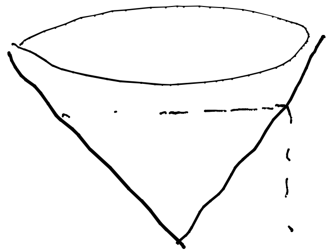
$$f(x,y) = \sqrt{x^2+y^2}$$

1) f è continua in $(0,0)$

2) È derivabile in $(0,0)$?

$$f_x(0,0) = \lim_{h \to 0} \frac{\overbrace{\sqrt{h^2+0^2} - \sqrt{0^2+0^2}}^{f(h,0)-f(0,0)}}{h}$$

$$= \frac{|h|}{h} \to \text{NON ESISTE}$$



$h>0$

$$\frac{\sqrt{h} - \sqrt{0}}{h} = \frac{1}{\sqrt{h}}$$

$$\lim_{h \to 0^+} \frac{1}{\sqrt{h}} = +\infty$$

$$f(x,y) = (x^2+y^2)^{\frac{3}{4}}$$

$$f_x(0,0) = \lim_{h\to 0} \frac{1}{h}\Big[\overbrace{(h^2)^{\frac{3}{4}}}^{f(h,0)} - 0\Big] = \frac{h^{3/2}}{h} = \lim_{h} h^{\frac{1}{2}} = 0$$

$|x|$ non è der. in 0

$|\underline{\underline{x^2}}| = x^2$ è der. in 0

$$\sqrt{|x^2 y|}^{\,1} \qquad \frac{3}{2} - \text{omog.}$$

$$\frac{(h^2k)^{\frac{1}{2}}}{\sqrt{h^2+k^2}}$$