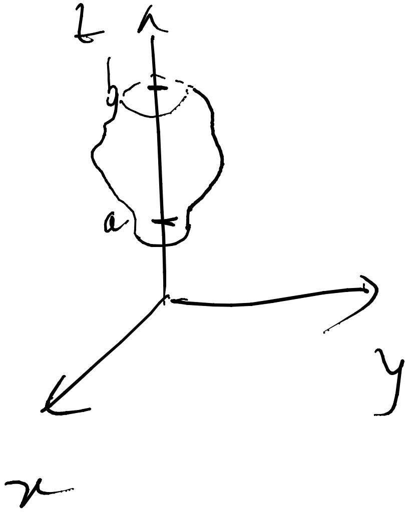


$y = f(z) \quad z \in [a, b]$

Vol — ogni fetta ha area $\pi f^2(z)$ e altezza dz

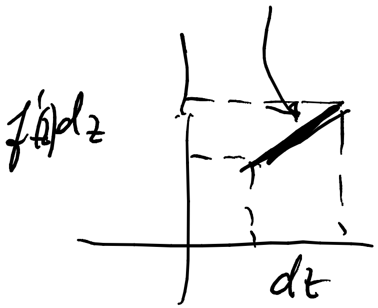
Volume ~~totale~~ $= \int_a^b \pi f^2(z)\, dz$

$\sqrt{dz^2 + f'(z)^2 dz^2}$

$y = \sin z$

Superficie laterale della fetta

Circonferenza $2\pi f(z)$



$$\int_a^b 2\pi f(z) \sqrt{1 + f'^2(z)}\, dz$$

① METODO FORZA BRUTA

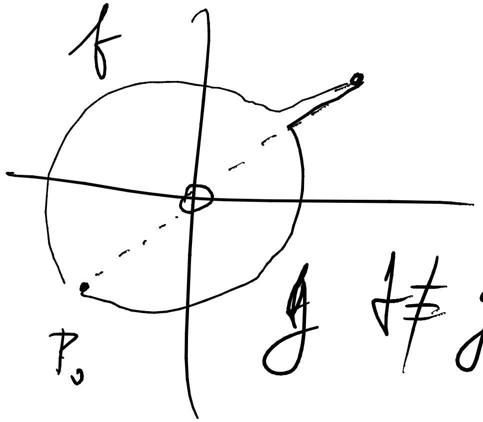
$$\begin{cases} f_x = A_1 \\ f_y = A_2 \end{cases}$$ integro

$$A \equiv \begin{pmatrix} y \\ x \end{pmatrix}$$

$f_x = y \Rightarrow f(x,y) = xy + c(y)$

$f_y = x$

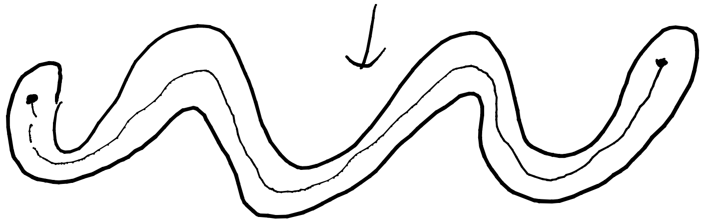
$f_y(x,y) = x + c'(y)$



$f \neq g$

$x + c'(y) = x \Rightarrow c'(y) = 0 \Rightarrow c = \text{cost.}$

$$\boxed{f(x,y) = xy + C}$$



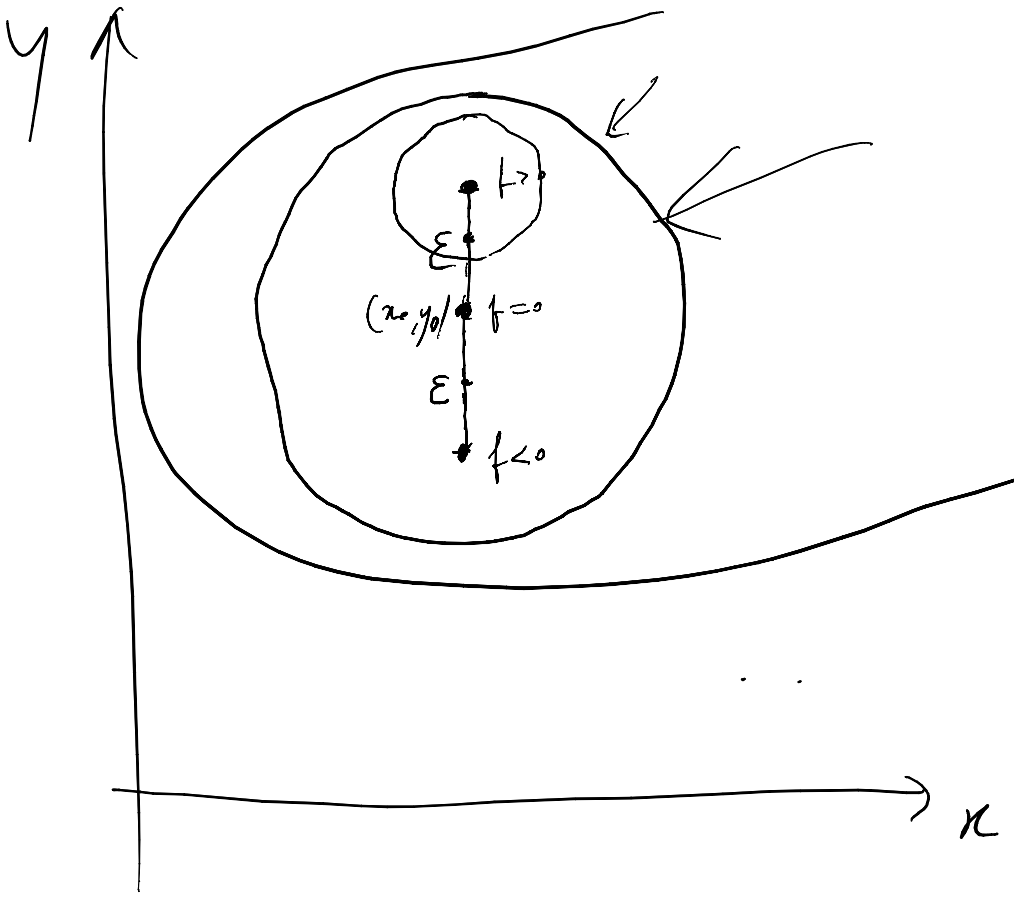
$A : \mathbb{R}^n \to \mathbb{R}^m$ lineare

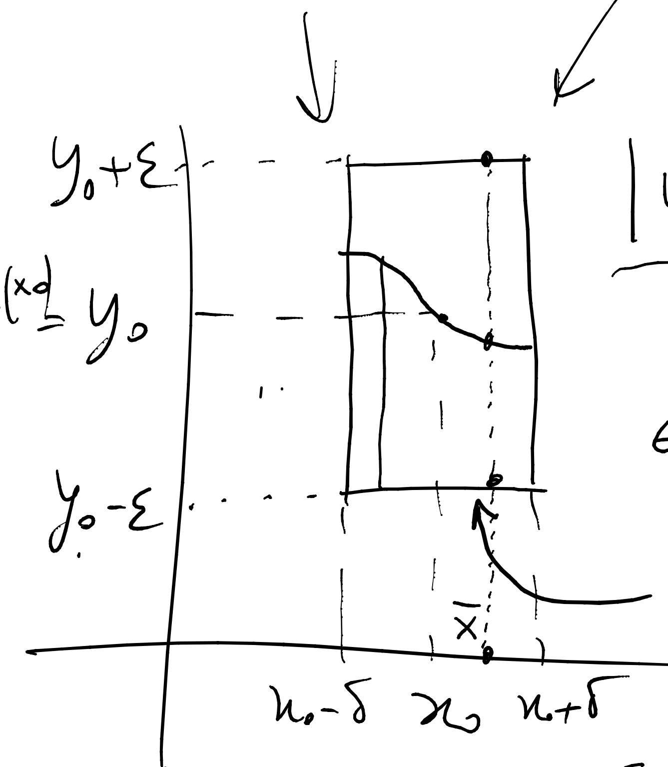
$$dA(x_0, w) = A(w)$$

$$\lim_{w \to 0} \frac{|A(x_0+w) - A(x_0) - A(w)|}{|w|} \overset{?}{=} 0$$

Il numeratore è id. nullo e quindi convergente a 0.

$\bar{x}, \bar{\varphi}(x)$





y_0

$$|\varphi(x) - \varphi(x_0)| < \varepsilon$$

$\varphi(x) \in$

$\in [y_0-\varepsilon, y_0+\varepsilon]$

δ sembra aggiustare la continuità

Th: Funzioni Implicite II

curve rettif. (

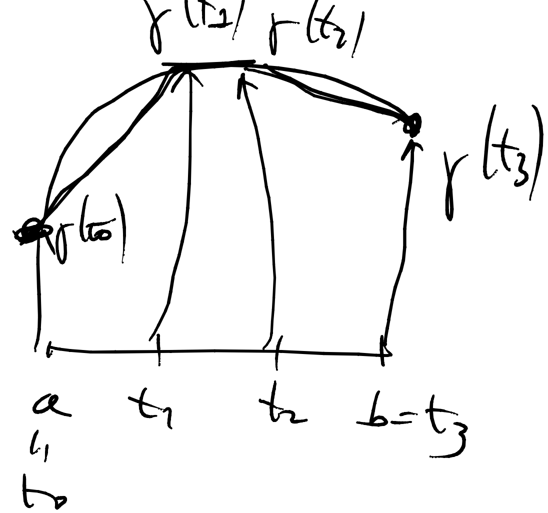
$\gamma : [a,b] \to \mathbb{R}^N$ $\longrightarrow$ $\Pi = \{t_0, t_1, \dots, t_n\}$

tale che $a = t_0 < t_1 < \dots < t_n = b$

partizione di $[a,b]$

Lunghezza della poligonale inscritta relativa a Π

$$\longrightarrow \quad \Lambda(\Pi) = \sum_{0}^{n} |\gamma(t_{i+1}) - \gamma(t_i)|$$



$$\longrightarrow \quad \Lambda(\gamma) = \sup_{\Pi} \Lambda(\Pi)$$

se $\Lambda(\gamma) < \infty$ γ è rettif. e $\Lambda(\gamma)$ è lunghezza

se $\Lambda(\gamma) = \infty$ γ è non rettif.

f è NON limitata in Ω se $\qquad$ $f : \Omega \to \mathbb{R}^p \quad \Omega \subseteq \mathbb{R}^n$

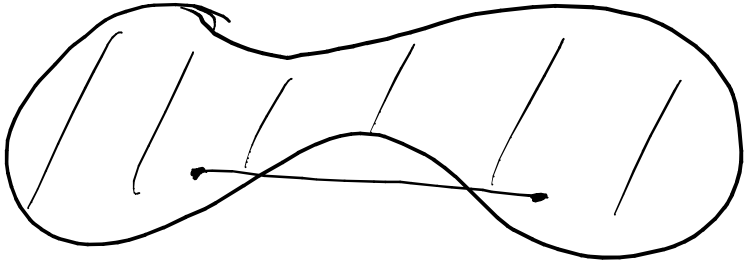
$$\forall K > 0 \ \exists x \in \Omega : \|f(x)\| > K$$

a_n è una successione NON limitata

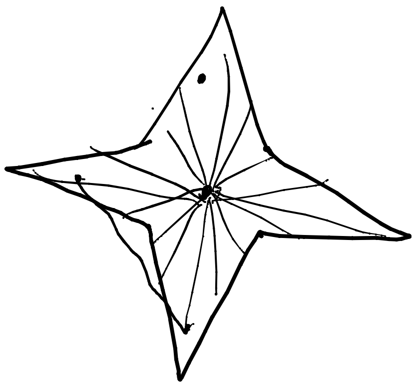
$$\forall k \geq 0 \ \exists \nu : |a_\nu| > k$$

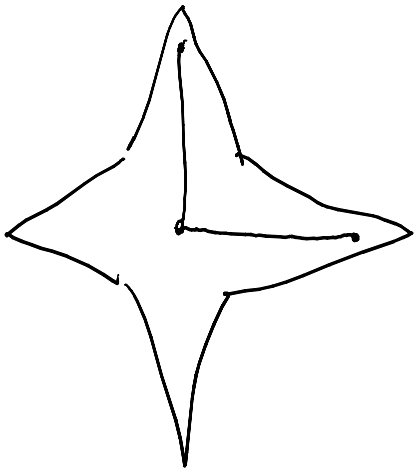
$$0, 1, 0, 2, 0, 3, 0, 4, \dots, 0, 2k, \dots$$

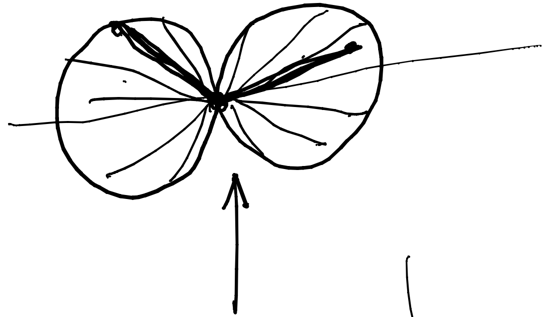
$$\exists K > 0 : \forall x \in \Omega \ \|f(x)\| \leq K$$

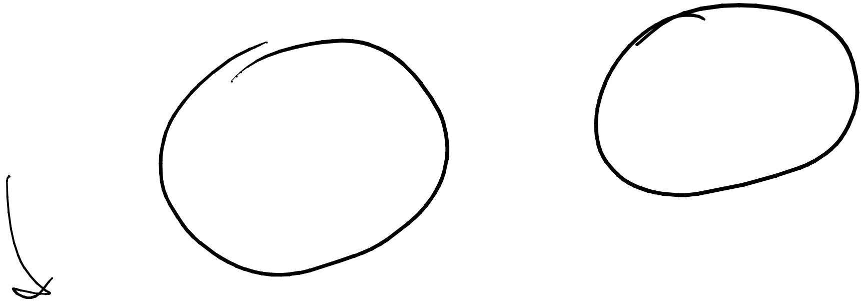


connesso ma non convesso

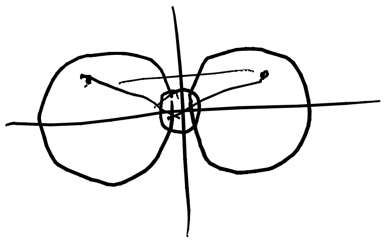






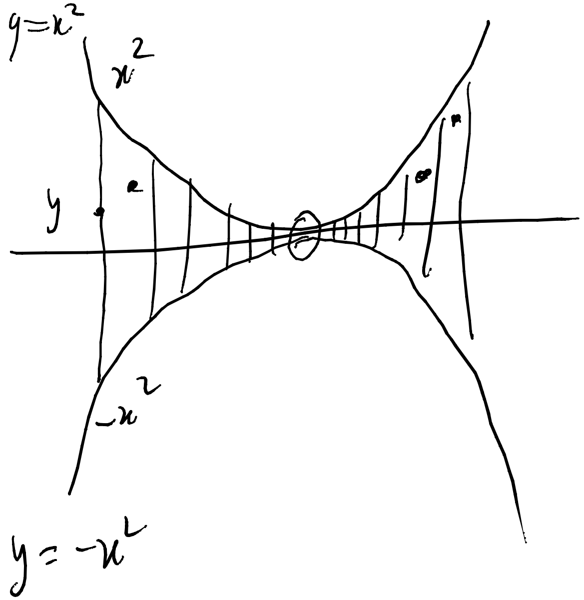


$$\{x^2+y^2-x<0\} \cup \{x^2+y^2+x<0\}$$



$\Omega = \{ |y| < x^2 \}$

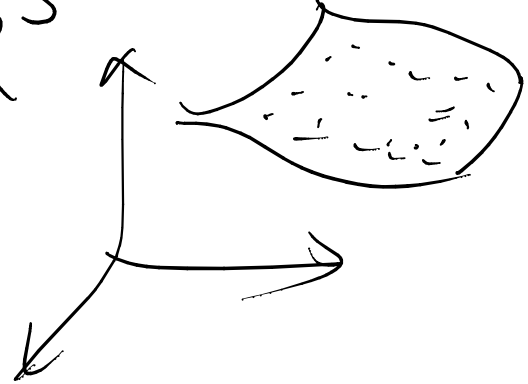
$-x^2 < y < x^2$



$(0,0) \notin \Omega$

Sconnesso

$\phi : \Delta \to \Sigma \subseteq \mathbb{R}^3$



$f : \Sigma \to \mathbb{R}$

$\gamma : [a,b] \to \Sigma \subseteq \mathbb{R}^n$

$f : \Sigma \to \mathbb{R}$

$$\int_\gamma f = \int_a^b \underbrace{f(\gamma(t))} \underbrace{|\dot{\gamma}(t)|\, dt}_{dl \quad dx}$$

$$\int_\phi f \equiv \int_\Delta \underbrace{f(\phi(u,v))} \underbrace{|\phi_u(u,v) \times \phi_v(u,v)|\, du\, dv}_{d\sigma \quad ds}$$

$A : \Omega \to \mathbb{R}^n \quad \Omega \subseteq \mathbb{R}^n$ di classe C^1 si dice irrotazionale se

$$(A_i)_{x_j} = (A_j)_{x_i} \quad \forall i,j$$

$\gamma : [a,b] \longrightarrow \mathbb{R}^N$

PARTIZIONE DI $[a,b]$

$\Pi = \{t_0, t_1, \dots, t_n \in [a,b] : a = t_0 < t_1 < \dots < t_n = b\}$

$$\Lambda(\Pi) = \sum_{0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)|$$

lunghezza della poligonale inscritta definita da Π

$$\Lambda(\gamma) = \sup_{\Pi} \Lambda(\Pi)$$

γ è rettificabile se $\Lambda(\gamma) < \infty$

e $\Lambda(\gamma)$ è la sua lunghezza

$\alpha = x\,dx + y\,dy$ $\quad A = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$ $\quad \dfrac{\partial A_1}{\partial y} \overset{?}{=} \dfrac{\partial A_2}{\partial x}$

$\dfrac{\partial x}{\partial y} = 0 \qquad \dfrac{\partial y}{\partial x} = 0$

α è chiusa (A è irrotazionale)

dom $A = \mathbb{R}^2$ convesso

$\Downarrow$

semplic. connesso

$$f(x,y): \begin{cases} f_x = x & \frac{1}{2}x^2 + c(y) \\ f_y = y & c'(y) = y \Rightarrow c(y) = \frac{1}{2}y^2 \end{cases}$$

$f(x,y) = \frac{1}{2}(x^2+y^2)$

$+ c$

$$\boxed{\alpha(x,\underbrace{dx}_{w}) = A_1(x)\underbrace{dx_1}_{w_1} + A_2(x)\underbrace{dx_2}_{w_2} + \dots + A_n(x)\underbrace{dx_n}_{w_n}}$$

$$A = \begin{pmatrix} A_1(x) \\ \vdots \\ A_n(x) \end{pmatrix}$$

$x \in \Omega$

$$(A_i)_{x_j} = (A_j)_{x_i} \quad \forall i,j$$

$\Omega \subseteq \mathbb{R}^n$

$\alpha : \Omega \times \mathbb{R}^n \to \mathbb{R}$

$w \to \alpha(x,w)$ è lineare $\forall x \in \Omega$

$\sin(x+y)\,dx - e^x\,dy$

$$\frac{\partial}{\partial y}\big(\sin(x+y)\big) = \cos(x+y)$$

$$\frac{\partial}{\partial x}\left(-e^x\right) = -e^x$$

sono diversi

$$\overset{A_1}{(xy+z)}\,dx + (y \sin t^2)\,dy + z\,dz$$

$$\frac{\partial A_1}{\partial x_2} = \frac{\partial (xy+z)}{\partial y} = x$$

$\neq$ non closed

$$\frac{\partial A_2}{\partial x_1} = \frac{\partial (y \sin t^2)}{\partial x} = 0$$

$$\frac{\partial A_1}{\partial x_3} = \frac{\partial (xy+z)}{\partial z} = 1$$

$\neq$

$$\frac{\partial A_3}{\partial x_1} = \frac{\partial z}{\partial x} = 0$$

$$\text{"}\nabla\text{"} \times A$$

$$\nabla \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

$$\phi : \Delta \to \mathbb{R}^3 \qquad \boxed{\phi \in C^1(\Delta)}$$

ϕ regolare

$$\nu(u,v) = \phi_u(u,v) \times \phi_v(u,v) \neq 0 \quad \underline{\forall u,v \in \Delta}$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \qquad \phi_u = \begin{pmatrix} (\phi_1)_u \\ (\phi_2)_u \\ (\phi_3)_u \end{pmatrix} \qquad \phi_v \; - - -$$

$$\phi' = \begin{pmatrix} (\phi_1)_u & (\phi_1)_v \\ (\phi_2)_u & (\phi_2)_v \\ (\phi_3)_u & (\phi_3)_v \end{pmatrix}$$

sfera standard

$$\phi(u,v) = \begin{pmatrix} \sin u \cos v \\ \sin u \sin v \\ \cos u \end{pmatrix}$$

ν normale esterna

ϕ regolare

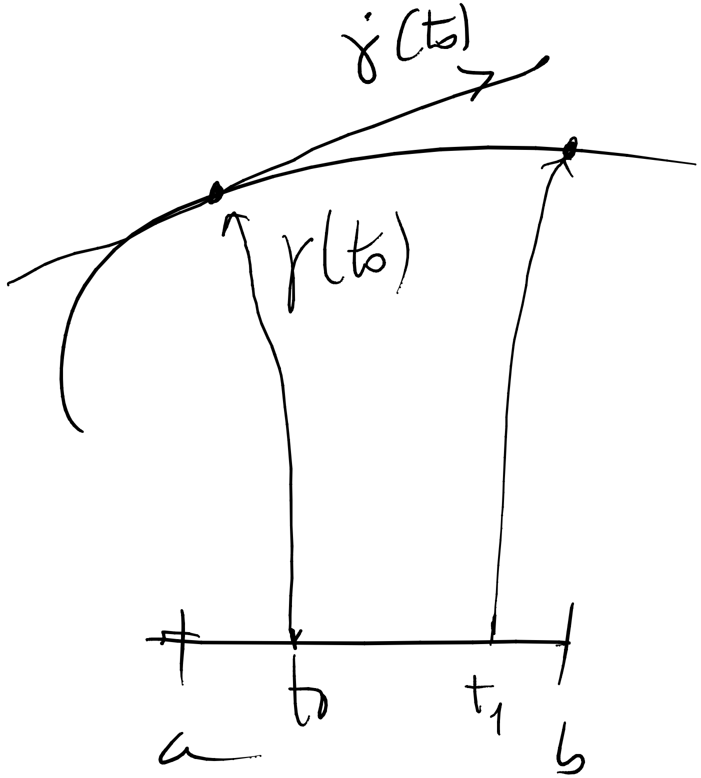
1) $\phi \in C^1(\Delta)$

2) $\phi_u \times \phi_v \neq 0 \quad \forall (u,v) \in \Delta$

3) ϕ iniettiva su $\mathring{\Delta}$

(punti interni a Δ)

$\gamma : [a,b] \to \mathbb{R}^n$ $\gamma(t_0)$ punto del sostegno (immagine) della curva



direzione della tangente

è $\dot\gamma(t_0)$

$a = \gamma_1(t)$

$b = \gamma_2(t)$

retta per $\gamma(t_0)$

in direzione $\dot\gamma(t_0)$

$$\boxed{\varphi(t) = \gamma(t_0) + t\,\dot\gamma(t_0)} \qquad \gamma(t) = \begin{pmatrix} a \\ b \end{pmatrix}$$

alternativa

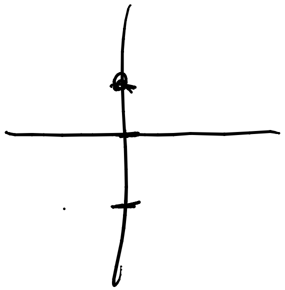
$$\sigma(t) = \gamma(t_0) + (t - t_0)\,\dot\gamma(t_0)$$

$x + y = 1$

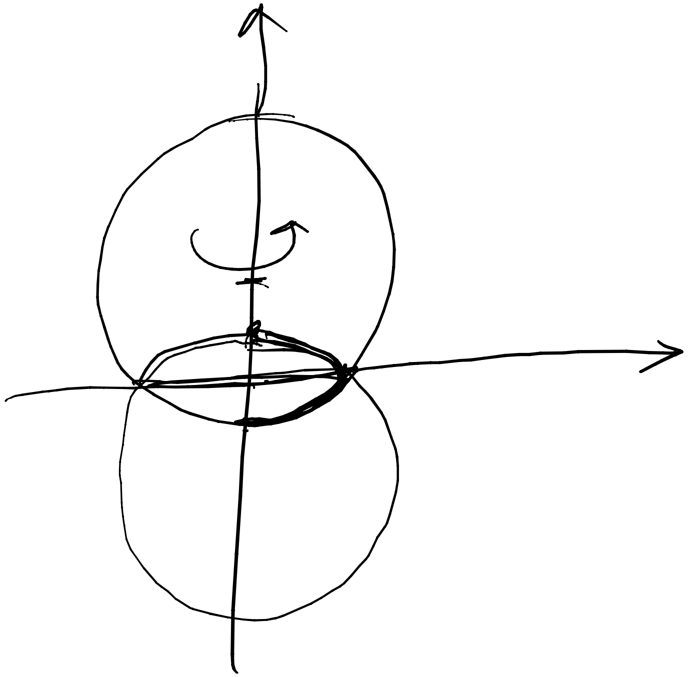
$2x + 2y = 2$

$$\{x^2+y^2+z^2-2z \le 3\} \cap \{x^2+y^2+z^2+2z \le 3\}$$

$$\rho^2 - 2\rho\cos\theta \le 3$$



$$\boxed{x^2+y^2+(z-1)^2 \le 4}$$



raggio 2

centro $(0,0,1)$

$$y^2 = 3+2z-z^2$$

$$y = \sqrt{3+2z-z^2}$$

$$x^2+y^2+z^2-2z \leq 3$$

$$x^2+y^2+z^2+2z \leq 3$$

ENRICO GIUSTI

ANALISI MATEMATICA II

BORINGHIERI