Note Title 6/18/2020

$h(t) = f(\gamma(t))$

$\gamma(t) \in \Omega = \mathrm{dom} f$
$\forall t \in [a,b]$

$q(w) = \dfrac{p(z_0 + w)}{p(z_0)}$

$h : [a,b] \to \mathbb{R}$

Memento

$\dfrac{|p(z_0+\bar{w})|}{|p(z_0)|} < 1$ tesi

La tesi è
$\exists \bar{w} : |q(\bar{w})| < 1$

$q(w) = 1 + \alpha_k w^k + w^{k+1} \tilde{q}(w)$

k minimo indice > 0
per cui $\alpha_k \neq 0$

$\Downarrow$ triang. in $\mathbb{C}$

$$\boxed{|q(w)| \leq |1 + \alpha_k w^k| + |w|^{k+1} |\tilde{q}(w)|}$$

1) e 2) $|1 + \alpha_k \bar{w}^k| = 1 - |\alpha_k| |\bar{w}|^k$

occorre scegliere w in modo da avere

1) $|\alpha_k w^k| < 1$ 2) $\alpha_k w^k$ reale e negativo

sono distinti

$\alpha_k \bar{w}^k \in \mathbb{R}$ e negativo

$\alpha_k \bar{w}^k$ è reale e negativo

$$|1+\alpha_k \bar{w}^k| = 1 - |\alpha_k||\bar{w}|^k$$

somma algebrica di numeri reali discordi. Quella di modulo max è 1

$$|q(\bar{w})| \leq 1 - |\bar{w}^k| \underbrace{\Big[\overbrace{|\alpha_k|}^{>0 \text{ perché } \alpha_k \neq 0} + \overbrace{|\bar{w}|}^{\to 0}\overbrace{|\tilde{q}(\bar{w})|}^{\to |\tilde{q}(0)|}\Big]}_{\text{perm. segno}}$$

$$\underbrace{\{1 > z > x^2\}}_{\Downarrow} \cap \{0 < y < 1 + z\}$$

$$z > 0 \qquad z \in \left]x^2, 1\right[$$

$$|x| < 1$$

$$\int_{-1}^{1} dx \int_{x^2}^{1} dz \int_{0}^{1+z} dy$$

$\lim_{\infty} \alpha(x)$

$$\boxed{\alpha(x) = \sum_{i,j=1}^{n} a_{ij} x_i x_j}$$

Se α è definita positiva $\lim = +\infty$

" " " " negativa $\lim = -\infty$

Altrimenti $\lim$ NON ESISTE (salvo che sia costante $(=0)$)

Semidefinita >0
$\lambda = 0$ e $\Lambda > 0$

$$\boxed{\lambda |x|^2 \le \alpha(x) \le \Lambda |x|^2}$$

λ: min autov. di α

Λ: max autov. α

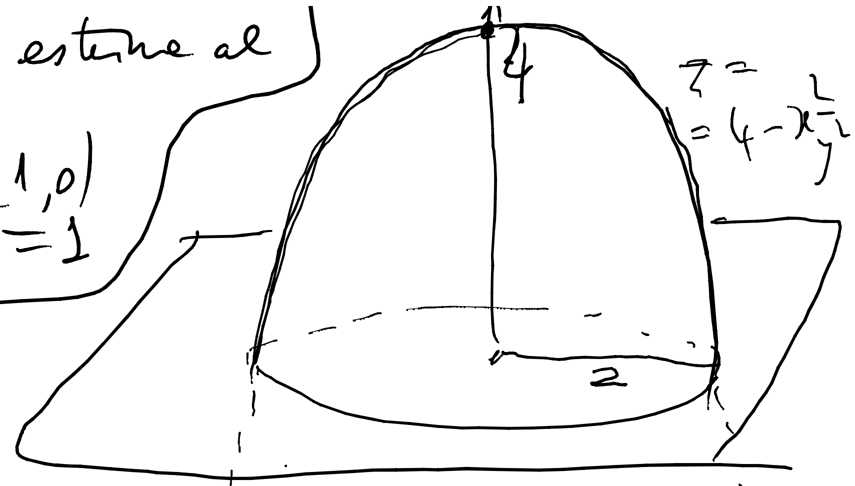
$\alpha(x,y) = x^2$
sull'asse x diverge a $+\infty$
sull'asse y $= 0$

l'uguaglianza vale sugli autovettori di λ (a sx) e Λ (a dx)

$$\Omega = \{ 0 \leq z < 4 - x^2 - y^2 \}$$ esterno al

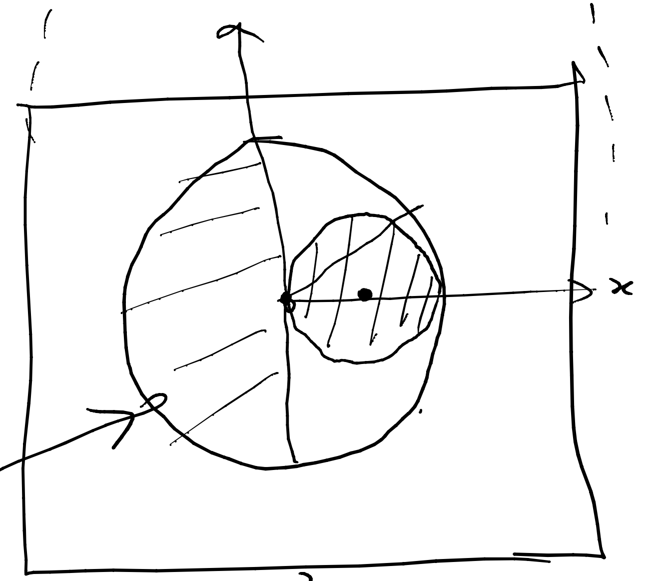
$$\{ (x-1)^2 + y^2 > 1 \}$$ centro $(1,0)$ raggio $= 1$

$x^2 + y^2 - 2x > 0$ cilindro



$$4 - x^2 - y^2 > 0$$

$$x^2 + y^2 < 4$$



$$\int_\Omega f(x,y)\,dx\,dy = \qquad \underline{f(x,y) = 4 - x^2 - y^2}$$

$$= \int_{\{x^2+y^2<4\} \cap \{x^2+y^2-2x>0\}} 4 - x^2 - y^2\,dx\,dy = \int d\theta \int d\rho\, \rho\,(4-\rho^2)$$

jacobiano coord. polari

$$x^2 + y^2 < 4 \Rightarrow 0 < \rho < 2$$

$$x^2+y^2-2x>0$$

$$\rho^2-2\rho\cos\theta>0$$

$\rho>0$

$$\boxed{\rho>2\cos\theta}$$

se $\cos\theta<0$ $\quad \rho>2\cos\theta$

$$\theta\in\left]\frac{\pi}{2},\frac{3}{2}\pi\right[$$

$$\boxed{2\cos\theta<\rho<2}$$

$$\underbrace{\int_{\frac{3}{2}\pi}^{\frac{\pi}{2}}d\theta\int_0^2 d\rho\,\rho(4-\rho^2)}_{\text{II e III quadrant}}+\underbrace{\int_0^{\pi/2}d\theta\int_{2\cos\theta}^{2}\rho(4-\rho^2)\,d\rho}_{\text{I quadrante}}+\underbrace{\int_{-\frac{\pi}{2}}^{0}d\theta\int_{2\cos\theta}^{2}\rho(4-\rho^2)\,d\rho}_{\text{IV quadr.}}$$

$f \in C^2(B_\delta(0))$

Hp. $\nabla f(x_0) = 0$ + ...

Taylor con resto di Peano (2 ordine)

$$f(x_0+w) = f(x_0) + \underbrace{\cancel{df(x_0,w)}}_{\text{Fermat}} + \frac{1}{2!}\sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\, w_i w_j + R_2(w)$$

$$f(x_0+w) - f(x_0) = \frac{1}{2}\sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\, w_i w_j + \underbrace{R_2(w)}_{} =$$

$$= |w|^2 \left[\underbrace{\frac{1}{2}\,\frac{\sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\, w_i w_j}{|w|^2}}_{\geq \frac{1}{2}\cdot\lambda > 0} + \underbrace{\frac{R_2(w)}{|w|^2}}_{\to 0 \text{ se } w \to 0} \right]$$

1) se l'Hessiana è def. p. > 0 $\quad \lambda > 0$

$$\boxed{\lambda |w|^2 \leq \sum_{i,j} f_{x_i x_j}(x_0)\, w_i w_j \leq \Lambda |w|^2}$$

$[\quad] > 0$ se

l'ε di $\lim_{w\to 0} \frac{R_2}{|w|^2}$ è scelto $< \frac{1}{2}\lambda$

2) def. < 0 $\qquad \varepsilon < \frac{|\Lambda|}{2}$

$$x^2+y^2+z^2=1 \qquad x^2+y^2<\frac{3}{4}$$

$$z=\sqrt{1-x^2-y^2}$$

sovrastante
$(z>0)$

$$\int_{x^2+y^2<\frac{3}{4}} \sqrt{1+z_x^2+z_y^2}\,dx\,dy =$$

$$=\int_{x^2+y^2<\frac{3}{4}} \sqrt{1+\left(\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2-y^2}}\right)^2+\left(\frac{-\cancel{2}y}{\cancel{2}\sqrt{1-x^2-y^2}}\right)^2}\,dx\,dy$$

$$=\int_{x^2+y^2<\frac{3}{4}} \sqrt{\frac{x^2}{1-x^2-y^2}+\frac{y^2}{1-x^2-y^2}+\underbrace{\frac{1-x^2-y^2}{1-x^2-y^2}}_{=1}}\,dx\,dy=$$

$$= \int_{x^2+y^2<\frac{3}{4}} \sqrt{\frac{x^2+y^2+1-x^2-y^2}{1-x^2-y^2}}\,dx\,dy = \int_{x^2+y^2<\frac{3}{4}} \frac{1}{\sqrt{1-x^2-y^2}}\,dx\,dy =$$

$$= \int_0^{2\pi} d\theta \int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-\rho^2}}\,\rho\,d\rho =$$ moltipl. e div. per -2

$$= -\frac{1}{2}\cdot 2\pi \int_0^{\sqrt{3}/2} \frac{d(1-\rho^2)}{\sqrt{1-\rho^2}} \underset{1-\rho^2=t}{=} -\pi \int_1^{1/4} \frac{1}{\sqrt{t}}\,dt =$$

$$= \pi \int_{\frac{1}{4}}^{1} \frac{1}{\sqrt{t}}\,dt$$

$$p(z) = \alpha_n z^n + \alpha_{n-1} z^{n-1} + \dots + \alpha_1 z + \alpha_0 =$$

$$= z^n \left[\alpha_n + \frac{\alpha_{n-1}}{z} + \dots + \frac{\alpha_0}{z^n}\right]$$

$$\frac{\alpha_{n-1}}{z} \to 0 \qquad \frac{\alpha_0}{z^n} \to 0$$

$$|p(z)| = |z|^n \left|\alpha_n + \text{roba che va a } 0\right|$$

$$\frac{|\alpha_n|}{2} \quad \text{Permanenza del segno} \quad |\alpha_n| > 0$$

per $|z| > M$

$$|p(z)| > \boxed{\frac{|\alpha_n|}{2} |z|^n} > \varepsilon \text{ per } |z| > \delta$$

$\alpha_i \neq 0$

$$\left|\frac{\alpha_i}{z^i}\right| < \varepsilon$$

$$\Updownarrow$$

$$|z^i| > \frac{\varepsilon}{|\alpha_i|}$$

$$|z|^n > \frac{2\varepsilon}{|\alpha_n|}$$

$$\Uparrow$$

$$|z| > \left(\frac{2\varepsilon}{|\alpha_n|}\right)^{\frac{1}{n}} = \delta$$

1) $\dfrac{1}{x^2+2y^2}$ 2) $\dfrac{1}{x^2-y^2}$ 3) $\dfrac{1}{-x^2-3y^2}$ Il loro modulo diverge a $+\infty$ in 0.

$w \in \mathbb{R}^n$

$$\lim_{0} f(w) = 0 \qquad \underbrace{\lim_{0} \frac{1}{|f(w)|} = +\infty}$$

$$\boxed{\lim_{x\to 0} \frac{1}{x} \text{ NON ESISTE}}$$

$$\underbrace{\frac{1}{|f(w)|} > \varepsilon > 0} \qquad \boxed{\begin{array}{l} t \to \frac{1}{t} \\ t > 0 \end{array}}$$

$\Updownarrow$

$$\boxed{|f(w)| < \frac{1}{\varepsilon}} \text{ vera per } \boxed{\begin{array}{l} |w| < \delta \\ w \neq 0 \\ w \in \operatorname{dom} f \left(\text{anche dom } \frac{1}{f}\right) \end{array}}$$

Il modulo del reciproco di un infinitesimo è un infinito nel caso 1) $\quad |x^2+2y^2| = x^2+2y^2$ e quindi

$$\lim_{0} \frac{1}{x^2+2y^2} \equiv \lim_{0} \frac{1}{|x^2+2y^2|} = +\infty$$

nel caso 3)

$$\lim_{0} \frac{1}{-x^2-3y^2} = -\lim_{0} \frac{1}{|-x^2-3y^2|} = +\infty$$

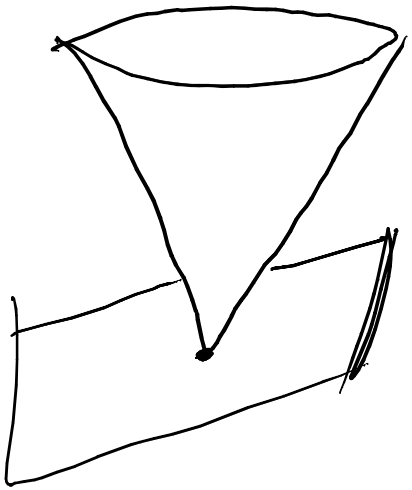
$= -\infty$

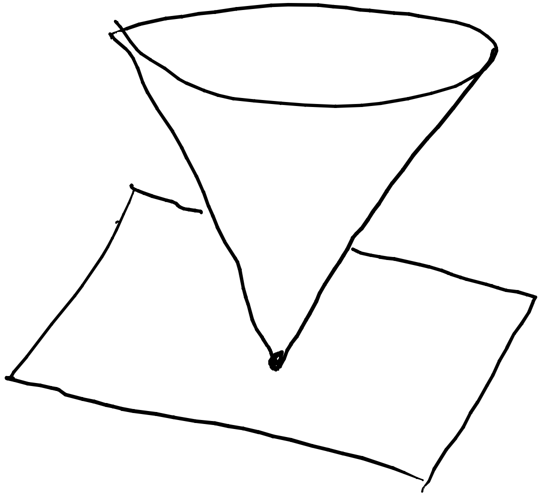
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

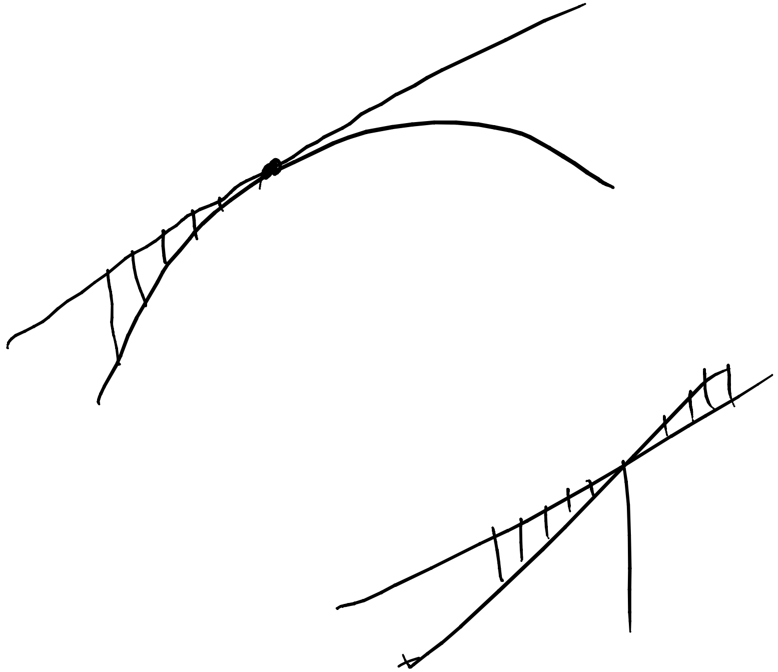
nel caso 2) la forma quadratica x^2-y^2 è indefinita e quindi il segno cambia $\Longrightarrow$ il lim <u>NON ESISTE</u> (come per $\lim_{0} \frac{1}{x}$)

$$\frac{f(x_0+w) - \big(f(x_0) + \underbrace{\nabla f(x_0)\, w}_{\text{affine}}\big)}{|w|} \longrightarrow 0 \quad \text{se } f \text{ è diff.}$$

$$z = f(x_0) + \nabla f(x_0)\, w \longleftarrow \text{piano per } (x_0, f(x_0))$$







$$z^2 = x^2 + y^2 \qquad z = \sqrt{x^2+y^2}$$

$$\lim_{\infty} \frac{1}{2x^2+y^2}$$

$$\lim_{\infty} \frac{1}{f(x)} = 0 \quad \text{se so che} \quad \lim_{\infty} f(x) = \infty$$

$$\left| \frac{1}{f(x)} \right| < \varepsilon \iff |f(x)| > \frac{1}{\varepsilon} \quad \text{sì se } |x| > \delta$$

$x \in \operatorname{dom} f$

$f(x) \neq 0$

$|x| < \delta$

$x \in \operatorname{dom} \frac{1}{f}$

I segni di forme quadratiche di regenti (e così quelle definite) sono importanti.

$$\lim_{\infty} \frac{1}{x^2}$$

$f(x,y) = \frac{1}{x^2}$

definita in $\mathbb{R}^2 \setminus$ asse y

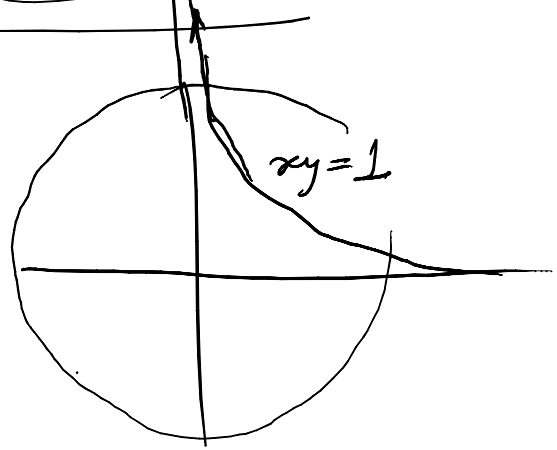
$\alpha(x,y) = x^2$ è semi def. ≥ 0

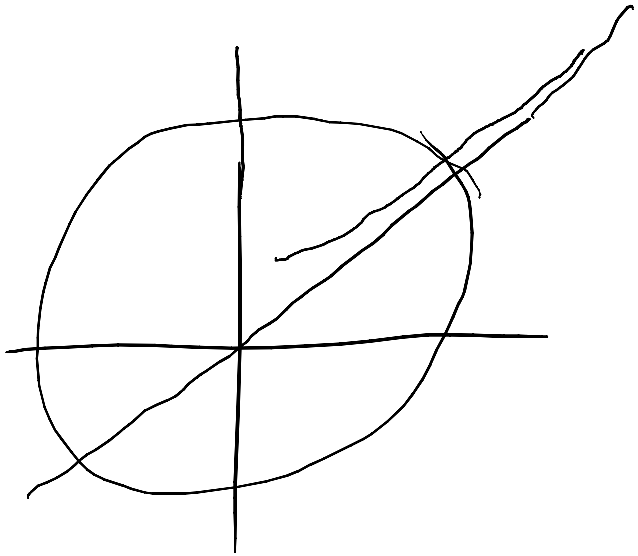
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$(x,y) = (x,0)$

(sull'asse x)

$$\frac{1}{x^2} \to 0$$





$$(x,y) = \left(\frac{1}{y}, y\right)$$

$$\lim_{\infty} f\left(\frac{1}{y}, y\right) =$$

$$= \lim_{\infty} \frac{1}{\left(\frac{1}{y}\right)^2} = \lim_{(x,y)\to\infty} y^2 = +\infty$$

$\frac{1}{x^2-y^2} \to 0$ sull'asse x

diverge approssimando $y=x$

$$\lim_{\infty} \underbrace{a_n z^n + \dots + a_1 z + a_0}_{p(z)} = \infty \qquad n \geq 1$$

$$p(z) \quad a_n z^n + \dots + a_0 = z^n \left[a_n + \frac{a_{n-1}}{z} + \dots + \frac{a_0}{z^n} \right]$$

$\frac{a_{n-1}}{z} \to 0$, $\frac{a_0}{z^n} \to 0$

<u>Nota</u> che $\lim_{\infty} \frac{a_k}{z^k} = 0$ perché

$$\left| \frac{a_k}{z^k} \right| = \frac{|a_k|}{|z|^k} < \varepsilon \quad \text{se e solo se} \quad |z|^k > \frac{|a_k|}{\varepsilon}$$

$$\text{e quindi se} \quad |z| > \underbrace{\left[\frac{|a_k|}{\varepsilon} \right]^{1/k}}_{\delta}$$

$$|p(z)| = |z|^n \left| a_n + \dots + \frac{a_0}{z^n} \right| \qquad \lim_{\infty} \left| a_n + \dots + \frac{a_0}{z^n} \right| = |a_n| > 0$$

Perm. segno $\left|a_n + \dots + \frac{a_0}{z^n}\right| > \frac{|a_n|}{2}$ se $|z| > \delta$

basta porre $\underline{\underline{\varepsilon < \frac{|a_n|}{2}}}$

$$|a_n| + \varepsilon \le \underbrace{\left|a_n + \dots \frac{a_0}{z^n}\right| \le |a_n| - \varepsilon}_{} \quad |z| > \delta$$

$> \frac{|a_n|}{2}$

$$\boxed{|p(z)| > |z|^n \frac{|a_n|}{2}} \text{ diverge}$$

$$|z|^n \frac{|a_n|}{2} > \varepsilon \qquad |z|^n > \frac{2\varepsilon}{|a_n|}$$

$$\Downarrow$$

$$|z| > \left[\frac{2\varepsilon}{|a_n|}\right]^{\frac{1}{n}}$$