Note Title 6/16/2020

somme di Riemann di

$$\int_a^b \gamma(t)\,dt \sim \sum_0^{n-1} \gamma(\xi_i)(t_{i+1}-t_i)$$

$$\left| \int_a^b \gamma(t)\,dt \right| \le \int_a^b \underbrace{|\gamma(t)|}_{\text{scalare}}\,dt$$

$\in \mathbb{R}^n$ $\quad a<b$

$$\begin{pmatrix} \sum_0^{n-1} \gamma_1(\xi_i)(t_{i+1}-t_i) \\ \\ \sum_0^{n-1} \gamma_n(\xi_i)(t_{i+1}-t_i) \end{pmatrix} = \sum_0^{n-1}(t_{i+1}-t_i)\begin{pmatrix} \gamma_1(\xi_i) \\ \vdots \\ \gamma_n(\xi_i) \end{pmatrix}$$

$t_0 < t_1 < \dots < t_n$

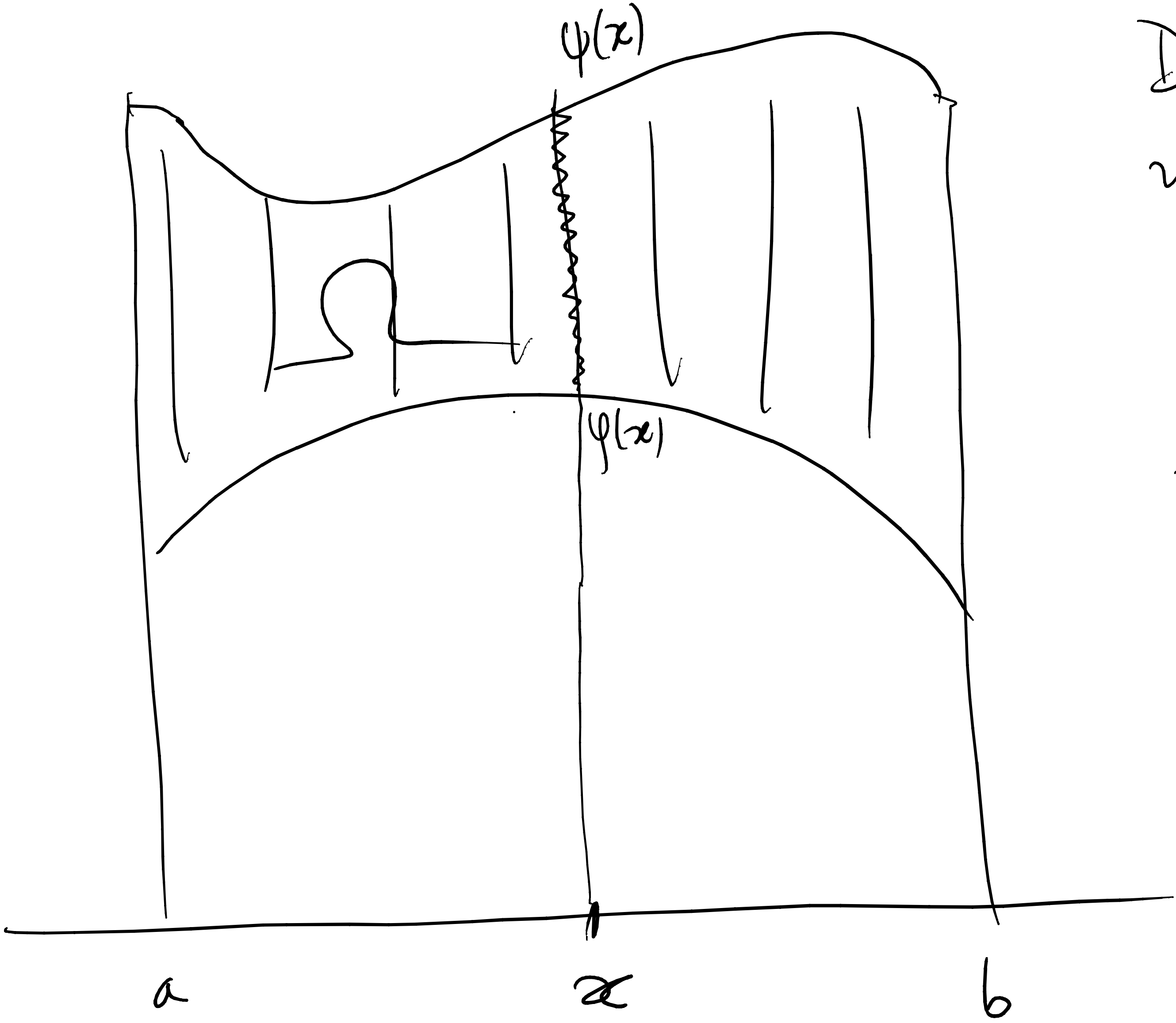
$$|\quad| \le \sum_0^{n-1}(t_{i+1}-t_i)\overbrace{|\gamma(\xi_i)|}^{\text{scalare}} \sim \int_a^b |\gamma(t)|\,dt$$

somme di Riemann di

$\boxed{f \le |f|}$ $\boxed{a<b}$

$-f \le |f| \Rightarrow \int_a^b f \le \int_a^b |f|$ $\quad \underline{\left|\int f\right| \le \int |f|}$

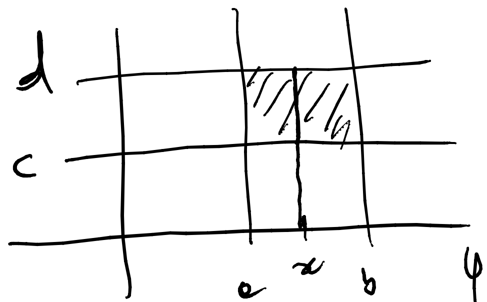
$-\int_a^b f = \int -f \le$



Domini normali rispetto ad x

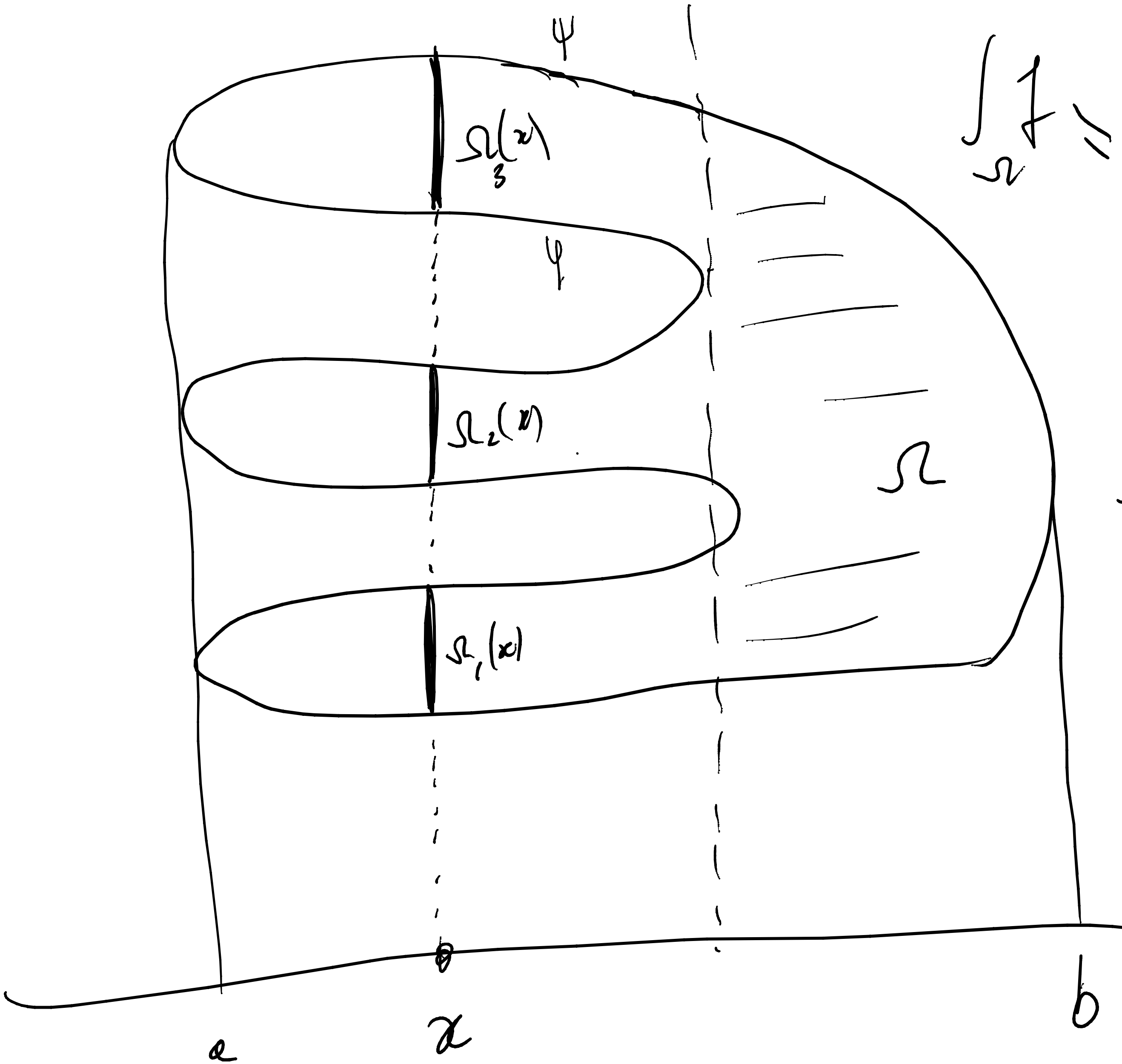
$x \in [a, b]$

$y \in [\varphi(x), \psi(x)]$

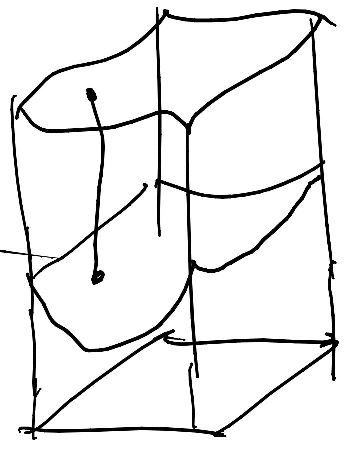


$\varphi(x) = c$
$\psi(x) = d$

$[a, b] = \Pi_x \Omega \qquad \Omega(x) = [\varphi(x), \psi(x)]$



$$\int_\Omega f = \int_a^b dx \left[\int_{\Omega_1(x)} f(x,y)\,dy + \int_{\Omega_2(x)} + \int_{\Omega_3(x)} \right]$$



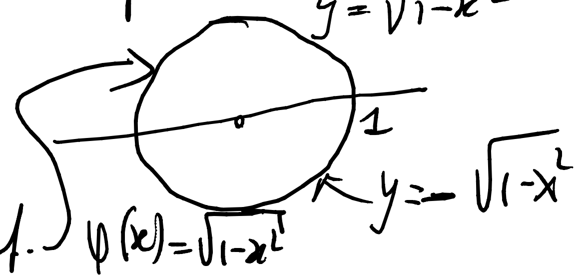
Estremi di $f : \Omega \to \mathbb{R}$ con Ω chiuso e limitato

(1) Studiare separatamente i punti interni su $\mathring{\Omega}$ ogni estremo x_0 (anche solo locale) verifica la condizione di Fermat $\boxed{\nabla f(x_0) = 0}$

(2) Studiare gli estremi su $\underline{\partial\Omega}$ (supposto un po' regolare)

Se $\partial\Omega$ si può decomporre in grafici cartesiani (pezzi di bordo del tipo $\underset{\in\partial\Omega}{}(x, \varphi(x))$ φ funzioni decenti

$x \to f(\overbrace{x, \varphi(x)}^{\in \partial\Omega})$ 1 variabile assume gli stessi valori di f sulle semicirconf.



b) $\exists \gamma : [a,b] \to \partial\Omega$ suriettiva

$x \to f(\gamma(x))$ assume tutti i valori di f su $\partial\Omega$

c) Se $\exists g : \mathbb{R}^2 \to \mathbb{R} : \quad \partial\Omega = \{(x,y) : g(x,y) = 0\}$

Lagrange $\underline{\text{estremi}}$ (liberi) di $f(x,y) + \lambda g(x,y) = \Phi$

$$\begin{matrix} \Phi_x \\ \\ \Phi_y \\ \\ \Phi_\lambda \end{matrix} \quad \begin{cases} f_x + \lambda g_x = 0 \\ f_y + \lambda g_y = 0 \\ g(x,y) = 0 \end{cases}$$

nel caso $\Omega = \{x^2+y^2 \leq 1\}$

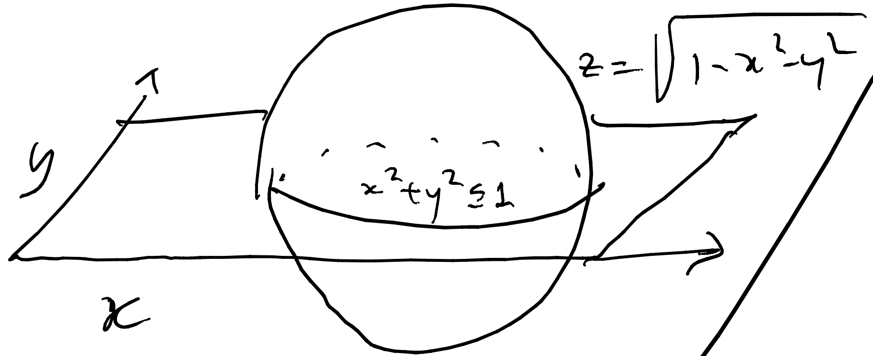
a) $(x, \sqrt{1-x^2}) \quad x \in [-1,1]$

$(x, -\sqrt{1-x^2}) \quad x \in [-1,1]$

b) $(\cos t, \sin t) \quad t \in [0, 2\pi]$

c) $g(x,y) = x^2+y^2-1$

$\Omega \{ x^2+y^2+z^2 \leq 1 \}$



$\partial \Omega$

a) $\left(x, y, \sqrt{1-x^2-y^2}\right)$ con $\underline{x^2+y^2 \leq 1}$

$\left(x, y, -\sqrt{}\right)$ "

$$(x,y) \to f\left(x, y, \sqrt{1-x^2-y^2}\right)$$

$$(x,y) \in \overline{B_1(0,0)}$$

b) $(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\theta \in [0,\pi] \quad \varphi \in [0, 2\pi]$

c $\quad g(x,y,z) = x^2+y^2+z^2-1$

$\Downarrow$

$$\begin{cases} x^2+y^2+z^2 = 1 \\ f_x + 2\lambda x = 0 \\ f_y + 2\lambda y = 0 \\ f_z + 2\lambda z = 0 \end{cases}$$

f è continua in x_0 se $\lim_{w \to 0} |f(x_0+w) - f(x_0)| = 0$

$$f(x_0+w) - f(x_0) = \underbrace{f(x_0+w) - f(x_0) - A(w)} + A(w) =$$

$$= |w| \underbrace{\frac{f(x_0+w) - f(x_0) - A(w)}{|w|}}_{\substack{w\to 0 \\ \text{ipot.}}} + \underbrace{A(w)}_{w \to 0}$$

0, $w \neq 0$, 0, 0, 0

$A = df$

$|w| \dfrac{A(w)}{|w|}$ o-ung.

$$A(w) = A\left(\sum w_i e_i\right) = \sum w_i \underbrace{A(e_i)}_{\in \mathbb{R}} \xrightarrow[w \to 0]{} 0$$

perché $w \to 0$ $w_i \to 0$ $\forall i$

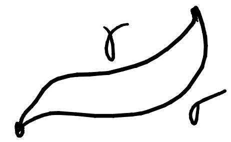
0

1) A è integrabile su Ω se $\forall \gamma$ chiusa in Ω $\int_\gamma A = 0$ (C.N.S. se $A \in C^0$)

2) Ω è sempl. connesso allora ogni γ chiusa in Ω è deform. in una σ costante

3) Th. invarianza omotopica se γ e σ sono omotope in Ω e A è irrotaz. in Ω $\Rightarrow \int_\gamma A = \int_\sigma A$

Sul Prodi An 2



4) da 2) e 3) $\int_\gamma A = \int_\sigma A = 0$ → σ è costante e $\dot{\sigma} \equiv 0$

$\Longrightarrow \int_{\text{ogni curva chiusa}} A = 0 \Leftrightarrow 1 \Leftrightarrow$ tesi.

$$\gamma(t_{i+1}) - \gamma(t_i) \underset{\text{Torric.}}{=} \int_{t_i}^{t_{i+1}} \dot\gamma(t)\,dt$$

$$f(b) - f(a) = \int_a^b f'(x)\,dx$$

$\underline{\underline{\text{se}}}$ f' è continua su $[a,b]$

$$\parallel$$

$$\gamma_1(t_{i+1}) - \gamma_1(t_i) = \int_{t_i}^{t_{i+1}} \dot\gamma_1(t)\,dt \qquad \dot\gamma_1 \text{ è continua in } [t_i, t_{i+1}]$$

$$\vdots$$

$$\gamma_N(t_{i+1}) - \gamma_N(t_i) = \int_{t_i}^{t_{i+1}} \dot\gamma_N(t)\,dt \qquad \dot\gamma_N \text{ è continua su } [t_i, t_{i+1}]$$

$$\gamma \text{ è } C^1 \Rightarrow \dot\gamma_i \in C^0 \quad \forall i = 1 \dots N$$

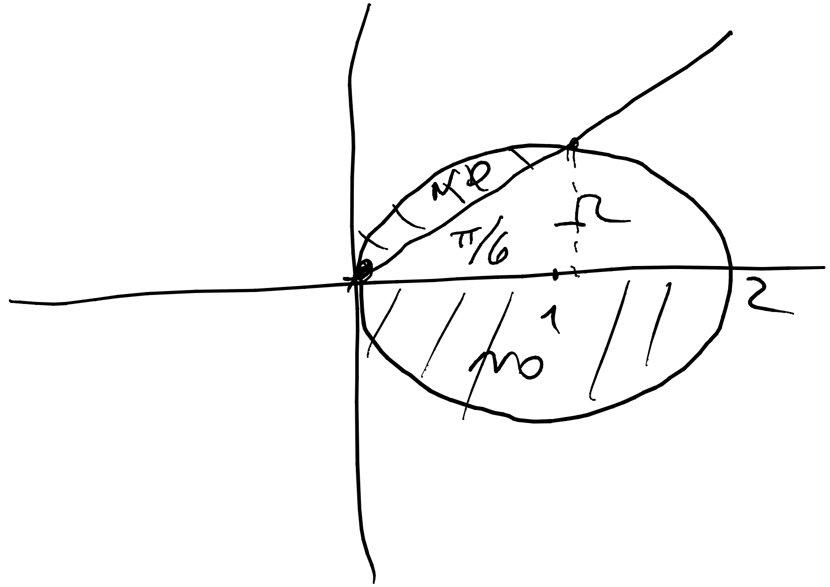
<u>area</u> $(1,0)$ raggio 1

$$\int_{\Omega} 1\, dx\, dy = \int_0^{\frac{\pi}{6}} d\theta \int_0^{2\cos\theta} 1 \cdot \rho \, d\rho =$$

jacob.

$$= \int_0^{\frac{\pi}{6}} \frac{1}{2} 4\cos^2\theta \, d\theta = \underbrace{\int_0^{\frac{\pi}{6}} 1 + \cos 2\theta \, d\theta}_{} =$$

$$\underline{2\cos^2\theta} - 1 = \cos 2\theta$$



$$(x-1)^2 + y^2 \le 1$$

$$x^2 + y^2 - 2x + \cancel{1} \le \cancel{1}$$

$$\boxed{0 < \rho} \Rightarrow \rho^2 \le 2\rho\cos\theta$$

$$\Downarrow$$

$$\rho < 2\cos\theta$$

$$(4x^2+3y^2)^{-\frac{1}{2}}(4x,\ 3y)$$

$\text{dom}\, A = \mathbb{R}^2 \setminus \{(0,0)\}$

<u>CONNESSO</u>

$$\begin{cases} f_x = \dfrac{4x}{\sqrt{4x^2+3y^2}} \\[2ex] f_y = \dfrac{3y}{\sqrt{4x^2+3y^2}} \end{cases} \Rightarrow f = \int \frac{4x\,dx}{\sqrt{4x^2+3y^2}} = \frac{1}{2}\int \frac{d_x(4x^2+3y^2)}{\sqrt{4x^2+3y^2}} =$$

$$(4x^2+3y^2)_x = 8x$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}\int t^{-\frac{1}{2}}\,dt =$$

$t = 4x^2+3y^2$

$$= \frac{1}{2}\left[\frac{1}{\frac{1}{2}}\, t^{\frac{1}{2}}\right] = \sqrt{4x^2+3y^2} + c(y)$$

$$\Downarrow$$

$$f_y = c'(y) + \frac{\cancel{6}^{\,3} y}{\cancel{2}\sqrt{4x^2+3y^2}}$$

$$\Downarrow$$

$$c'(y) = 0 \Rightarrow c = \text{cost.}$$

$$\boxed{f(x,y) = \sqrt{4x^2+3y^2} + C}$$

$$\underline{\lambda |x|^2} \le \underbrace{\sum_{i,j} a_{ij} x_i x_j}_{\alpha(x)} \le \underline{\Lambda |x|^2}$$

Se $\alpha(x)$ è definita $> 0 \iff \lambda > 0$

$$\alpha(x) \ge \underset{>0}{\lambda} |x|^2 \to +\infty$$

$+\infty$

$\lim_{x\to\infty}$ $\quad |x| \to +\infty$

segue th. confronto

$\alpha \to +\infty$

α def. $< 0 \overset{}{\underset{\text{def.}}{\iff}} \Lambda < 0$

$$\alpha(x) \le \underbrace{\underset{<0}{\Lambda} \underbrace{|x|^2}_{+\infty}}_{-\infty}$$

confr. $\alpha(x) \to -\infty$

(1) <u>α indefinita</u> $\Longleftrightarrow$ $\lambda < 0$ $\quad \Lambda > 0$

se u è un autovettore di λ } di norma 1
e v è un autov. di Λ } accade che

$$\alpha(u) = \lambda \qquad \alpha(v) = \Lambda$$

$$\lim_{t \to +\infty} \alpha(tu) = \underbrace{\lambda}_{<0} t^2 \longrightarrow -\infty \qquad \lim_{t \to +\infty} \alpha(tv) = \underset{\substack{\downarrow \\ +\infty}}{t^2} \underset{>0}{\Lambda} \to +\infty$$

$$|tu| \to +\infty$$

$$|tv| \to +\infty$$

il $\lim_{\infty} \alpha$ NON esiste

Caso semidefinito

α semidef. ≥ 0 $\qquad \lambda = 0 \qquad \Lambda > 0$

u versore nell'autospazio di $\lambda = 0$

v " " " Λ

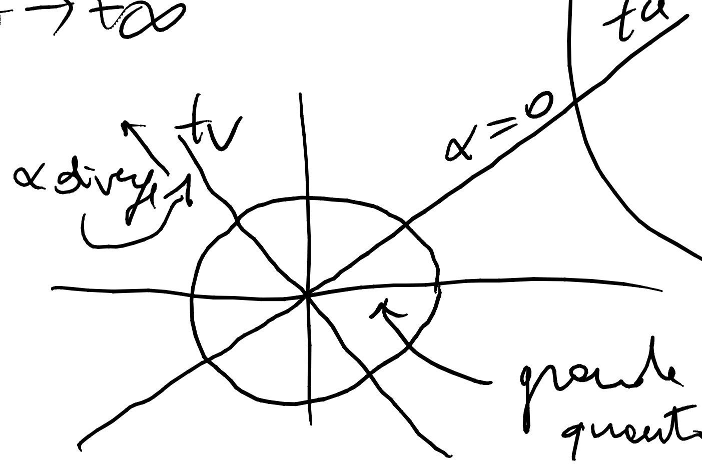
$\alpha(u) = 0 \qquad \alpha(v) = \Lambda > 0$

$\lim_{\infty} 4x^2 + 3y^2$

$\begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}$

def. > 0

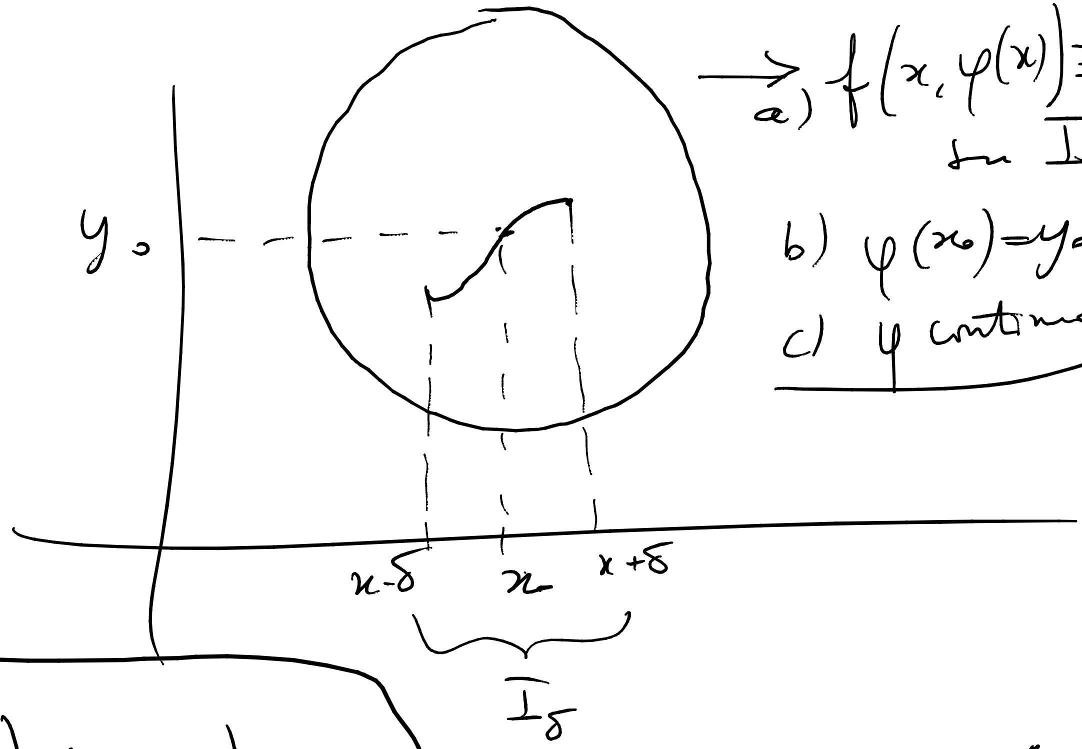
$\lim_{t \to +\infty} \alpha(tu) = 0 \qquad \lim_{t \to +\infty} \alpha(tv) = \lim_{+\infty} t^2 \Lambda = +\infty$



grande quanto voglio

NON ESISTE

$f \in C^1(B_\rho(x_0,y_0))$



$\to$ a) $f(x,\varphi(x)) \equiv 0$ in I_δ

b) $\varphi(x_0)=y_0$

c) φ continua in $\bar{I}_\delta$

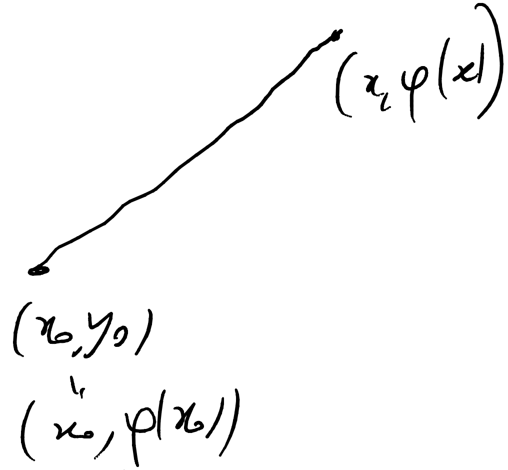
$$\overset{0}{\underset{a)}{\|}}\ \ \overset{0}{\underset{a)}{\|}}\ \ \overset{y_0\ b)}{}$$

$$f(x,\varphi(x)) - f(x_0,\varphi(x_0)) =$$

$$= f_x\Big(\underbrace{x_0+\xi(x-x_0)}_{\to x_0},\ \varphi(x_0)+\xi(\underbrace{\varphi(x)-\varphi(x_0)}_{\to 0\ c)})\Big)(x-x_0) +$$

$$+ f_y\Big(\qquad\qquad \underbrace{\qquad}_{\varphi(x_0)} \qquad\qquad\Big)(\varphi(x)-\varphi(x_0))$$

$$\frac{\varphi(x)-\varphi(x_0)}{x-x_0} = -\frac{f_x}{f_y}\Big(\qquad\Big)$$



$h(x) = f(x,\varphi(x))$

$h(x)-h(x_0) = h'(\xi)(x-x_0)$ $\xi \in [x_0,x]$ (oppure $[x,x_0]$)

$$\int_{\Omega} f_x \, dx\,dy = \int_{\partial\Omega^+} f \nu_1 \, ds \qquad \int_{\partial\Omega^+} f\,dy$$

$$\int_{\Omega} f_y \, dx\,dy = \int_{\partial\Omega^+} f \nu_2 \, ds \qquad -\int_{\partial\Omega^+} f\,dx$$

$$\int_{\Omega}$$

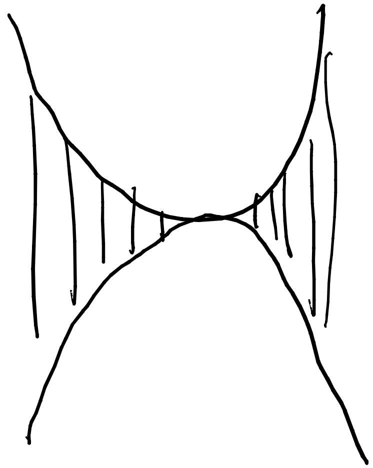
$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \qquad \int_{\Omega} \overbrace{(A_1)_{x_1} + (A_2)_{x_2}}^{\operatorname{div} A} \, dx\,dy = \int_{\partial\Omega^+} \underbrace{A_1 \cdot \nu_1 + A_2 \nu_2}_{A\,\nu} \, ds$$

$\boxed{\text{prodotto scalare}}$

$$\{\, |y| \le x^2 \,\}$$

$$-x^2 \le y \le x^2$$

$$\{(x,y) : -x^2 < y < x^2$$



$$|a| < 1$$

$$-1 < a < 1$$

$$|y| \le |x|$$

$$\Updownarrow$$

$$-|x| \le y \le |x|$$