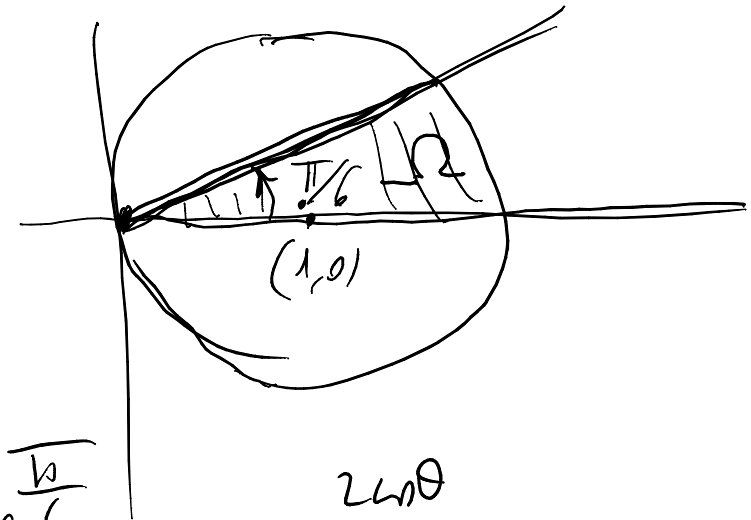
Note Title 6/4/2020



$$\text{Area } \Omega = \int_\Omega 1 =$$

$$= \int d\theta \int d\rho \, \rho = \int_0^{\pi/6} d\theta \int_0^{2\cos\theta} \rho \, d\rho =$$

$$(x-1)^2 + y^2 < 1$$

circonferenza di centro $(1,0)$ e raggio 1

$$x^2 + y^2 - 2x < 0$$

$$\rho^2 < 2\rho\cos\theta \overset{\rho>0}{\Longrightarrow} \boxed{\rho < 2\cos\theta}$$

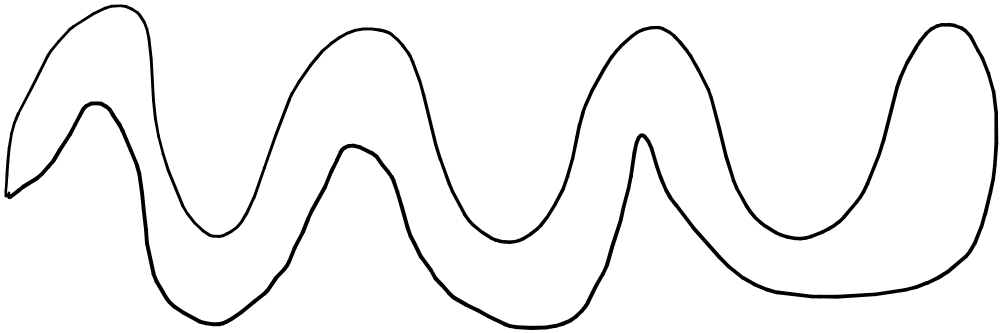
$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

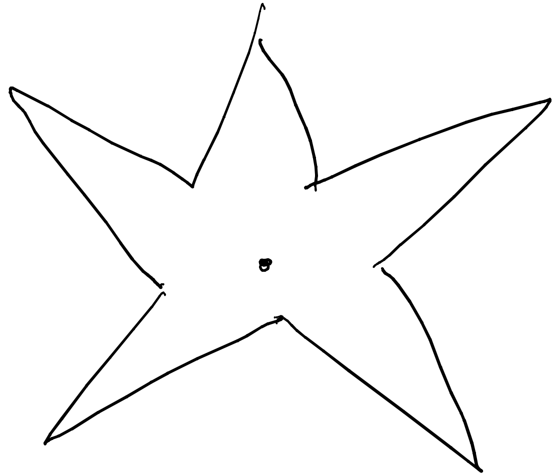
$$\theta \in \left[0, \frac{\pi}{6}\right]$$

$$= \int_0^{\frac{\pi}{6}} d\theta \; \frac{1}{2}\rho^2 \Big|_0^{2\cos\theta} =$$

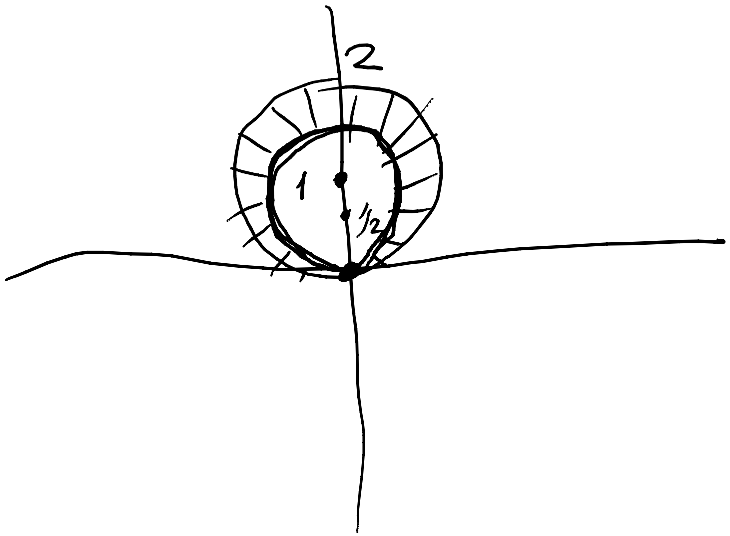
$$= \int_0^{\frac{\pi}{6}} \frac{1}{2} 4\cos^2\theta \, d\theta =$$

$$= 2\int_0^{\frac{\pi}{6}} \cos^2\theta \, d\theta = 2\left[\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \ldots\ldots$$





$$\{x^2+y^2-2y<0\} \cap \{x^2+y^2-y \geq 0\}$$



$$x^2+\left(y-\frac{1}{2}\right)^2 \geq \frac{1}{4}$$

$\left(0, \frac{1}{2}\right)$ raggio $\frac{1}{2}$

$$\begin{cases} y_1 = f_1(x_1 \ldots x_n) \\ \vdots \\ y_n = f_n(x_1 \ldots x_n) \end{cases}$$

$$f = \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} \qquad f : \mathbb{R}^n \to \mathbb{R}^n$$

$$\boxed{\begin{cases} f_1(x_1 \ldots x_n) - y_1 = 0 \\ \\ f_n(x_1 \ldots x_n) - y_n = 0 \end{cases}}$$

$\exists \ g_1 \ldots g_n$:

$$\begin{cases} x_1 = g_1(y_1 \ldots y_n) \\ \vdots \\ x_n = g_n(y_1 \ldots y_n) \end{cases} \; ?$$

$$x_i = \varphi_i(y_1 \ldots y_n)$$

$$g = \varphi$$

$$f(x,y) = xy \qquad \{x^2+y^2 \le 9\} = \Omega$$

Area graph f su Ω $= \displaystyle\iint_{x^2+y^2\le 9} \sqrt{1+y^2+x^2}\,dx\,dy =$

$$= \frac{1}{2}\int_0^{2\pi} d\theta \int_0^3 \sqrt{1+\rho^2}\; 2\rho\; d\rho = \frac{1}{\cancel{2}}\, \cancel{2}\pi \int_0^3 \sqrt{1+\rho^2}\; d(1+\rho^2) =$$

è la derivata di

$1+\rho^2 = t$

$$= \pi\, \frac{2}{3} \left(1+\rho^2\right)^{3/2} \Big|_0^3 = \dots$$

$$\frac{2xy\,dx}{(1+x^2)^2} - \frac{dy}{(1+x^2)} \qquad \text{dom } \mathbb{R}^2$$

$$\begin{cases} f_x = \dfrac{2xy}{(1+x^2)^2} \overset{?}{=} \boxed{\dfrac{y\,(2x)}{(1+x^2)^2}} + c'(x) \\[2ex] f_y = -\dfrac{1}{1+x^2} \Longrightarrow -\dfrac{y}{1+x^2} + c(x) \end{cases}$$

derivando risp. a x

dom. convesso aperto

$$c'(x) = 0 \Longrightarrow c = \text{costante}$$

$$\boxed{\frac{y}{1+x^2} + c}$$

← TUTTE LE PRIMITIVE

CAMPI E FORME III

$f(x) = \sqrt{x} \quad [0, 1] \qquad f'(x) \; \frac{1}{2\sqrt{x}}$

$$\int_0^1 \sqrt{1 + \frac{1}{4x}} \, dx = \frac{1}{2}\int_0^1 \sqrt{\frac{4x+1}{x}} \, dx =$$

$$\underset{\sqrt{x}=t}{=} \frac{1}{2}\int_0^1 \frac{1}{t} \sqrt{4t^2+1} \, \underbrace{2t\,dt}_{dx} = \int_0^1 \sqrt{4t^2+1} \, dt$$

$\sqrt{4x+1}$

$\frac{1}{\sqrt{x}}$

$x = t^2 \quad 4x+1 = 4t^2+1$

$dx = 2t\,dt$

$2t = u$

$u = \sinh v$

$$\rho = \sin^5 \frac{\theta}{5} \qquad \theta \in \left[0, \frac{5\pi}{6}\right] \qquad \Lambda = \int_{\theta_0}^{\theta_1} \sqrt{\dot\rho^2 + \rho^2 \dot\theta^2}$$

$$\begin{cases} \rho(t) = \sin^5 \frac{t}{5} & \dot\rho(t) = 5\left(\sin^4 \frac{t}{5}\right)\left(\cos\frac{t}{5}\right)\frac{1}{5} \\ \theta(t) = t & \dot\theta = 1 \end{cases}$$

$$\sin^4 \frac{t}{5} \cos\frac{t}{5}$$

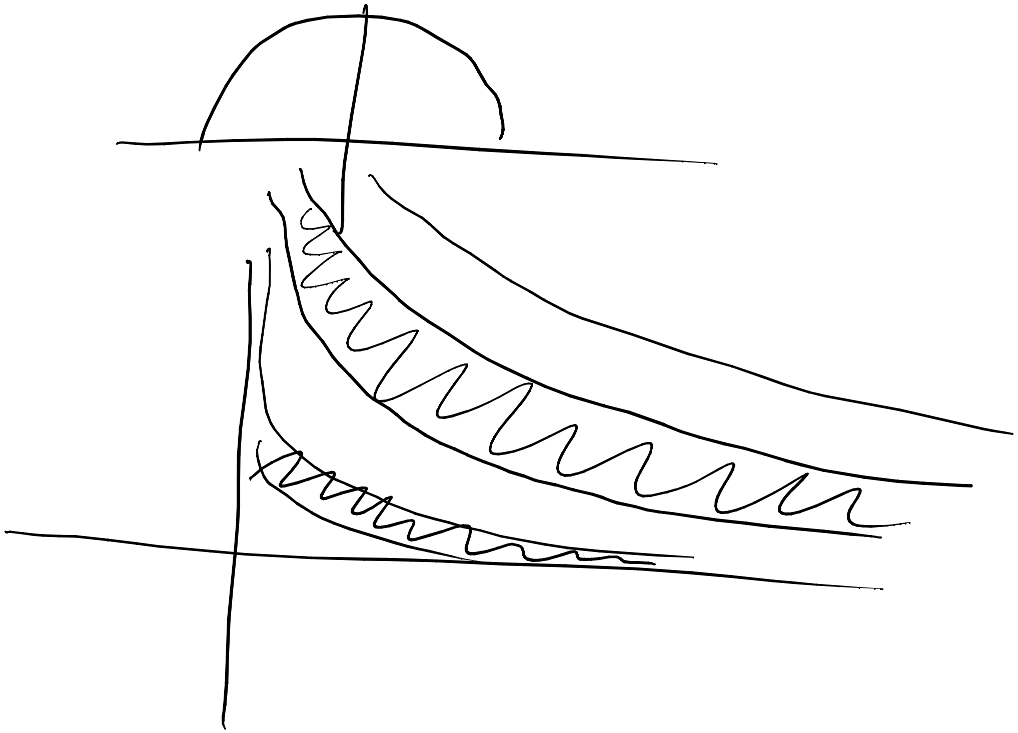
$$\int_0^{\frac{5}{6}\pi} \sqrt{\sin^8 \frac{t}{5} \cos^2 \frac{t}{5} + \sin^{10} \frac{t}{5}}\, dt =$$

$$= \int_0^{\frac{5}{6}\pi} \sin^4 \frac{t}{5}\, dt$$

$$\sin^2 \alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$

$$\sin^4 \alpha = \left(\qquad \right)^2$$

$$\{(x,y) : \sin(xy) > 0\}$$



$$0 + 2k\pi < xy < \pi + 2k\pi$$

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f continua su Ω

$\Omega \; \{x \in \Omega : f(x) > 0\}$ è aperto perché

se $x_0 \in \Omega \iff \underline{f(x_0) > 0} \Longrightarrow \exists\, \delta : f(x) > 0$ su $\underline{B(x_0, \delta)}$

perm-segno

$$\boxed{B(x_0, \delta) \subseteq \Omega}$$

$$T(x,y) = \big(\underset{\alpha}{xy},\ \underset{\beta}{\tfrac{x}{y}}\big)$$

$$\boxed{T : \mathbb{R}^2 \setminus \{y=0\} \to \mathbb{R}^2}$$

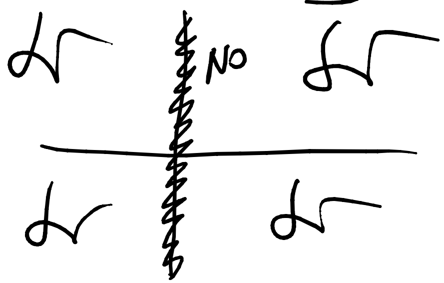
$$\frac{\partial(\alpha,\beta)}{\partial(x,y)} = \begin{pmatrix} y & x \\ \frac{1}{y} & -\frac{x}{y^2} \end{pmatrix}$$

matrice jacobiana

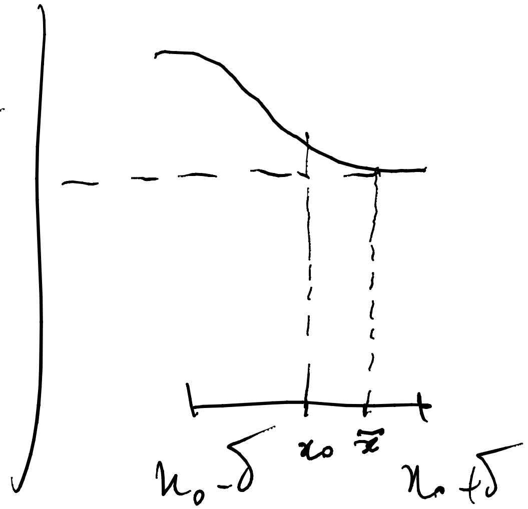
$$\det J = y \cdot \left(-\frac{x}{y^2}\right) - \frac{x}{y} = -2\,\frac{x}{y}$$

$$\det T' = 0 \iff x = 0$$

det jacobiano



$\varphi(\bar x) = \bar y$

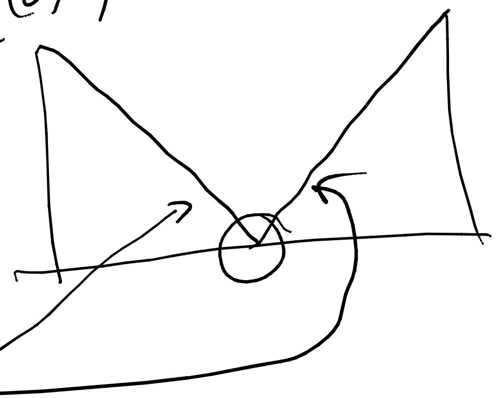


Si riapplica il th. DINI con punto centrale $(\bar x, \bar y)$

$$\gamma(t) = \begin{cases} t \\ |t| \end{cases} \qquad t \in [-1, 1]$$

vettori "tangenti" in $(0,0)$

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



regolare a ~~tratti~~

$-1, 0, 1$

$[-1, 0]$ $\gamma(t) = \begin{pmatrix} t \\ -t \end{pmatrix}$

$[0, 1]$ $\gamma(t) = \begin{pmatrix} t \\ t \end{pmatrix}$

$$\begin{cases} y_1 = f_1(x_1 \dots x_n) \\ \vdots \\ y_n = f_n(x_1 \dots x_n) \end{cases}$$

ϕ inversa della f

$\exists\ \varphi_1 \dots \varphi_n$:

$$\begin{cases} x_1 = \varphi_1(y_1 \dots y_n) \\ \vdots \\ x_n = \varphi_n(y_1 \dots y_n) \end{cases}$$

$$\begin{cases} \overbrace{f_1(x_1 \dots x_n) - y_1}^{g_1(x_1, \dots x_n)} = 0 \\ \vdots \\ f_n(x_1 \dots x_n) - y_n = 0 \end{cases}$$

$g(x,y) = f(x) - y$

si

se

$$\det \frac{\partial(f_1 \dots f_n)}{\partial(x_1 \dots x_n)} \neq 0$$

Th. DINI II

$$\begin{cases} g_1(x_1 \dots x_n, y_1 \dots y_m) = 0 \\ \vdots \\ g_m(x_1 \dots x_n, y_1 \dots y_m) = 0 \end{cases}$$

$$\det \frac{\partial (g_1 \dots g_m)}{\partial (y_1 \dots y_m)} \neq 0$$

$\exists \ \varphi_1 \dots \varphi_m$:

$$g(x, \varphi(x)) \equiv 0$$

in un intorno del punto iniziale

$$\boxed{g(x_0, y_0) = 0}$$

$-y\,dx + x\,dy$ forme neppure chiuse

prodotto scalare

$$\alpha(x,y;w_1,w_2) = -y\,w_1 + x\,w_2 = \boxed{\begin{pmatrix} -y \\ x \end{pmatrix}}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\underbrace{-y}_{\mathrm{I}}\;\underbrace{w_1}_{\mathrm{I}} + \underbrace{x}_{\mathrm{II}}\;\underbrace{w_2}_{\mathrm{II}} \qquad A(x,y)\cdot w$$

$$\boxed{\alpha(x,w) = A(x)w}$$

$$\begin{cases} f_x = A_1 \\ f_y = A_2 \end{cases} \qquad \boxed{\nabla f = A}$$

Se $A \in C^0$

i due problemi sono equivalenti

$\nabla f = A(x)$

$df = A(x)w$

$$\alpha(x,w) = A(x)w$$

$$\phi(\rho, \theta) = (\rho \cos\theta, \rho \sin\theta, \theta) \qquad \left(1, \frac{\pi}{2}\right)$$

$$\phi_\rho = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \qquad \phi_\theta = \begin{pmatrix} -\rho \sin\theta \\ \rho \cos\theta \\ 1 \end{pmatrix}$$

$$\nu = \phi_\rho \times \phi_\theta = \begin{pmatrix} \sin\theta \\ -\cos\theta \\ \rho \end{pmatrix} \qquad \nu\left(1, \frac{\pi}{2}\right) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \leftarrow$$

$$\phi\left(1, \frac{\pi}{2}\right) = \left(0, 1, \frac{\pi}{2}\right) \leftarrow$$

$$\boxed{1(x-0) + 0(y-1) + 1\left(z - \frac{\pi}{2}\right) = 0}$$