Note Title 5/28/2020

k $\boxed{f(x,y)=x^5+xy^2}=k$

$$\begin{cases} f_x = 5x^4+y^2 \\ f_y = 2xy \end{cases}$$

$f_y=0$ se $x=0$ oppure $y=0$

$f_x=0 \iff x=y=0$

UNICO PUNTO CRITICO È $(0,0)$

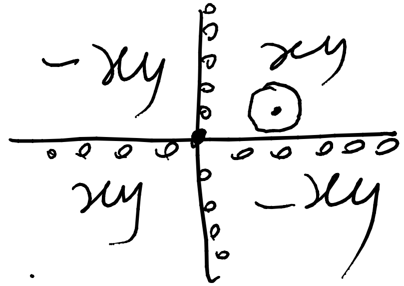
Il livello k critico (in cui NON si può applicare DINI) è $f(0,0)=0$

$\boxed{k=0}$ livello critico

$f(x,y)=|xy|$ diff. $(0,0)$ ma non è derivabile differenziabile

diff. in $(0,0)$

$$f(x,y)=\begin{cases} xy & xy>0 \\ -xy & xy<0 \\ 0 & xy=0 \end{cases}$$ assi



$$f_x(0,0)=\lim_{h\to 0}\frac{\overset{=|h0|}{f(h,0)}-\overset{=|0|}{f(0,0)}}{h}=0$$

$$f_y(0,0)=0$$

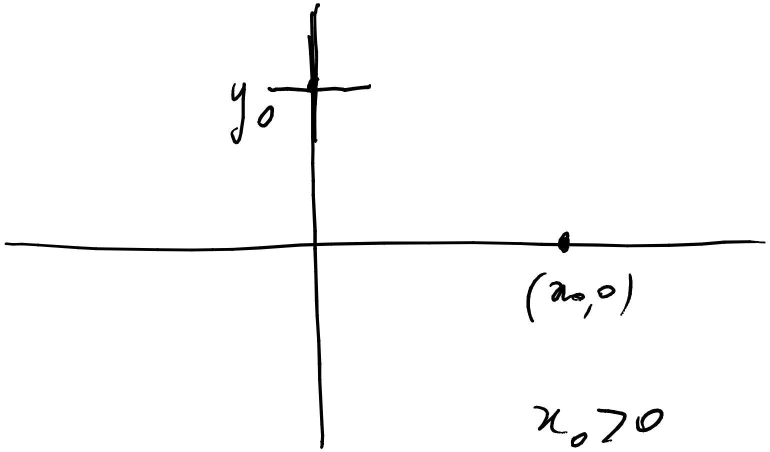
$$\nabla f(0,0)=\binom{0}{0} \qquad df(0,0;h,k)\overset{?}{=}0h+0k=0$$

se $x_0y_0\neq 0$ f è diff in (x_0,y_0) perché è C^∞

$$f \text{ è diff. se } \lim_{\substack{(h,k)\to 0\\ w}}\frac{\overset{f((0,0)+w)}{|hk|}-\overset{=f(0,0)}{0}-\overset{df?}{0}}{\underset{|w|}{\sqrt{h^2+k^2}}}=\lim_{h,k\to 0}\frac{|hk|^{\,2\text{-omog}}}{\sqrt{h^2+k^2}_{\,1\text{-omog.}}}=0$$

1-omog. denom. non si annulla su $h^2+k^2=1$

f è differenz. $df(0,0;h,k)=0$



$x_0 > 0$

$$f_x(x_0,0) = 0$$

perché f è costante sull'asse x (e fa 0)

$$f_y(x_0,0) = \lim_{k\to 0} \frac{f((x_0,0)+k(0,1)) - \overset{=0}{f(0,0)}}{k} =$$

$$f(x_0,k)$$

$$= \lim_{k\to 0} \frac{1}{k}\cdot |x_0 k| = |x_0| \underbrace{\lim_{k\to 0}\frac{|k|}{k}}_{\text{NON ESISTE}}$$

$f_y(x_0,0)$ NON ESISTE

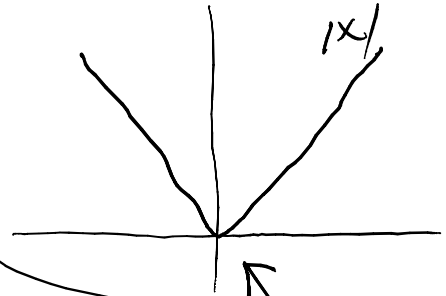
$$\begin{cases} \boxed{\gamma \in C^1[a,b]} \\ \dot\gamma(t) \neq 0 \quad \forall t \end{cases}$$

$$\gamma(t) = \begin{pmatrix} t \\ |t| \end{pmatrix}$$

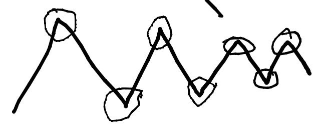
NON È C^1 perché la $\underline{\text{II}}$ componente $|t|$ non è derivabile in 0.

$\dot\sigma(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ σ non è una curva regolare ma è una funzione C^1

$$\sigma(t) = \begin{pmatrix} t^3 \\ |t^3| \end{pmatrix}$$



sostegno di γ e σ coincidono col grafico di $f(x) = |x|$



$$\dot\sigma(t) = \begin{pmatrix} 3t^2 \\ \end{pmatrix}$$

limitato da 1 in modulo

$$\frac{d}{dt}|t|^3(0) = \lim_{h\to 0} \frac{|h|^3 - |0|}{h}$$

$$= \lim_{h\to 0} \frac{|h|^3}{h} = \lim_{0} \frac{|h|^3}{h^3}\frac{h^3}{h} \to 0 \qquad 0 = $$

$$\lim_{(0,0)} \underbrace{\frac{x^2+y^2}{x-y}}_{f(x,y)} \qquad x^2+y^2 \to \text{2-om.} \qquad x-y \to \text{1-om.}$$

$$\mathrm{dom}\, f = \mathbb{R} \setminus \{y = x\}$$

$$\boxed{f \text{ è 1-omogenea}}$$

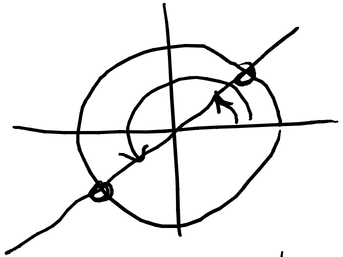
$$\mathrm{dom}\, f \cap \{x^2+y^2 = 1\} = \{x^2+y^2=1 \,;\, y \neq x\}$$

È LIMITATO, MA NON È CHIUSO

$$\left(\cos\frac{\pi}{4}, \sin\frac{\pi}{4}\right)$$

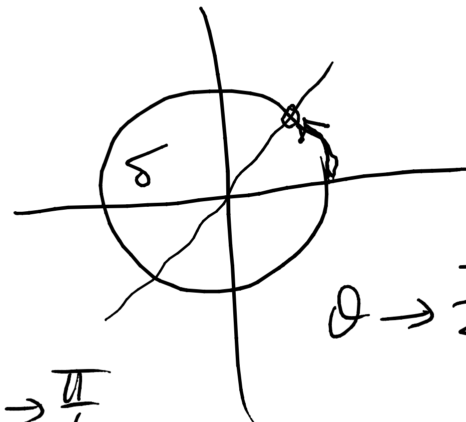
$$x = \cos\theta \qquad y = \sin\theta$$

$$\theta \neq \frac{\pi}{4}, \frac{\pi}{4}+\pi$$



$$f(\rho\cos\theta, \rho\sin\theta) = \frac{\rho^{\cancel{2}}(\cos^2\theta+\sin^2\theta)}{\cancel{\rho}(\cos\theta-\sin\theta)} = \frac{\rho}{\underbrace{\cos\theta-\sin\theta}_{\to 0^{\pm} \text{ se } \theta \to \frac{\pi}{4}}} \qquad \rho \text{ costante}$$

$|f|$ diverge e quindi non è limitata.



$$\theta \to \frac{\pi}{4}$$

$$\lim_{(0,0)} \frac{x\sin y}{(x^2+y^2)^{2/3}} = \lim_{(0,0)} \frac{\cancel{x}\sin y}{\cancel{xy}} \boxed{\frac{xy}{(x^2+y^2)^{2/3}}}$$

$x \neq 0$

$(x,y) \to (0,0)$

1

$\frac{2\text{-ordy.}}{\frac{4}{3}\text{-ordy}}$

$\to 0$ limitde

on $x^2+y^2=1$

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f $x^3+y^3-xy = 3$ Π

$f_x = 3x^2 - y$

$f_y = 3y^2 - x$

$f_x = 0 \Rightarrow \boxed{y = 3x^2}$ subst. in $f_y = 0$

$3(\underbrace{3x^2}_{y})^2 - x = 0$

$27x^4 - x = 0$

$x = 0 \Rightarrow y = 0$ $(0,0)$

$x^3 = \frac{1}{27}$ $x = \frac{1}{3}$

$\left(\frac{1}{3}, 3\left(\frac{1}{3}\right)^2\right) = \left(\frac{1}{3}, \frac{1}{3}\right)$

$f(0,0) = 0$ $\quad$ $(0,0)$ NON app. a Π

$f\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{27} + \frac{1}{27} - \frac{1}{9} \neq 3$ $\quad$ $\left(\frac{1}{3}, \frac{1}{3}\right) \notin \Pi$

$$f(x_0+w) = \boxed{\sum_{|k|\le N} \frac{1}{k!} f^{(k)}_{x_k}(x_0)\, w^k} + R_N(w)$$

$$\| \sum_{k=0}^{N} \frac{1}{k!} f^{(k)}_{x_k}(x_0)\, w^k \|$$

$x_0, w \in \mathbb{R}^n$

$k = (k_1, k_2, \dots, k_n)$. $k_i \in 0, 1, \dots, n$

$|k| = k_1 + k_2 + \dots + k_n$

$k! = k_1!\, k_2! \dots k_n!$

$$f^{(k)}_{x_k} = f_{x_1^{k_1} x_2^{k_2} \dots x_n^{k_n}}$$

$w^k = w_1^{k_1} w_2^{k_2} \dots w_n^{k_n}$.

$$\boxed{\frac{\partial^3 f}{\partial x_1^2 \partial x_2}}$$

$k_1 = 2$

$k_2 = 1$

$k_i = 0$

$i > 2$

$$f(x_0+w) = \underset{|k|=0}{f(x_0)} + \underbrace{\sum_{|k|=1} f_{x_k}(x_0)\, w^k}_{Df(x_0)\,w} + \sum_{|k|=2} \frac{1}{k!} f^{(2)}_{x_k}(x_0)\, w^k$$

$$\left.\begin{matrix} (1, 0, \dots, 0) \\ (0, 1, 0 \dots 0) \\ \vdots \\ (0, 0 \dots 0, 1) \end{matrix}\right\} \text{n multi indici di modulo } 1$$

$k = (2, 0, 0 \dots 0)$

$\frac{1}{k!} = \frac{1}{2!}$

$k = (1, 1, 0, \dots 0)$

$k! = 1!\,1! = 1$

$\gamma : [0,1] \longrightarrow \Omega$ continua, chiusa

$h(\lambda, t) = (1-\lambda)\gamma(t)$ continua

$\gamma(t)$ $\quad \gamma : [0,1] \to \Omega$ $\quad \gamma(0) = \gamma(1)$

continuità

$$\boxed{\begin{array}{l} f \quad \alpha\text{-omog.} \qquad \alpha < 0 \\[1em] \dfrac{1}{f} \quad -\alpha\text{-omog.} \quad \text{omog. di grad.} > 0 \end{array}}$$

$$g: \frac{1}{\sum a_{ij} x_i x_j} \qquad -2\text{-omogenea}$$

$$\lim_{0} f(x) = 0$$

$$|g| > \varepsilon \iff \underbrace{\left|\sum a_{ij} x_i x_j\right|}_{\text{continua in } 0} < \frac{1}{\varepsilon}$$

$\exists \delta : |x| < \delta \quad x \neq 0 \Rightarrow$

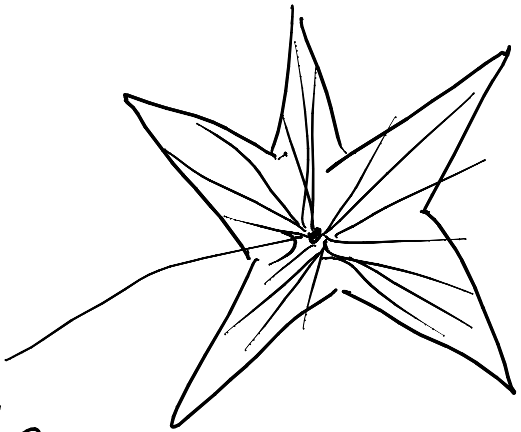
$$\lim_{0} \frac{1}{|f(x)|} = +\infty$$

$$\frac{1}{|f(x)|} > \varepsilon > 0$$

$$|f(x)| < \frac{1}{\varepsilon}$$

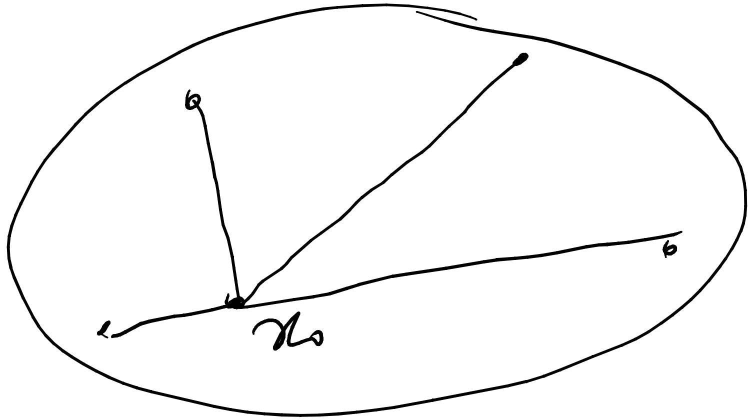
$x \neq 0 \quad |x| < \delta$

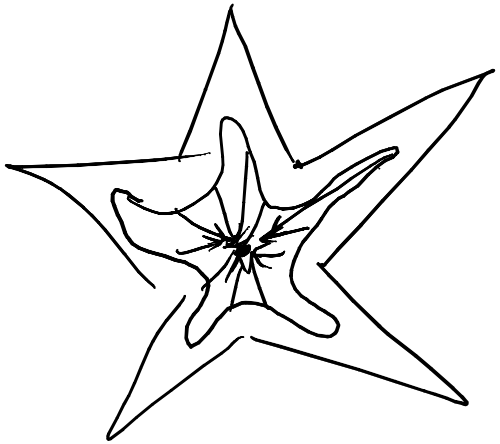
Ω stella $\Longrightarrow \exists\, x_0 : \forall x \in \Omega \quad \overrightarrow{x_0 x} \subseteq \Omega$



x_0

POLO





$\gamma : [0,1] \to \Omega \qquad \underline{\Omega \text{ stella}} \quad \boxed{x_0 = 0}$

$h(\lambda, t) = \underbrace{(1-\lambda)\gamma(t)}_{\text{punti del segmento } \overrightarrow{0\gamma(t)}}$

$\forall \gamma$ chiuse

$$\int_\gamma A = \int_\sigma A = 0 \quad \sigma \text{ costante}$$

$\sigma \to$ i costante

semplici

Invarianza omotopica

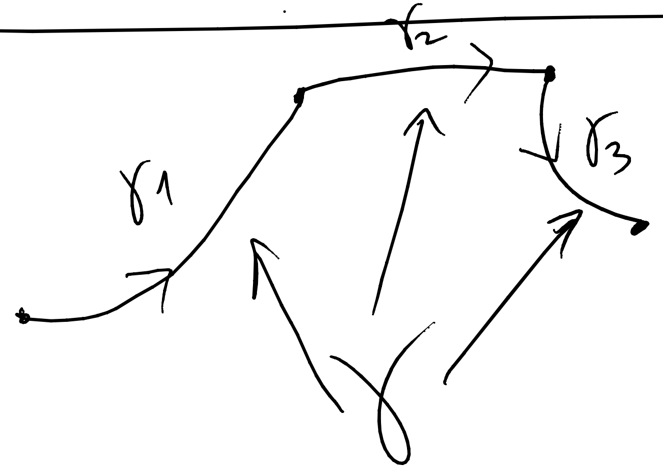
$$\boxed{f(x) = \int_{\gamma_{x_0 \to x}} A}$$

$\gamma_1(t) = x + t e_1$

$t \in [0,h]$

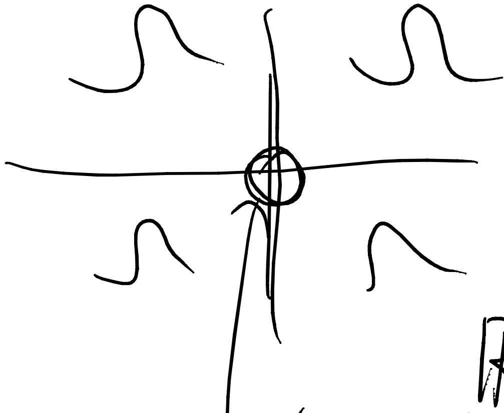
$f(x+he_1) = \int_{\gamma \oplus \sigma} A$

$f(x)$



$$\boxed{\int_\gamma A = \int_{\gamma_1} A + \int_{\gamma_2} A + \int_{\gamma_3} A}$$

$$\left(\frac{x}{x^2+y^2} , \frac{-y}{x^2+y^2} \right) = A(x,y)$$



No

$\mathbb{R}^2 - \{(0,0)\}$ è un aperto connesso

$$\boxed{\text{dom } A \ ?}$$

f una primitiva

tutte le altre sono del tipo

$$f + \begin{cases} c_1 & \text{in } \Omega_1 \\ c_2 & \text{in } \Omega_2 \\ \vdots & \\ c_k & \text{in } \Omega_k \end{cases}$$

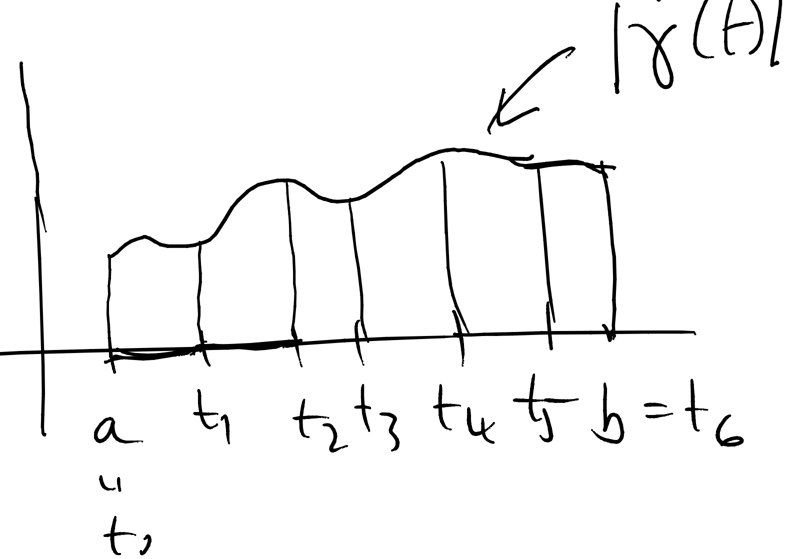
dove $\text{dom} A = \bigcup_{i=1}^{k} \Omega_i$

e Ω_i è aperto connesso

$$\Lambda(\pi) = \sum_{0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)| = \sum_{0}^{n-1} \underbrace{\left| \int_{t_i}^{t_{i+1}} \dot\gamma(s)\, ds \right|}_{\text{nnme}} \underset{\text{Tz Ceny.}}{\leq}$$

$$\leq \sum_{0}^{n-1} \int_{t_i}^{t_{i+1}} \underbrace{|\dot\gamma(s)|\, ds}_{\text{nnme}} =$$

$$= \underline{\underline{\int_a^b |\dot\gamma(s)|\, ds = M}}$$



$$\Lambda(\pi) \leq M \quad \forall \pi = \{t_0 < t_1 < \dots < t_n\}$$

$$M \geq \boxed{\sup \Lambda(\pi) = \Lambda(\gamma)}$$