Note Title 5/21/2020

$$H(w) = \sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\, w_i w_j$$ Forma Hessiana

$\nabla f(x_0) = 0$ per Fermat

$$\to f(x_0+w) - f(x_0) = \cancel{\nabla f(x_0) w} + \frac{1}{2}\sum_{i,j=1}^{n} f_{x_i x_j}(x_0) w_i w_j + R_2(w) =$$

$$\boxed{\lim_{w\to 0} \frac{R_2(w)}{|w|^2} = 0}$$

$$= \underbrace{|w|^2}_{\geq 0} \left[\frac{1}{2} \frac{\sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\, w_i w_j}{|w|^2} + \underbrace{\frac{R_2(w)}{|w|^2}}_{\to 0} \right]$$

$\lambda \leq \quad \leq \Lambda$

Poiché, dal th. Clairaut-Schwarz la forma quadratica in w $\sum_{i,j=1}^{n} f_{x_i x_j}(x) w_i w_j$ è simmetrica e dal th. spettrale segue che

$$\boxed{\lambda |w|^2 \leq \sum_{1}^{n} f_{x_i x_j}(x_0) w_i w_j \leq \Lambda |w|^2}$$

λ min autov.
Λ max autovalore

Se $\lambda > 0$ la forma è def. > 0 $\qquad \varepsilon < \lambda$ allora

$[\ \ ] > \lambda$ e per $|w| < \delta$ $\dfrac{|R_2(w)|}{|w|^2} < \varepsilon < \lambda \implies$

$[\ \ ] > 0$ per $|w| < \delta$ $\implies$ segno di $f(x_0 + w) - f(x_0)$ è > 0 per $|w| < \delta$

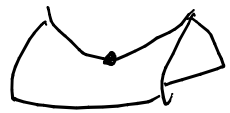
Se $\Lambda < 0$ stesso discorso prendendo $\varepsilon < |\Lambda|$

il segno di $f(x_0 + w) - f(x_0)$ è ≤ 0

se $|w| < \delta$ $\left(\implies \left|\dfrac{R_2(w)}{|w|^2}\right| < \varepsilon < |\Lambda|\right)$

$\lambda < 0$ e $\Lambda > 0$ due volte lo stesso discorso

$\varepsilon < |\lambda|$ $(\lambda < 0)$ $\qquad \varepsilon < \Lambda$ $(\Lambda > 0)$



$$\Gamma = \{(x,y)\in\mathbb{R}^2 : x^2+y^2-1=0\}$$

$\boxed{\text{esti } xy \quad \pi}$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$$\begin{cases} \nabla f = \sum \lambda_i \nabla g_i \\ \left.\begin{array}{l} g_1 = 0 \\ \vdots \\ g_k = 0 \end{array}\right\} \text{che il punto stia sull'intersez. dei vincoli} \end{cases}$$

$$f(x,y)=\sqrt{|x^3y|} \qquad (0,0)$$

1. f è continua in $(0,0)$ perché composta di funzioni continue definite dappertutto

2. Se f fosse differenziabile $\quad df(\underset{x_0}{(0,0)},\underset{w}{(h,k)}) = \nabla f(0,0)\,w$

$$f_x(0,0)=\lim_{h\to 0}\frac{f(\overset{\downarrow}{h,0})-\overset{0}{f(0,0)}}{h}=\lim_{h\to 0}\frac{0-0}{h}=0 \qquad \to (0,0)+h(1,0)$$

$f_y(0,0)=0$ idem

Si poteva vedere subito perché f è costante ($=0$) sugli assi coordinati

$\exists\,\nabla f(0,0) \Rightarrow$ Il candidato df è

$$\nabla f(0,0)\overbrace{(h,k)}^{w}=f_x(0,0)h+f_y(0,0)k=0$$

③ $$\lim_{(h,k)\to(0,0)} \frac{f(h,k) - f(0,0) - \nabla f(0,0)(h,k)}{\sqrt{h^2+k^2}} =$$

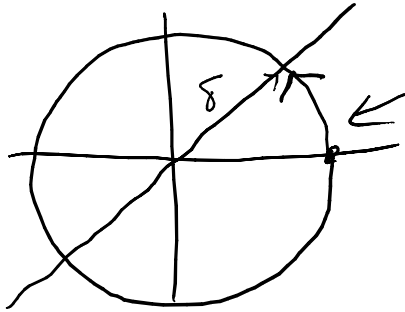
$$= \lim_{(h,k)\to(0,0)} \frac{\sqrt{|h^3k|} - 0 - 0}{\sqrt{h^2+k^2}}$$

$$\sqrt{\left|\frac{h^3k}{h^2+k^2}\right|} \xrightarrow{?} 0$$

2-omog sul compatto $x^2+y^2=1$

$$\lim_{(0,0)} \frac{2x^2+y^2}{x-y}$$

1-omogenea



$(\delta\cos\theta\ ,\ \delta\sin\theta)$

$\theta \to \frac{\pi}{4}^-$

$$\lim_{\theta\to\frac{\pi}{4}^-} \frac{2\delta^2\cos^2\theta+\delta^2\sin^2\theta}{\delta(\cos\theta-\sin\theta)}$$

$$2\cos^2\theta+\sin^2\theta = 1+\cos^2\theta \to 1+\frac{1}{2}=\frac{3}{2}$$

$$\lim_{\theta\to\frac{\pi}{4}^-} \delta\,\frac{2\cos^2\theta+\sin^2\theta}{\cos\theta-\sin\theta} = +\infty$$

$$\cos\theta-\sin\theta \to 0^+$$

$A:\Omega\to\mathbb{R}^N \qquad \Omega\subseteq\mathbb{R}^N \qquad \gamma:[a,b]\to\Omega$ di classe C^1

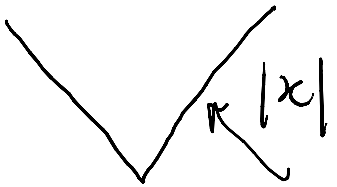
$$\boxed{\int_\gamma A \equiv \int_a^b A(\gamma(t))\,\dot\gamma(t)\,dt}$$

$$\left(-\frac{y}{x^2+y^2},\ \frac{x}{x^2+y^2}\right) = A(x,y) \longleftarrow$$

$$A:\mathbb{R}^2\setminus\{(0,0)\}\longrightarrow\mathbb{R}^2$$

Campo vettoriale su $\mathbb{R}^2-\{(0,0)\}$

$$\begin{cases} \dfrac{x}{|x|} & x\neq 0 \\ 0 & x=0 \end{cases}$$



$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

$$f : \Omega \to \mathbb{R}^n \qquad \Omega \subseteq \mathbb{R}^n$$

$$\boxed{\begin{cases} y_1 = f_1(x_1 \dots x_n) \\ y_2 = f_2(x_1 \dots x_n) \\ \vdots \\ y_n = f_n(x_1 \dots x_n) \end{cases}} \qquad (*)$$

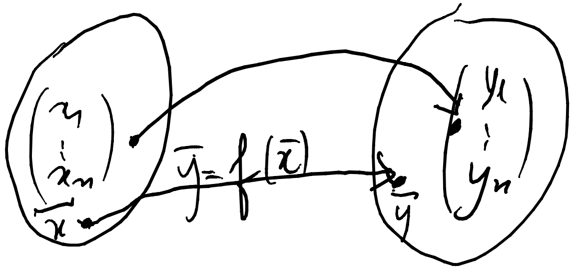
y_i espressi in funzione di $x_1 \dots x_n$

f_i componenti scalari del vettore f

$$f(x) = \begin{pmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{pmatrix}$$

$$(x_1 \dots x_n) \longrightarrow f(x_1, \dots, x_n)$$

$\exists \bar{x}$



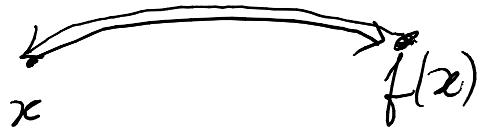
Dato $\bar{y}$ $\exists \bar{x} : f(\bar{x}) = \bar{y}$

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$$

$$(\rho, \theta) \longrightarrow (x, y) = f(\rho, \rho)$$

$$f^{-1}(y) = x$$

$$f^{-1}(f(x)) = x$$



Dati $y_1 \dots y_n$ $\exists$ $x_1 \dots x_n$: $f(x_1, \dots, x_n) = (y_1, \dots y_n)$

Th. funzioni implicite nel caso vettoriale

$$g_1(x_1, x_2, \dots, x_n, y_1, y_2, \dots y_n) = f_1(x_1 \dots x_n) - y_1$$

$$g_i(x_1 \dots x_n; y_1 \dots y_n) = f_i(x_1 \dots x_n) - y_i$$

$$g_n(x_1 \dots x_n, y_1, \dots y_n) = f_n(x_1 \dots x_n) - y_n$$

$$\begin{cases} g_1(\quad) = 0 \\ \vdots \\ g_n(\quad) = 0 \end{cases}$$

va risolto esplicitando $x_1 \dots x_n$ in funzione di $(y_1 \dots y_n)$ (termini noti)

II disp. Th. DINI

$$\boxed{\det \frac{\partial(g_1 \dots g_n)}{\partial(x_1 \dots x_n)} \neq 0}$$

si può fare in vicinanza di una soluzione nota

∂g_i

$$\begin{array}{cc} & \\ & \begin{pmatrix} - & - & - & & \\ - & - & - & & \\ - & - & - & - & \dfrac{\partial g_i}{\partial x_j} \\ & & & & \\ \end{pmatrix} \end{array}$$

i-esima →

↑ j-esima

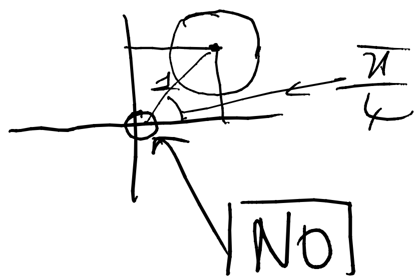
$$\frac{\partial g_i(x_1 \dots x_n, y_1 \dots y_m)}{\partial x_j} = \boxed{\frac{\partial f_i(x_1 \dots x_n)}{\partial x_j}}$$

$\rho = \sqrt{x^2+y^2}$ $\quad \theta = \left\{ \text{DIAVOLO A QUATTRO} \right.$

$\boxed{x = \rho \cos\theta \quad y = \rho \sin\theta}$

$$\frac{\partial(x,y)}{\partial(\rho,\theta)} = \begin{pmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{pmatrix}$$

$$\det \frac{\partial(x,y)}{\partial(\rho,\theta)} = \rho\cos^2\theta + \rho\sin^2\theta = \boxed{\rho}$$



ARRIVO $\left(1, \frac{\pi}{4}\right) \xleftarrow{\begin{cases}\rho = \sqrt{x^2+y^2} \\ \theta = \operatorname{arctg} \frac{y}{x}\end{cases}} \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\underline{\rho \quad \theta} \xrightarrow{} x \quad y$

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$$

$$\boxed{y_i = f_i(x_1 \ldots x_n) \quad i = 1 \ldots n}$$

$$\det \frac{\partial f_i(x_1, \ldots, x_n)}{\partial(x_1 \ldots x_n)}(x_1^0, \ldots x_n^0) \neq 0$$

$$\underline{y_i^0 = f_i(x_1^0, \ldots x_n^0)}$$

allora $\exists\, \varphi_i : B_\delta(x_0^1, \ldots x_0^n)$

$$\boxed{x_i = \varphi_i(y_1 \ldots y_n) \quad i = 1 \ldots n}$$

δ abbastanza piccolo

$$f(x,y) = \arctan x \sin y$$

$$f_x = \frac{\sin y}{1+x^2} \in C^0 \qquad f_y = \arctan x \cos y \in C^0$$

$$\sqrt{|x^3 y|}$$

$|\sin x|$

$k\pi$ non è derivabile

non c'è derivata

$$\lim_{h\to 0} \frac{|\sin h| - 0}{h} = \lim_{h\to 0} \frac{|\sin h|}{|h|/h} \; |h|$$

$$\boxed{|x^4| = x^2}$$

$$\sqrt{x^2} = |x|$$

$$\sqrt{x^4} = x^2$$

$$\lim_{h\to 0} \frac{|\sin h|}{|h|} \frac{|h|}{h} = \lim_{h\to 0} \underbrace{\left|\frac{\sin h}{h}\right|}_{\downarrow\, |1|=1} \frac{|h|}{h}$$

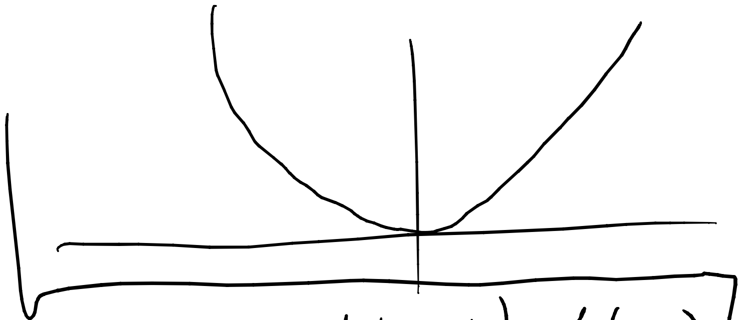
$\frac{|h|}{h} = 1 \quad h>0$

$= -1 \quad h<0$

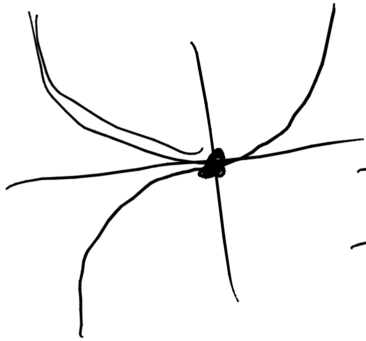
unendlich

USARE LA DEF. DI DERIVATA

$$= \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$$

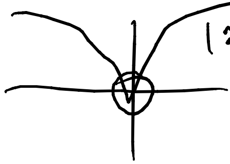


$|x^2|$



$(x^3)'(0)=0$

$|x|^{1/2}$



$x - x_0 = w$

$$f(x_0 + w) = f(x_0) + Df(x_0)\,w + \frac{1}{2!}\sum_{i,j=1}^{n} f_{x_i x_j}(x_0)\,w_i w_j + \dots$$

n dim stesso

$$+ \dots + \frac{1}{N!}\sum_{i_1, i_2, \dots i_N = 1}^{n} f_{x_{i_1} x_{i_2} \dots x_{i_N}}(x_0)\, w_{i_1} \dots w_{i_N} + R_N(w)$$

$$\boxed{\lim_{w \to 0} \frac{R_N(w)}{|w|^N} = 0}$$

a_{ij}

matrice associata alla forma

$\sum a_{ij} w_i w_j$

forma quadratica

Matrice hessiana

$$\left(\frac{\partial^2 f}{\partial x_i \partial x_j}(x_0) \right)$$

Forma quadratica hessiana

$$\to \ |\alpha_k w^k| < 1 \qquad \Big|\underbrace{1}_{>0} + \underbrace{\alpha_k w^k}_{<0}\Big| = \underbrace{1 - |\alpha_k w^k|}_{<1}$$

reale negativo

$$\to \ |w| < \frac{1}{|\alpha_k|^{1/k}}$$

$$|q(w)| \le |1 + \alpha_k w^k| + |w|^{k+1} |\tilde{q}(w)| =$$

$$= 1 - |\alpha_k| |w|^k + |w|^{k+1} |\tilde{q}(w)| =$$

$$= 1 - |w|^k \Big[\underbrace{|\alpha_k|}_{>0} + \underbrace{|w|}_{\downarrow\, 0} |\tilde{q}(w)| \Big]$$

$$\xrightarrow{w \to 0} |\alpha_k| \Rightarrow [\ ] > 0 \quad |w| \text{ piccolo}$$

$$\lambda |x|^2 \le \sum a_{ij} x_i x_j \le \Lambda |x|^2$$

$$\boxed{\lambda \le \frac{\sum a_{ij} x_i x_j}{|x|^2} \le \Lambda}$$

Segno della forma quadratica

$$|w|^2 \left[\frac{\sum f_{x_i x_j}(x_0) w_i w_j}{|w|^2} + \frac{R_2(w)}{|w|^2} \right] \qquad \left|\frac{R_2(w)}{|w|^2}\right| < \varepsilon$$

$\geq \lambda$ $\qquad$ $\leq \Lambda$

0-sup.

$\varepsilon < \lambda$ def. ≥ 0

$\varepsilon < |\Lambda|$ def. < 0

$$\lim_{x \to x_0} -\frac{f_x}{f_y}\Big(\underbrace{\overbrace{x_0 + \underbrace{\xi(x-x_0)}_{\to 0}}^{\to x_0}}_{\to 0},\ \underbrace{\overbrace{\varphi(x_0) + \xi\,(\overbrace{\varphi(x)-\varphi(x_0)}^{\to 0})}^{\to 0}}_{\to \varphi(x_0)}\Big)$$

$\xi_x \in [0,1]$

φ continua

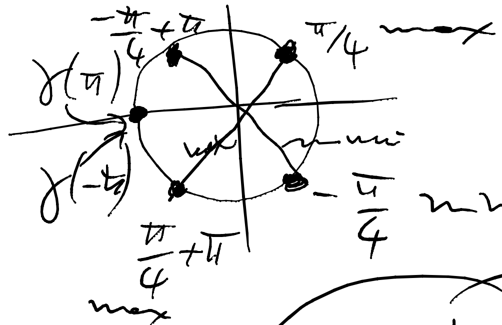
th ① C^0

se f_x e f_y è continua e $f_y \to \neq 0$

$$-\frac{f_x}{f_y}(\underbrace{\bar{x},\ \bar{y}}_{\to x_0,\ \varphi(x_0)}) \longrightarrow -\frac{f_x(\underline{x_0, \varphi(x_0)})}{f_y(\underline{x_0, \varphi(x_0)})}$$

$$\boxed{(x_0+\xi(x-x_0),\ \varphi(x_0)+\xi(\varphi(x)-\varphi(x_0))) \to (x_0, \varphi(x_0))}$$

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \qquad t \in [-\pi, \pi]$$



$\frac{1}{2}\sin 2t$ ha massimo se $\sin 2t = 1$

ha minimo se $\sin 2t = -1$

Estremi vincolati

$$2t = -\frac{\pi}{2}$$

$$2t = -\frac{\pi}{2} + 2\pi$$

$$t = -\frac{\pi}{4}$$

$$t = -\frac{\pi}{4} + \pi$$

$$2t = \frac{\pi}{2} \qquad t = \frac{\pi}{4} \qquad t = \frac{\pi}{4} + \pi$$

$$2t = \frac{\pi}{2} \qquad 2t = \frac{\pi}{2} + 2\pi$$

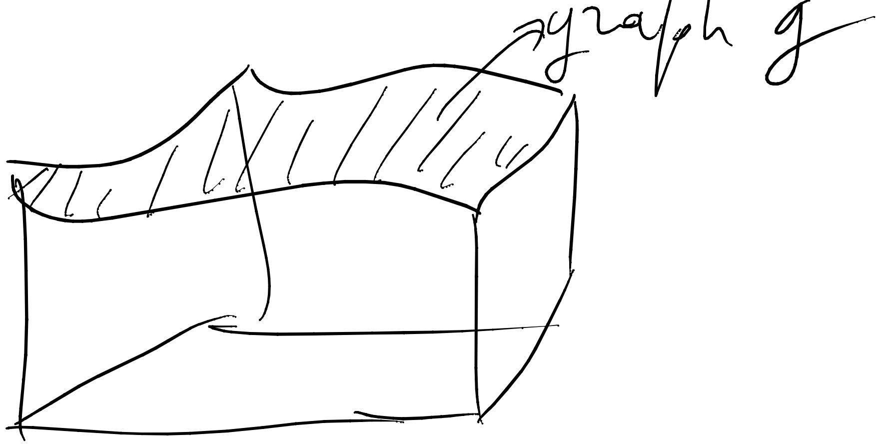
$$t = \frac{\pi}{4} \qquad t = \frac{\pi}{4} + \pi$$

$$f(x,y)$$

$$\boxed{\theta \to f(\cos\theta, \sin\theta) = h(\theta)}$$

$[-\pi, \pi]$ oppure $[0, 2\pi]$

$z = g(x_1, x_2, \dots, x_n)$



estremi di f sul grafico

$f(x_1, x_2, \dots, x_n, z)$

estremi di $f(x_1, \dots, x_n, g(x_1, \dots x_n))$

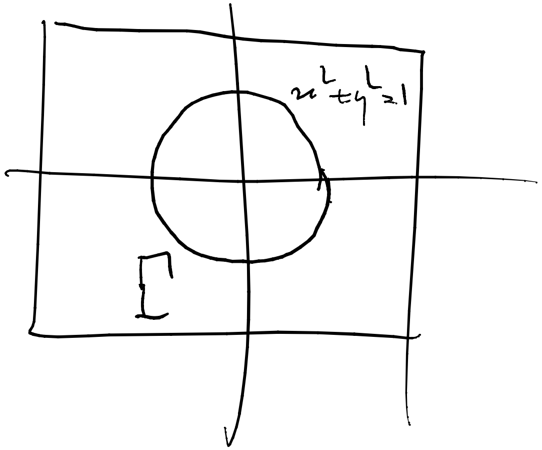
$(u,v) \to \overline{f(\phi(u,v))}$

2 variabili

$\Phi(u,v) \quad \phi : \mathbb{R}^2 \to \mathbb{R}^3 \quad \max_{\phi} \min_{\phi} f(x,y,z)$

$x = \phi_1(u,v) \quad z = \phi_3(u,v)$

$y = \phi_2(u,v)$



$$g(x,y) = x^2+y^2-1$$

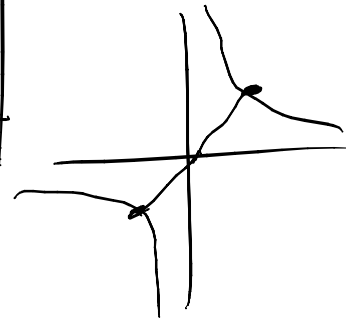
$$\begin{cases} \nabla f + \lambda \nabla g = 0 \\ g(x,y) = 0 \longleftarrow (x,y)\in\Gamma \end{cases}$$

$$\boxed{g(x_1, x_2, \ldots x_n) = 0}$$

$$\boxed{xy = 1}$$

$$\begin{cases} \min f(x,y) = x^2+y^2 \\ g(x,y) = xy-1 = 0 \end{cases}$$

$$\begin{array}{l} \min |(x,y)| \\ xy = 1 \end{array}$$



$$|f(x)-f(x_0)|<\varepsilon$$

$$f(x_0)-\varepsilon\le f(x)<f(x_0)+\varepsilon$$

$=0$

$\varepsilon \le f(x_0)$

$|x-x_0|<\delta$ $(\cancel{x=x_0})$

continuità

convergente