

$$\forall v \neq 0 \quad |u| \geq |u_v| \quad \forall u \in X$$

$$|u|^2 = \left| \underbrace{u - u_v}_{\perp v} + \underbrace{u_v}_{\parallel v} \right|^2 = |u - u_v|^2 + |u_v|^2 \geq |u_v|^2$$

$\perp v$ per il th. proiezione
 $\parallel v$ la def. di proiezione

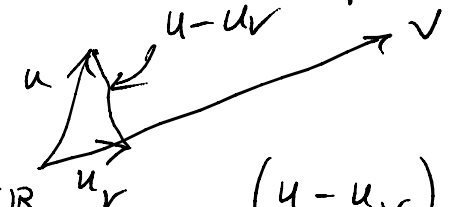
$$a \perp b \quad (ab=0)$$

$$|a+b|^2 = |a|^2 + |b|^2$$

$$a^2 + ab + ba + b^2$$

$$0 \quad 0$$

Th. della proiett.



Da $|u|^2 \geq |u_v|^2$ prend. le radici square

AL_2.1 lo spazio euclideo \mathbb{R}^n

$$u_v = \frac{uv}{|v|^2} v$$

$\in \mathbb{R}$ paralleli
 $(u - u_v) \perp v = 0$ ortogonale

$$x_y = \frac{x \cdot y}{|y|^2} y =$$

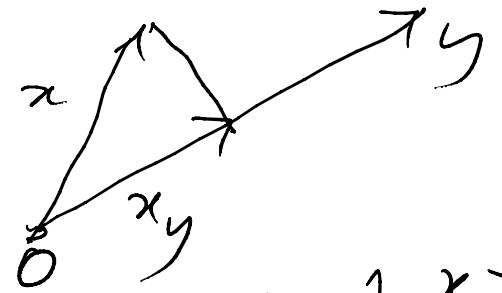
$$x = (x_1 \dots x_n)$$

$$y = (y_1 \dots y_n) \neq (0, \dots, 0)$$

$$x \cdot y = \sum_{i=1}^n x_i y_i$$

$$|y|^2 = \sum_{i=1}^n y_i^2$$

$$= \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n y_i^2} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$



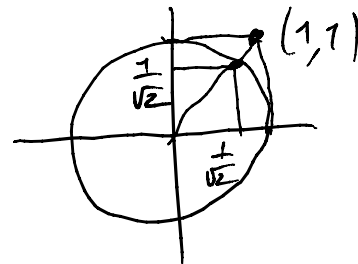
$$\frac{x}{|x|} = \begin{cases} -1 & x > 0 \\ 1 & x < 0 \end{cases}$$

want
 $x = 0$

$$\frac{(1, 1)}{|(1, 1)|} = \frac{1}{\sqrt{2}} (1, 1) = \quad x \in \mathbb{R}^n$$

$\frac{x}{|x|}$ versore di x

$$= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$



AL-2.1

Spazio euclideo reale

$|a|^2 = a a$
 $|a| = \sqrt{a a}$

$\rightarrow |x+y|^2 =$

$= (x+y)(x+y) \stackrel{\text{bilineare}}{=}$

$= \underline{xx} + xy + yx + \underline{yy} =$

$= |x|^2 + |y|^2 + \underline{2xy}$

key sono
ortogonali

- $a(b+c) = ab + ac$ } Bilineare
- $a(\lambda b) = \lambda ab$ }
- $ab = ba$ Simmetria
- $aa \geq 0$
- $aa = 0 \Leftrightarrow a = 0$
- $\rightarrow aa \text{ def. } > 0$

$(x+y)(x+y) = (x+y)x + (x+y)y =$

$\begin{matrix} (x+y) \\ a \end{matrix} \begin{matrix} (x+y) \\ b \quad c \end{matrix} = (x+y)x + (x+y)y =$

$\stackrel{\text{lineare}}{=} x($

$$|x - \lambda y|^2 = \underbrace{(x - \lambda y)}_a \underbrace{(x - \lambda y)}_a = aa = |a|^2$$

$$= xx - x(\lambda y) - (\lambda y)x + (-\lambda y)(-\lambda y) =$$

$$= \underset{||}{|x|^2} + \lambda^2 |y|^2 - 2\lambda xy \geq 0$$

$$\frac{\Delta}{4} \leq 0$$

$$(xy)^2 - |x|^2 |y|^2 \leq 0$$

Schwarz

$$|\cdot| : X \rightarrow \mathbb{R}$$

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norme \rightarrow $\|x\| \in \mathbb{R}$

modulus in \mathbb{R}

$$\|x\| = |x| = \text{norme de } x$$

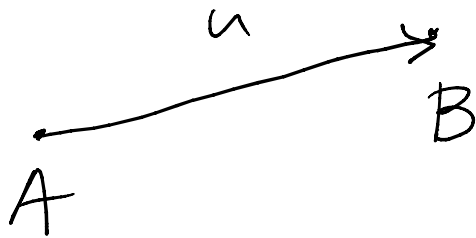
$$|x| \geq 0 \quad \underbrace{|\|x\||}_{\geq 0} = \|x\|$$

$$|\lambda x| = |\lambda| |x|$$

$\lambda \in \mathbb{C}$ in \mathbb{C}

norme \rightarrow $\lambda \in \mathbb{R}$ or \mathbb{C}

norme \rightarrow $x \in X$



$$u \approx \vec{AB}$$

$$|\lambda x| = |\lambda| |x|$$

$$|xy| \leq |x| |y| \quad \text{Schwarz}$$

\rightarrow pointe
scalare