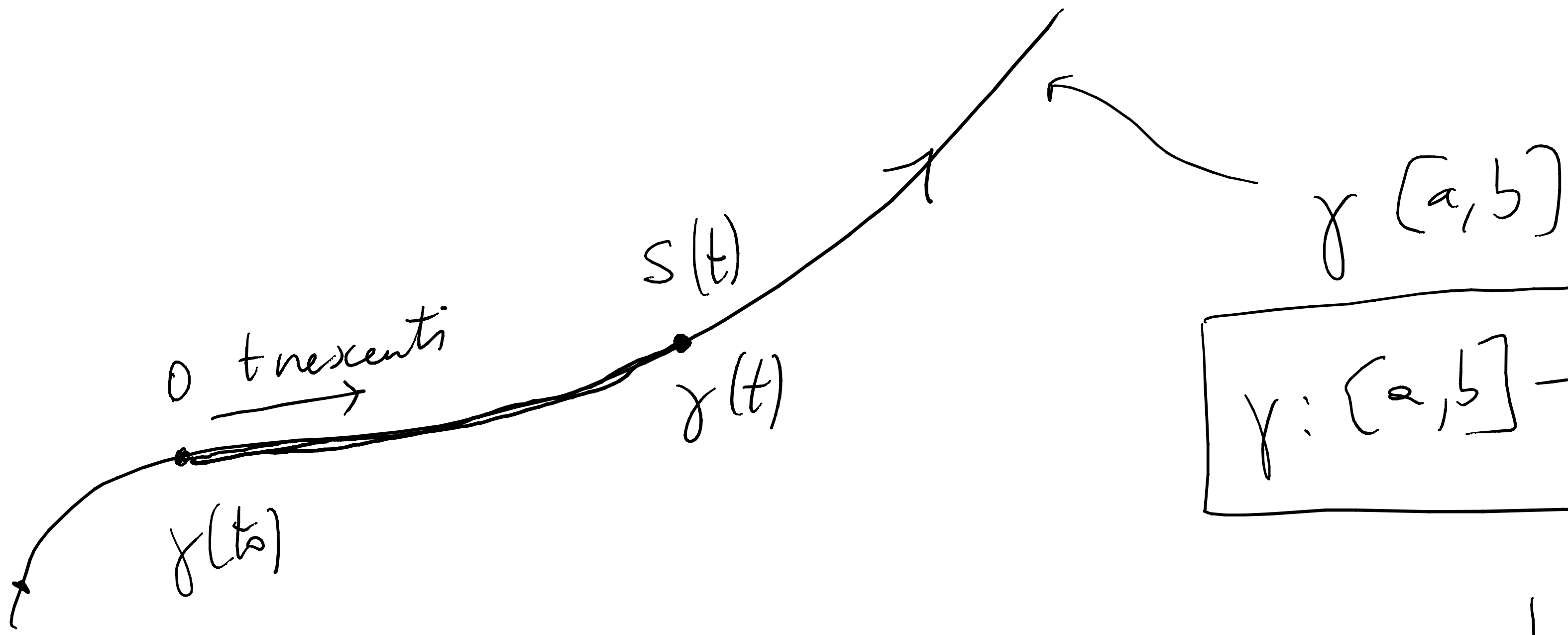
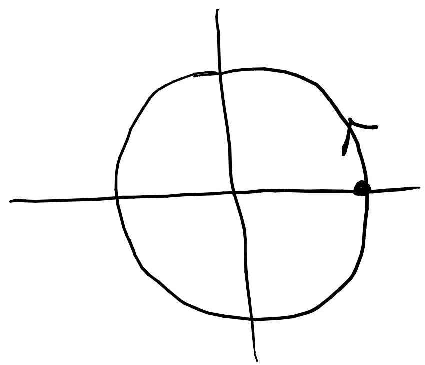


$$\boxed{\gamma : [a,b] \to \mathbb{R}^n}$$

$L(\gamma)$ calcolato fra (t_0, t)

$$\boxed{s(t) = \int_{t_0}^{t} |\dot{\gamma}(s)|\, ds}$$

$$\dot{s}(t) = \boxed{|\dot{\gamma}(t)|}$$



$f(x,y) = y^x$ $\qquad \boxed{f(x,y) = 1}$ è . . . . $\qquad y^x = e^{x \lg y}$ $\qquad \boxed{y > 0}$

$$\nabla f(x,y) = \nabla \left(e^{x \lg y} \right) =$$

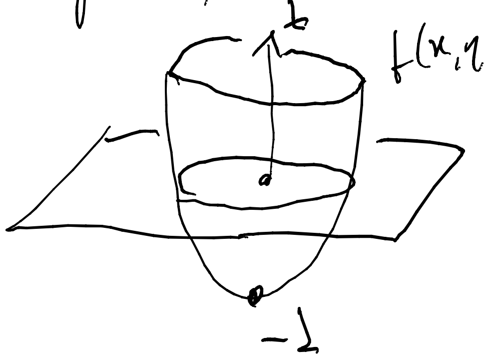
$$= \Big(\underbrace{e^{x \lg y} \lg y}_{f_x} , \underbrace{e^{x \lg y} \frac{x}{y}}_{f_y} \Big) = \underbrace{y^x}_{>0} \left(\lg y , \frac{x}{y} \right) \underset{\text{se}}{=} (0,0)$$

$$\boxed{y = 1 \text{ e } x = 0}$$

Un solo punto critico $(x_0, y_0) = (0,1)$

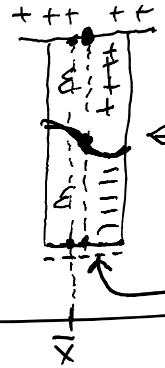
livello critico $f(0,1) = 1^0 = 1$. Dunque il livello $f(x,y) = 1$ è critico.

Nelle ipotesi del th. Dini la funzione $y \to f(x,y)$ può avere, al più, un unico zero.



$f(x,y) = x^2+y^2-1$

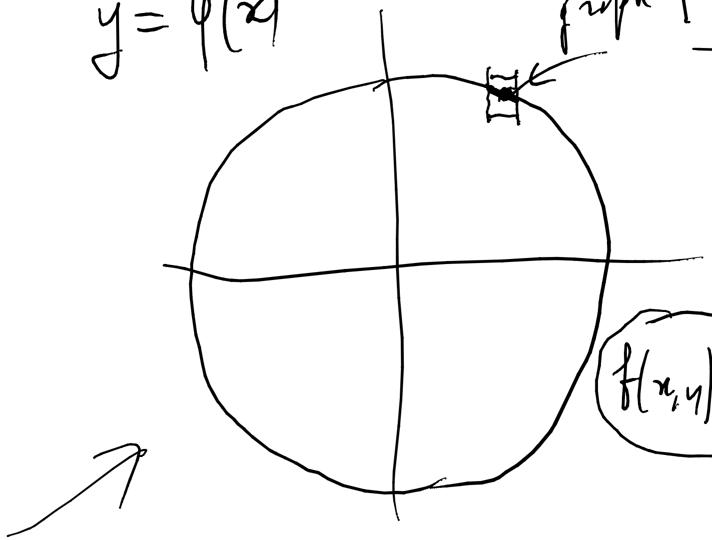
$\to$ ε della continuità



$\varphi(\bar{x})$ è l'unico zero di

$y \to f(\bar{x}, y)$

$y=\varphi(x)$



$$\boxed{|\varphi(x)-y_0|<\varepsilon}$$

$$\boxed{f(x,\varphi(x))=0}$$

$f(x,y)=x^2+y^2-1$

$$\forall \varepsilon\ \exists \delta\ \ |x-x_0|<\delta \Rightarrow |\varphi(x)-\overset{y_0}{\varphi(x_0)}|<\varepsilon$$

$$X = x^2 y \qquad (X,Y) = T(x,y) \qquad T : \mathbb{R}^2 \to \mathbb{R}^2$$

$$Y = x^2 + y^2 \qquad \exists\, T^{-1} : (x,y) = T^{-1}(X,Y) \; ?$$

$$T\underbrace{(x,y)}_{\in \mathbb{R}^2} = \begin{pmatrix} x^2 y \\ x^2 + y^2 \end{pmatrix} \in \mathbb{R}^2$$

jacobiano

$$\det \frac{\partial(X,Y)}{\partial(x,y)} = \begin{vmatrix} 2xy & x^2 \\ 2x & 2y \end{vmatrix} = 4xy^2 - 2x^3 = $$

$$= 2x\left(2y^2 - x^2\right)$$

si annulla se: $x = 0$ oppure $2y^2 = x^2$ cioè $\sqrt{2}\,|y| = |x|$

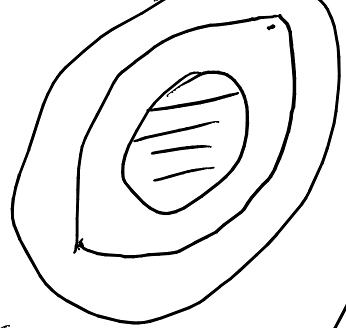
fuori dalle rette $x = 0$; $y\sqrt{2} = x$ $\;-y\sqrt{2} = x$ si può applicare l'inversione locale

$$\left[\frac{\partial(f_1, f_2, \dots, f_m)}{\partial(x_1, x_2, \dots, x_n)}\right]_{ij} = \frac{\partial f_i}{\partial x_j}$$

$f_1, f_2, \dots, f_m : \Omega \to \mathbb{R}$

$\Omega \subseteq \mathbb{R}^n$

La Jacobiana ha, sulla riga i, tutte le derivate parziali della i-esima componente scalare, mentre ha sulla colonna j le derivate parziali di tutte le funzioni $f_1, \dots, f_m$ rispetto alla variabile x_j



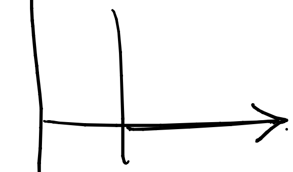
NON è sempl. connesso

f è α-omogenea $\quad \underline{\underline{|f| \ge k > 0}}$ su $\{|x| = 1\} \cap \operatorname{dom} f = S$

allora $\lim_{0} f = \infty$

se f è α-omog. $\alpha < 0$ allora $\frac{1}{f}$ è $-\alpha$-omog.

$-\alpha > 0$



$\boxed{|f| \ge K > 0}$ su S allora

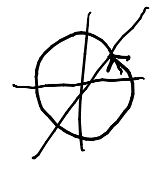
$$\underline{\underline{\frac{1}{|f|} \le \frac{1}{K}}} \text{ su } \underline{\underline{S}}$$

$$\frac{x-y}{x^2+y^2}$$

f continua $\quad f(x) \neq 0$ se $\underline{x \neq 0}$

$\frac{1}{f}$ continua

$$\frac{x^2+y^2}{x-y} \not\to 0$$



$f(x, \varphi(x)) \equiv 0 \quad \underline{[x_0-\delta,\ x_0+\delta]}$

$\varphi(x_0) = y_0$

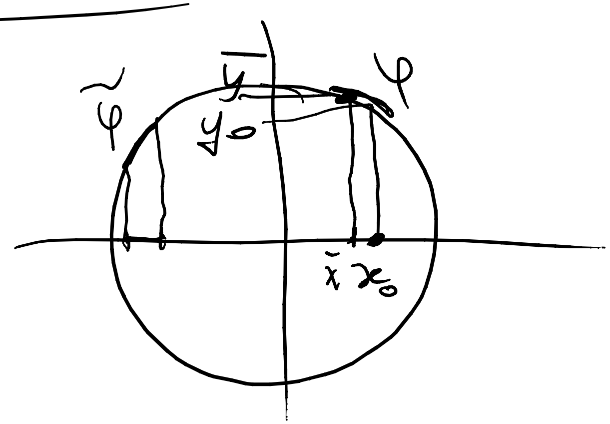
$\varphi(x)$ è l'unico zero della funzione $y \to f(x,y)$

$$\Big|\varphi(x) - \underbrace{\varphi(x_0)}_{y_0}\Big| < \varepsilon < \bar{\varepsilon}$$

$$|x - x_0| < \delta$$

Ma, se $\bar{\varepsilon} < \varepsilon$

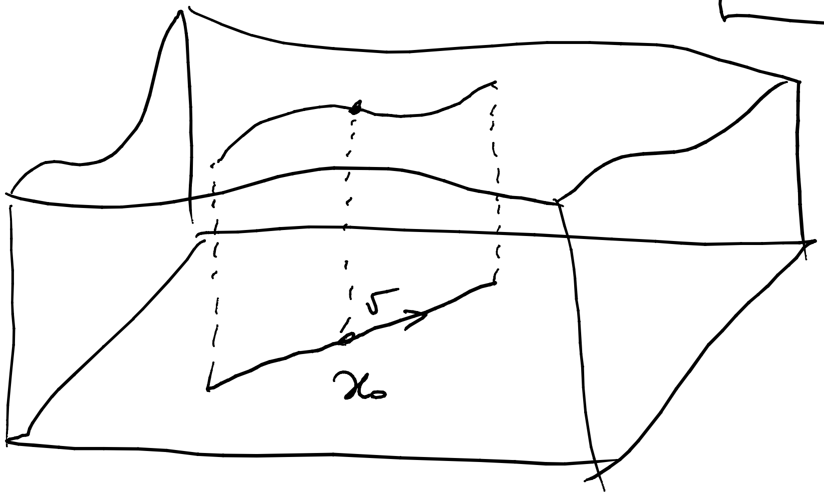
$$|\varphi(x) - y_0| < \bar{\varepsilon} \quad ?$$



$$\frac{\partial f}{\partial v}(x_0) = h'(0) \qquad h(t) = f(x_0 + tv)$$

$$\lim_{k \to 0} \frac{h(k) - h(0)}{k} =$$

$$= \boxed{\lim_{k \to 0} \frac{f(x_0 + kv) - f(x_0)}{k}}$$



$$\lim_{t \to 0} \frac{f(x_0 + tv) - f(x_0)}{t} = \frac{\partial f}{\partial v}(x_0)$$

se $\exists$ finito

$$\frac{\partial f}{\partial x_1}(x_0) \equiv \frac{\partial f}{\partial e_1}(x_0) = \lim_{t \to 0} \frac{f(x_0 + te_1) - f(x_0)}{t}$$

$$e_1 \to (1, 0, \dots, 0)$$

$$te_1 = (t, 0, 0, \dots, 0) \qquad x_0 = (x_0^1, x_0^2, \dots, x_0^n)$$

$$\lim_{t \to 0} \frac{f(x_0^1 + t, x_0^2, x_0^3, \dots x_0^n) - f(x_0^1, \dots, x_0^n)}{t}$$

$$x_0^1$$

$$\frac{d}{dt}\varphi(t) \qquad \frac{t \to f(x_0^1 + t, x_0^2, \dots, x_0^n)}{\text{nel punto } t = 0} =$$

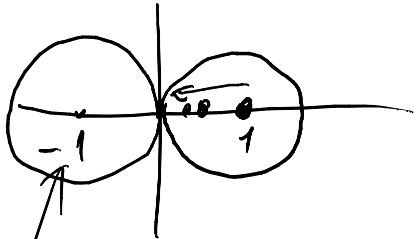
$$t \to f(t, x_0^2, \dots x_0^n)$$

$(0,0)$

$x^2 - 2x + y^2 = 1$

$(x-1)^2 + y^2 = 1 =$

$= \{x^2 + y^2 - 2x < 0\}$



$\{x^2 + y^2 + 2x < 0\}$

$(0,0)$ rispetto a $\underbrace{B((1,0),1)}_{B_1} \cup$

$\cup \underbrace{B((-1,0),1)}_{B_2}$

$(0,0)$ è il $\lim_{\varepsilon \to 0} (\varepsilon, 0)$

$(0,0) \notin B_1$

$(0,0) \notin B_2$

$\left(\frac{1}{n}, 0\right) \to (0,0)$

$f \in C^N(B(x_0,\delta))$ $\quad w \in \mathbb{R}^n \quad |w|<\delta$ $\quad$ n numero delle variabili indip.

$$f(x_0+w) = \sum_{k=0}^{N} \frac{1}{k!} \sum_{i_1,\dots,i_k=1}^{n} \frac{\partial^{(k)} f}{\partial x_{i_1}\partial x_{i_2}\cdots\partial x_{i_k}}(x_0)\, w_{i_1} w_{i_2}\cdots w_{i_k} + \underbrace{R^N(w)}_{\text{resto}}$$

x_1 oppure x_2 oppure ... x_n

N grado del polinomio di Taylor

$$= \boxed{\sum_{|k|\le N} \frac{1}{k!}\, \partial^k f(x_0)\, w^k} + \underbrace{R^N(w)}_{\text{resto}}$$

$$\boxed{k = (k_1, k_2, \dots, k_n)}$$

dim spazio

$$\boxed{|k| = \underbrace{k_1 + k_2 + \dots + k_n}_{\text{numero totale di derivate}}}$$

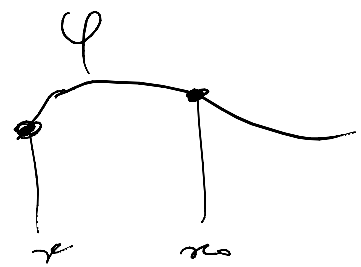
$$\boxed{k! = k_1!\, k_2! \cdots k_n!}$$

$k_1 = 5$ vuol dire 5 derivate rispetto a x_1

$$\boxed{\partial^k f(x_0) = \frac{\partial^{|k|} f(x_0)}{\partial x_1^{k_1}\partial x_2^{k_2}\cdots\partial x_n^{k_n}}} \qquad \boxed{w^k = w_1^{k_1} w_2^{k_2} \cdots w_n^{k_n}}$$

$$\underbrace{f(x,\varphi(x))}_{=0} - \underbrace{f(x_0,\varphi(x_0))}_{=0} = f_x(x_0+\xi(x-x_0),\dots)\underbrace{(x-x_0)}_{x\to x_0} +$$

$$+\underbrace{f_y(\qquad)}_{\neq 0 \text{ ipotesi}}(\varphi(x)-\varphi(x_0))$$



$$\frac{\varphi(x)-\varphi(x_0)}{x-x_0} = -\frac{f_x(x_0+\xi(x-x_0),\ y_0+\xi(\varphi(x)-\varphi(x_0)))}{f_y(\qquad)}$$

$x\to x_0$ per la continuità di φ $\varphi(x)\to\varphi(x_0)$

e quindi $(x_0+\xi(x-x_0),\ y_0+\xi(\underbrace{\varphi(x)-\varphi(x_0)}_{\text{tende a 0 perché } \varphi \text{ è continua.}}))\longrightarrow(x_0,y_0)$