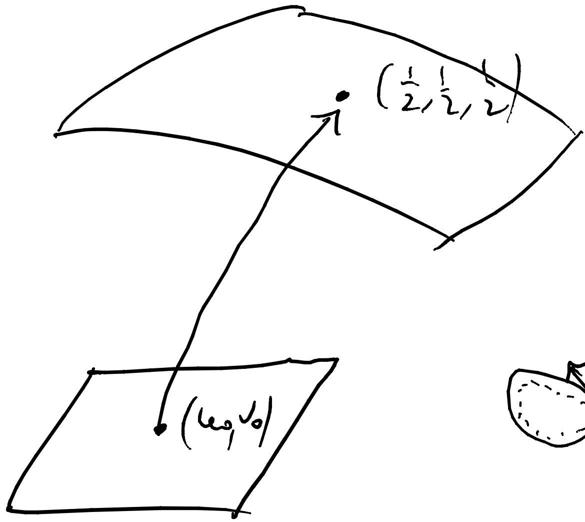
$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

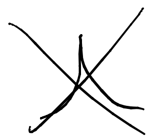
$\phi(u,v) = (\sin u \cos v, \sin u \sin v, \cos^2 v)$

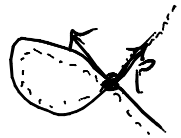
$\phi : [0,1] \times [0,1]$

$\exists\, u_0, v_0 : \phi(u_0,v_0) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$?

$(u_0,v_0) \in [0,1] \times [0,1]$







$$\begin{cases} \sin u \cos v = \frac{1}{2} \\ \sin u \sin v = \frac{1}{2} \\ \cos^2 v = \frac{1}{2} \end{cases}$$

$\Longrightarrow (\sin u)\frac{1}{\sqrt{2}} = \frac{1}{2}$

$\sin u = \frac{1}{\sqrt{2}} \Rightarrow u = \frac{\pi}{4}$

$[0,1]$

$\cos v = \frac{1}{\sqrt{2}}$

$v = \frac{\pi}{4}$ unico in $[0,1]$

$u_0 = \frac{\pi}{4}$

$v_0 = \frac{\pi}{4}$

$t_1, t_2 \to P$

$N = \phi_u\left(\frac{\pi}{4}, \frac{\pi}{4}\right) \times \phi_v\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

$$\left|\int_a^b f(t)\,dt\right| \le \int_a^b |f(t)|\,dt \qquad \text{falsa se } b<a$$

ed è immediata altrimenti $(a<b)$ perché

$$\boxed{f(t) \le |f(t)|} \qquad -f(t) \le |f(t)| \qquad a<b$$

$$\int_a^b f(t) \le \int_a^b |f(t)| \qquad \int_a^b -f(t) \le \int_a^b |f(t)|$$

$$\left|\int_a^b f(t)\,dt\right| \le \int_a^b |f(t)|\,dt$$

$$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$\left|\int_0^{2\pi} \gamma(t)\,dt\right| = 0$$

$$\int_0^{2\pi} |\gamma(t)| = \int_0^{2\pi} 1 = 2\pi$$

$$\int \gamma(t) \approx \sum_1^n (t_{i+1}-t_i)\,\gamma(\xi_i)$$

$$|\qquad| \underset{\text{dis. triang.}}{\le} \sum_1^n \left|\underbrace{(t_{i+1}-t_i)}_{>0}\gamma(\xi_i)\right| = \sum_1^n (t_{i+1}-t_i)\,|\gamma(\xi_i)| \approx \int_a^b |\gamma(t)|\,dt$$

$\gamma(t) = \begin{pmatrix} t \\ t \end{pmatrix}$ $\quad (0,1)$

~~Au~~ Andrini Luca

$$\left| \int_0^1 \begin{pmatrix} t \\ t \end{pmatrix} dt \right| = \left| \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right| = \frac{1}{\sqrt{2}}$$

esempio con $=$

$$\int_0^1 \left| \begin{pmatrix} t \\ t \end{pmatrix} \right| dt = \int_0^1 \sqrt{t^2+t^2}\, d = \sqrt{2} \int_0^1 t = \sqrt{2}\,\frac{1}{2} = \frac{1}{\sqrt{2}}$$

$\gamma(t) = \begin{pmatrix} t \\ 0 \end{pmatrix}$ $\quad (0,1)$ $\quad \int_0^1 \begin{pmatrix} t \\ 0 \end{pmatrix} dt = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}$ $\quad \left| \int_0^1 \begin{pmatrix} t \\ 0 \end{pmatrix} \right| = \frac{1}{2}$

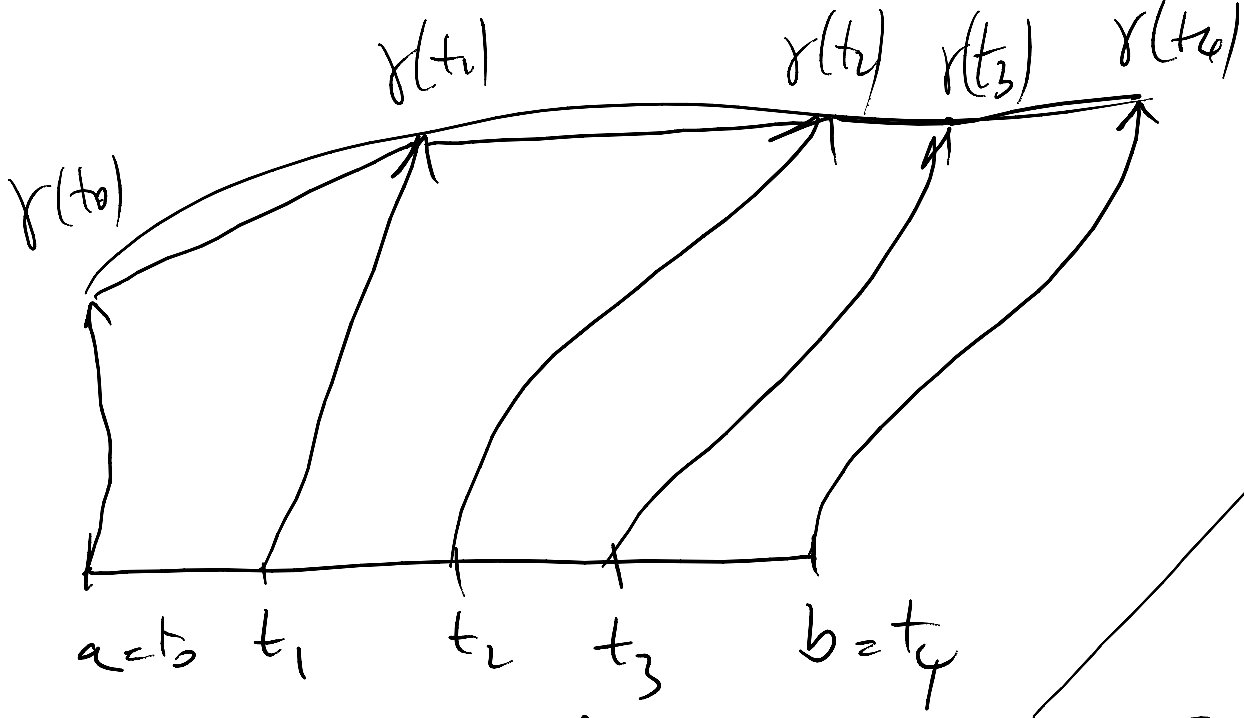
$$\int_0^1 |\gamma(t)|\, dt = \int_0^1 \sqrt{t^2+0}\, dt = \int_0^1 t = \frac{1}{2}$$

$\rho = \theta$ caso partic. di $\rho = f(\theta)$ caso partic. di $\begin{cases} \rho = f(t) \\ \theta = g(t) \end{cases}$

Spir. Archimede $\quad f(\theta) = \theta$

$\rho = t \qquad f(t) = t$

$\theta = t \qquad g(t) = t$



$\Pi = \{t_0, t_1, \dots, t_4\}$ partizione di $[a,b]$

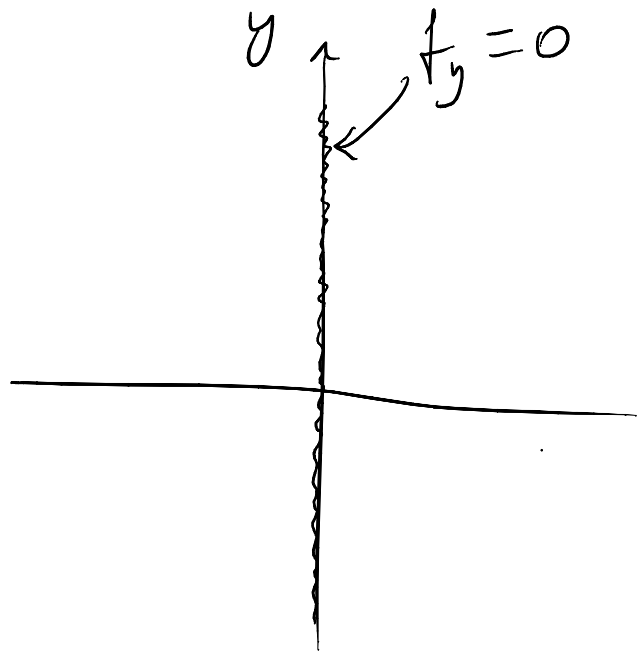
$\Lambda(\Pi)$

$\Pi \to$ poligonale $\to$

$\to$ lunghezza della poligonale $=$

$\to \sum_{0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)|$

$x^3y - x = 1$ $\quad f(x,y) = 1 \quad f(x,y) = x^3y - x$



$$\begin{cases} f_x(x,y) = 3x^2y - 1 \\ \underline{f_y(x,y) = x^3} \end{cases} \Rightarrow f_y = 0 \text{ se } \underline{\underline{x=0}}$$

sostit. $x=0$ in $f_x = 0$ si ottengono gli eventuali punti singolari.

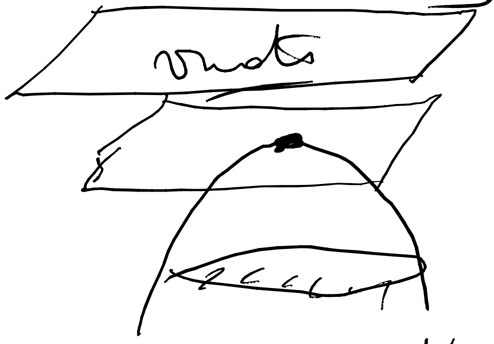
sost. $\underline{\underline{(0,y)}}$ in $3x^2y - 1$

$$f_x(0,y) \equiv -1 \quad \forall y$$

Quindi, se si annulla f_y non si annulla f_x.

Dini funz. dall'asse y per esplic. y in funz. di x

Sull'asse y, x è sempre funzione di y (localm. x Dini)



$$\exists\,(x,y)\in\mathbb{R} : \; x^3y - x = 1 \qquad x+y=1$$

$$\Downarrow$$

$$y=0 \Rightarrow x=-1$$

$$f(0,0) = x^3y - x\big|_{(0,0)} = 0$$

quindi $\to (-1,0) \in$ insieme di livello dato $f=1$

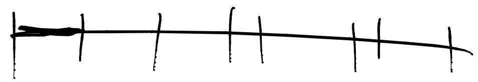
$$\left|\int_a^b \gamma(t)\,dt\right| \approx \left|\sum_1^n (t_{i+1}-t_i)\,\gamma(\xi_i)\right| \underset{\text{t. triang.}}{\leq} \sum_1^n \big|\underbrace{(t_{i+1}-t_i)}_{>0}\,\gamma(\xi_i)\big| =$$

$$= \sum (t_{i+1}-t_i)\,|\gamma(\xi_i)| \approx \int_a^b |\gamma(t)|\,dt$$

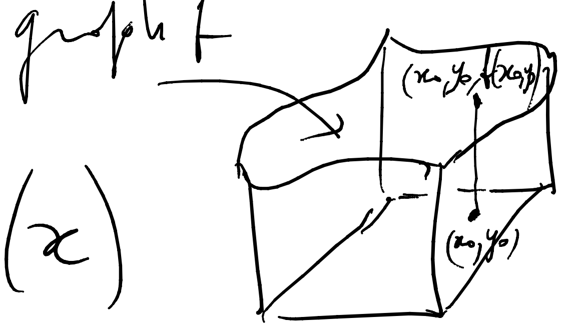
scalare

$|\gamma(t)|$

C^0 compost. di f. continue

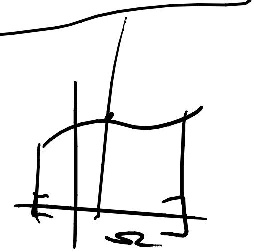


$$f : \Omega \to \mathbb{R}$$



$$\operatorname{graph} f = \{ \underbrace{(x,y) \in \Omega}_{x \in \Omega} \times \mathbb{R} : y = f(x) \}$$

$$f : X \to Y \qquad \boxed{\operatorname{graph} f = \{ (x,y) \in X \times Y : y = f(x) \}}$$



$\Omega \subseteq \mathbb{R}^2$ graph f è una superficie in $\mathbb{R}^3$ (cartesiana)

$x = x_0 \in \Omega$

$a_n = n!$ $\operatorname{graph} a_n = \{ (n,m) \in \mathbb{N} \times \mathbb{N} : m = n! \}$

$xy = 1 \qquad f(x,y) = 1$

$f_x = y$

$f_y = x$

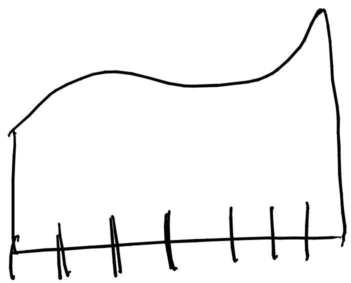
$(0,0)$ è l'unico punto critico (singolare) (stazionario)

$f(0,0) = 0$

L'unico punto critico non sta nell'insieme $f = 1$

$$\Lambda(\{t_0, t_1, \dots, t_n\}) \le \int_a^b |\dot\gamma(t)|\,dt = K$$

$$\sum_0^{n-1} \underbrace{|\gamma(t_{i+1}) - \gamma(t_i)|}_{\left|\int_{t_i}^{t_{i+1}} \dot\gamma(t)\,dt\right|} \le \underbrace{\sum_0^{n-1} \int_{t_i}^{t_{i+1}} |\dot\gamma(t)|\,dt}_{\int_a^b |\dot\gamma(t)|\,dt}$$



$$\forall \Pi \quad \Lambda(\Pi) \le K \Rightarrow K \text{ è maggiorante}$$

e quindi $\sup \Lambda(\Pi)$ è finito

inoltre $\sup \Lambda(\Pi)$ è il MINIMO MAGGIORANTE $\Rightarrow$

minimo maggiorante $\to \Lambda(\Pi) \le K \leftarrow$ maggiorante

$$\int \sqrt{1+t^2}\,dt \underset{t=\sinh u}{=}$$

$$\int_0^1 \sqrt{1}$$

$\sqrt{1-t^2}$ $t=\sin u$ oppure $\cos u$

$$\left(\frac{e^u+e^{-u}}{2}\right)^2 =$$

$\sqrt{t^2-1}$ $t=\cosh u$ $= \frac{1}{4}\left[e^{2u}+e^{-2u}+2\right]$

$\sqrt{-1-t^2}$ NON VUOL DIRE NULLA

$\sqrt{1+t^2}$ $t=\sinh u$

AN1

$$\int \sqrt{1+4t^2}\,dt \underset{2t=u}{=} \boxed{\frac{1}{2}\int \sqrt{1+u^2}\,du} \underset{u=\sinh v}{=}$$

$t = u/2$

$dt = \frac{du}{2}$

$\{ \binom{x}{y} \in \mathbb{R}^2 ; \exists t \in [a,b] \ \binom{x}{y} = \gamma(t) \}$

$\mathrm{Im}\,\gamma$ $\boxed{\text{graph } f = \gamma[a,b]}$

$\{(t, f(t))\}$

grafico cartesiano

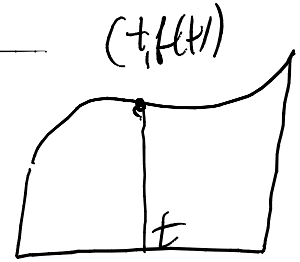
$$\boxed{\Lambda(\gamma) = \int_a^b |\dot\gamma(t)|\,dt}$$

$$\boxed{\gamma(t) = \binom{t}{f(t)}} \quad t \in [a,b]$$

$$\int_a^b \sqrt{1+f'(t)^2}\,dt \qquad \dot\gamma = \binom{1}{f'(t)}$$

$$\begin{cases} \rho = f(t) \\ \theta = g(t) \end{cases} \qquad \begin{cases} x = f(t)\cos g(t) \\ y = f(t)\sin g(t) \end{cases}$$

$$\int \sqrt{f'^2 + f^2 g'^2}\,dt$$

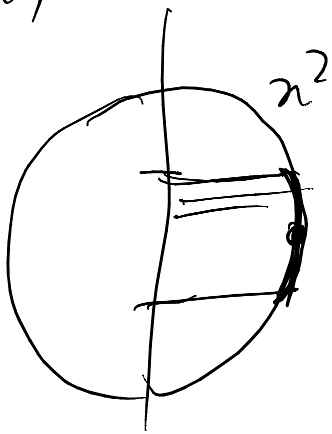


$$\begin{cases} x = t \\ y = f(t) \end{cases}$$

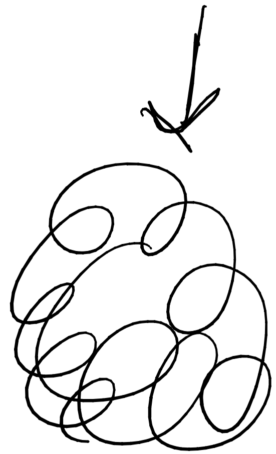
$$\begin{cases} x = 2t+1 \\ y = t-1 \end{cases} \qquad t \in [\ \ ]$$

$x^2+y^2=1$

$x = \varphi(y)$

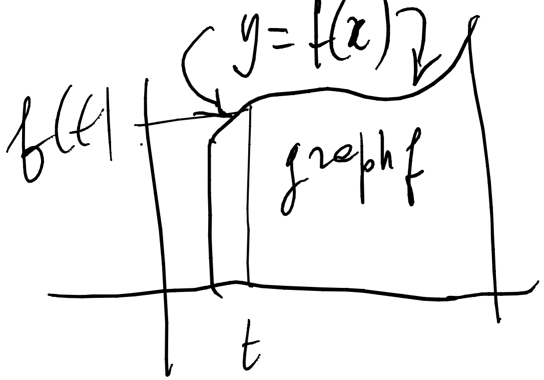


$$\gamma(t) = \begin{pmatrix} 2t+1 \\ t-1 \end{pmatrix}$$



$f(\varphi(y), y)$

$$\gamma : [a,b] \to \mathbb{R}^n$$



$$\gamma(t) = \begin{pmatrix} t \\ f(t) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$y = f(x)$ esplicita

$t \in [a,b]$ $\quad \{\gamma(t) : t \in [a,b]\}$

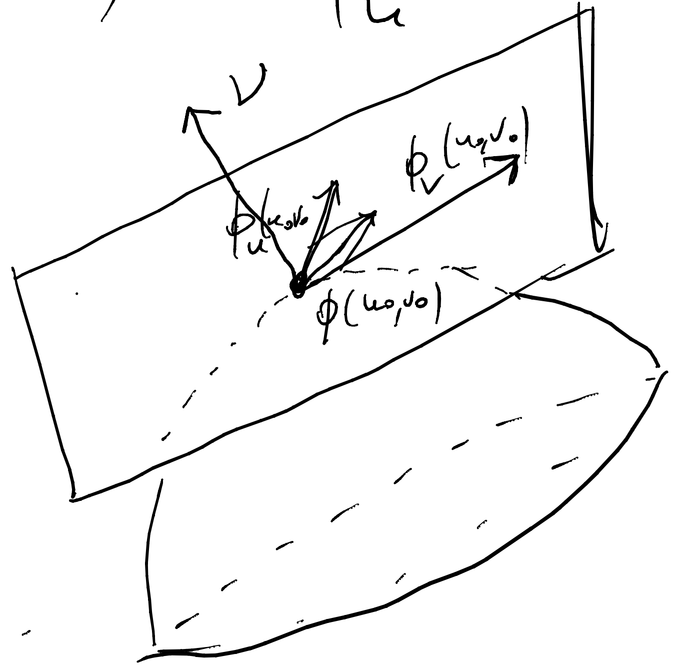
$y = f(t)$

$\phi(u,v)$ $\boxed{\phi : \Delta \to \mathbb{R}^3 \quad \Delta \subseteq \mathbb{R}^2}$

$\phi_u(\underline{u_0,v_0})$ $\phi_v(\underline{u_0,v_0})$ spostamenti sul piano tangente in $\phi(\underline{u_0,v_0})$

$$\boxed{\psi(\alpha,\beta) = \phi(u_0,v_0) + \alpha\,\phi_u(u_0,v_0) + \beta\,\phi_v(u_0,v_0)}$$

$(\alpha,\beta) \in \mathbb{R}^2$



$\phi(u_0,v_0) +$

$+ < \phi_u(u_0,v_0), \phi_v(u_0,v_0)$

$$\underbrace{\phi_u(u_0,v_0) \times \phi_v(u_0,v_0)}$$ sono tangenti

$= \nu(u_0,v_0)$ è perpend. al piano tangente

$\Rightarrow$ è il vettore normale standard in $\phi(u_0,v_0)$

$$\nu(u_0,v_0) = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \qquad \phi(u_0,v_0) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\boxed{n_1(x-a) + n_2(y-b) + n_3(z-c) = 0}$$

Eq. implicita del piano tangente