Note Title 5/6/2020

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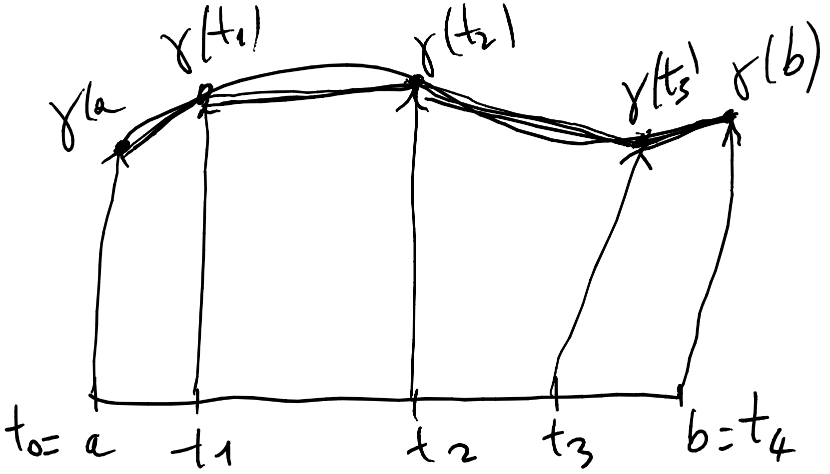
$\gamma : [a,b] \longrightarrow \mathbb{R}^N$ curva parametrica. $\gamma[a,b]$ sostegno di γ ($\mathrm{Im}\,\gamma$)

Π partizione di $[a,b]$ $\quad \Pi = \{\underline{a = t_0 < t_1 < \cdots\cdots < t_{n-1} < t_n = b}\}$

$\{t_0, t_1, \ldots, t_{n-1}, t_n\}$: $t_i < t_{i+1}$ $\forall i = 0 .. n-1$.



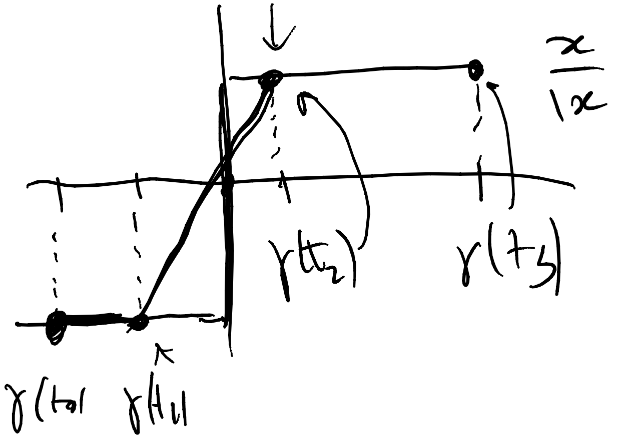
$$\Lambda(\Pi) \equiv \underbrace{\sum_0^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)|}$$

$$\Lambda(\gamma) \equiv \sup_{\substack{\Pi \text{ partiz.} \\ \text{di } [a,b]}} \Lambda(\Pi)$$

γ si dice rettificabile se $\Lambda(\gamma) = \sup_{\Pi} \Lambda(\Pi) < \infty$

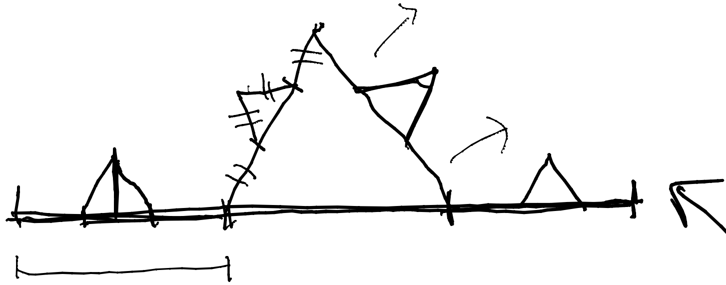
non rettificabile se $\underline{\Lambda(\gamma) = +\infty}$

$\gamma(t) \begin{pmatrix} t \\ f(t) \end{pmatrix}$



γ_0 segmento lung. 1

γ_1 4 segmenti lunghi 1/3



$\gamma_2 =$

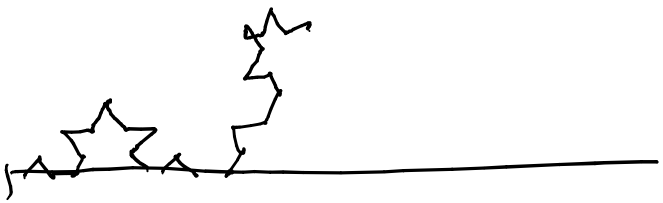
γ_0 ha lunghezza 1

γ_1 ha lunghezza $\frac{4}{3}$

γ_2 $\frac{16}{9}$

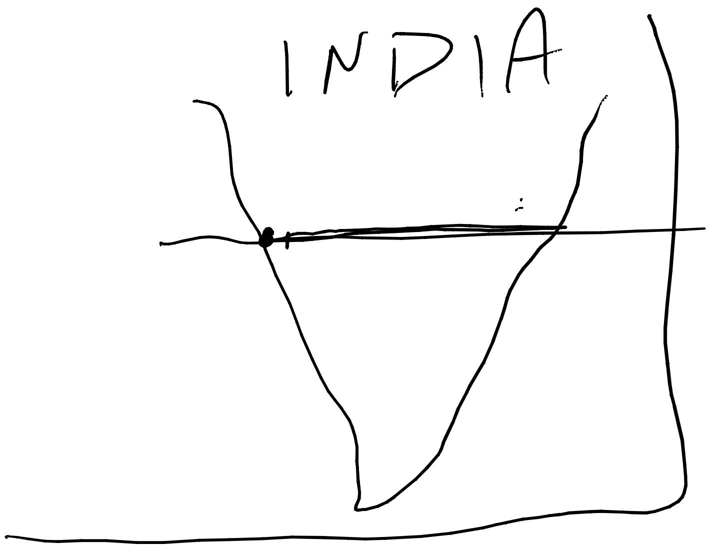
$\vdots$

γ_n ha lunghezza $\left(\frac{4}{3}\right)^n$



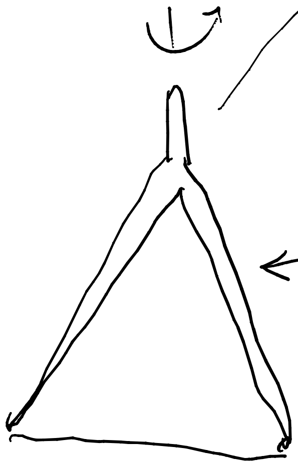
KOCH

"Fiocco di neve"



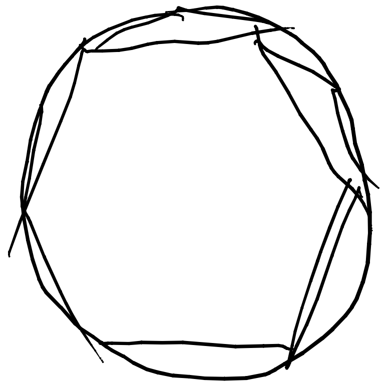
"CALCOLO" della lunghezza delle coste occidentali della

SCOZIA



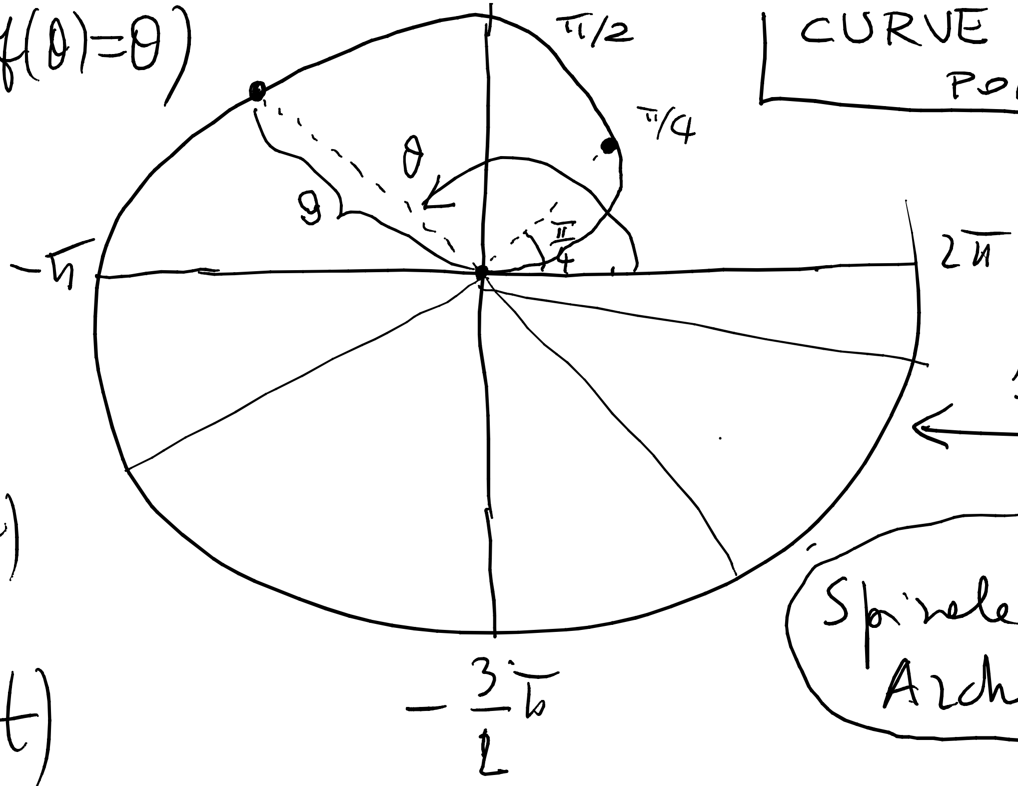
compasso a punte fisse, per riportare distanze

rappresenta una distanza reale in scala



CURVE IN COORDINATE POLARI

$\rho = \theta \quad (f(\theta) = \theta)$



$$\rho = f(\theta)$$

$$\gamma(t) = \begin{cases} \rho = f(t) \\ \theta = g(t) \end{cases}$$

$$\Lambda(\gamma) = \int_a^b |\dot{\gamma}(t)|\,dt$$

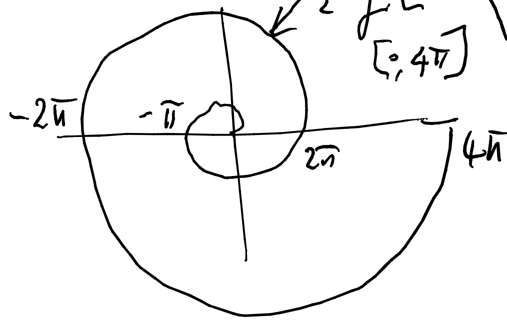
COORDINATE CARTESIANE

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$$

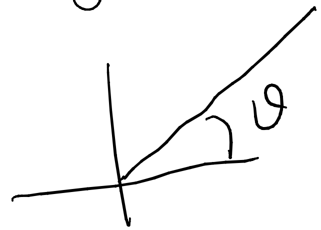
1 "giro" $[0, 2\pi]$

Spirale di Archimede

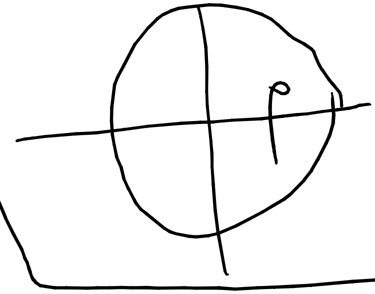
2 "giri" $[0, 4\pi]$



$\theta = \omega t$



$\rho = \omega t$



$$\rho = \theta \qquad \begin{aligned} x &= \rho\cos\theta \overset{\rho=\theta}{=} \theta\cos\theta \\ y &= \rho\sin\theta = \theta\sin\theta \end{aligned}$$

$$\boxed{\varphi(\theta) = \begin{pmatrix} \theta\cos\theta \\ \theta\sin\theta \end{pmatrix}} \qquad \theta \in [0, 2\pi]$$

$$\dot\varphi = (\cos\theta - \theta\sin\theta,\ \sin\theta + \theta\cos\theta)$$

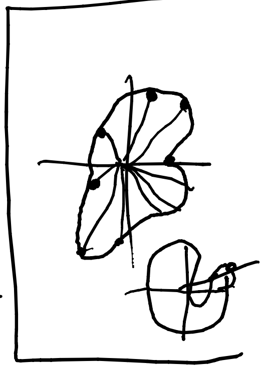
$$l = \int_0^{2\pi} \underbrace{|\dot\varphi(\theta)|\, d\theta}_{dl} = \int_0^{2\pi} \sqrt{1+\theta^2}\, d\theta$$

$$|\dot\varphi(\theta)| = \sqrt{(\cos\theta - \theta\sin\theta)^2 + (\sin\theta + \theta\cos\theta)^2} = \sqrt{1+\theta^2}$$

$$\rho = f(\theta) \qquad \begin{aligned} x &= f(\theta)\cos\theta \\ y &= f(\theta)\sin\theta \end{aligned}$$

$$f : \mathbb{R} \to \mathbb{R}^+$$

$$\varphi(\theta) = (f(\theta)\cos\theta,\ f(\theta)\sin\theta) = f(\theta)(\cos\theta, \sin\theta)$$



$$\dot\varphi(\theta) = \dot f(\theta)\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} + f(\theta)\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} = \begin{pmatrix} \dot f(\theta)\cos\theta - f(\theta)\sin\theta \\ \dot f(\theta)\sin\theta + f(\theta)\cos\theta \end{pmatrix}$$

$$|\dot\varphi(\theta)| = \sqrt{[\dot f(\theta)]^2 + f^2(\theta)}$$

$$\gamma(t) = \begin{pmatrix} f(t) \\ g(t) \end{pmatrix} \iff \begin{matrix} \rho = f(t) \\ \theta = g(t) \end{matrix} \qquad \left.\begin{matrix} \rho \\ \theta \end{matrix}\right\} \text{ funzioni del parametro } t$$

$$t \in [a,b]$$

$$x(t) = \rho \cos\theta$$

$$y(t) = \rho \sin\theta$$

$$\sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(\dot{\rho}\cos\theta - \rho\dot{\theta}\sin\theta)^2 + (\dot{\rho}\sin\theta + \rho\dot{\theta}\cos\theta)^2} =$$

$$= \sqrt{\dot{\rho}^2 + \rho^2\dot{\theta}^2}$$

$$\Lambda(\gamma) = \int_a^b \sqrt{\dot{f}^2(t) + f^2(t)\,\dot{g}^2(t)}\,dt$$

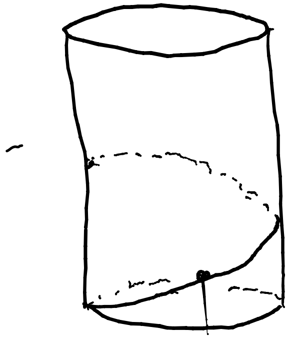
$$\rho = f(\theta) \qquad \begin{cases} \theta = t \\ \rho = f(t) \end{cases} \qquad \boxed{\dot{\theta} = 1 \quad \dot{\rho} = \dot{f}(t) \quad \rho = f(t)}$$

$\boxed{\text{ELICA CILINDRICA}}$

$$\begin{cases} \rho = 1 \\ \theta = t \\ z = t \end{cases} \qquad t \in [0, 2\pi]$$

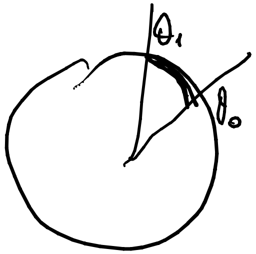
$$\begin{cases} x = 1 \cos t \\ y = 1 \sin t \\ z = t \end{cases} \qquad t \in [0, 2\pi]$$

equazioni polari d'un'ELICA cilindrica



$$\varphi(t) = (\cos t, \sin t, t) \qquad \dot{\varphi}(t) = (-\sin t, \cos t, 1)$$

$$\Lambda(\gamma) \equiv \Lambda(\varphi) = \int_0^{2\pi} \underbrace{\sqrt{1^2 + \sin^2 t + \cos^2 t}}_{|\dot{\varphi}|^2} \, dt = \int_0^{2\pi} \sqrt{2} \, dt = \boxed{2\sqrt{2}\,\pi}$$



$$R(\theta_1 - \theta_0)$$