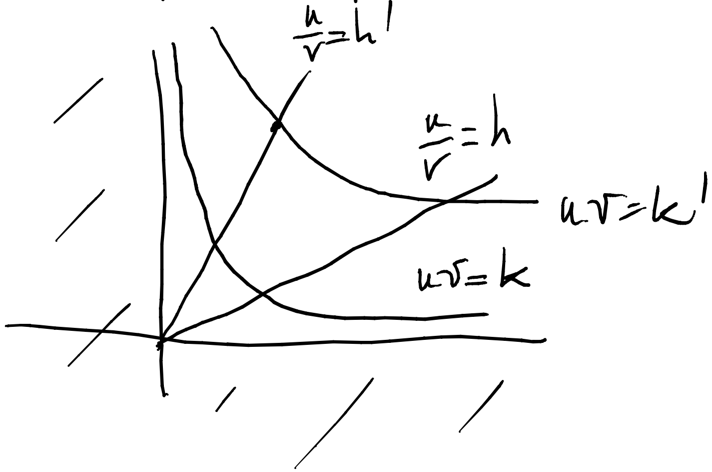
Note Title 5/5/2020

$$(u,v) \xrightarrow{T} \Big(\underset{\substack{\shortparallel \\ x}}{uv}, \underset{\substack{\shortparallel \\ y}}{\frac{u}{v}} \Big) \qquad v \neq 0$$

$$\det \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} v & u \\ \frac{1}{v} & -\frac{u}{v^2} \end{vmatrix} = -\frac{u}{v} - \frac{u}{v} = -2\frac{u}{v}$$

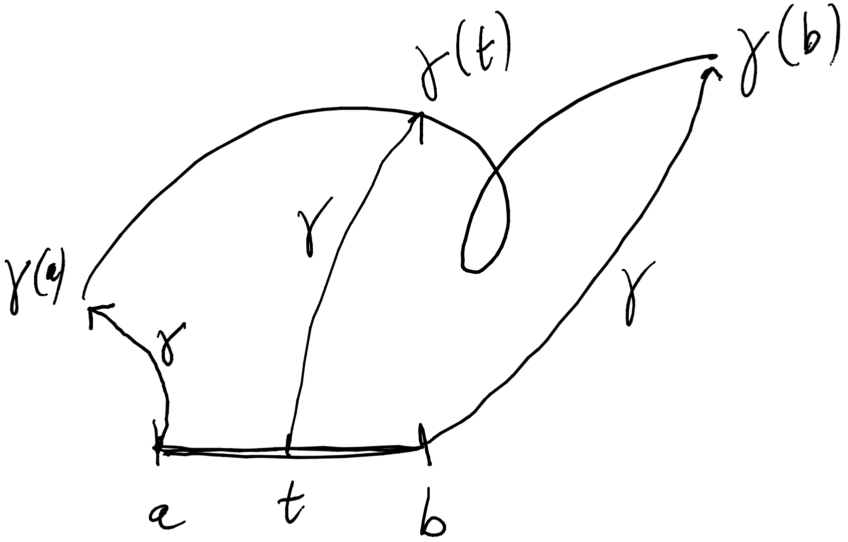
Si può applicare il th. inversione locale se $u \neq 0$

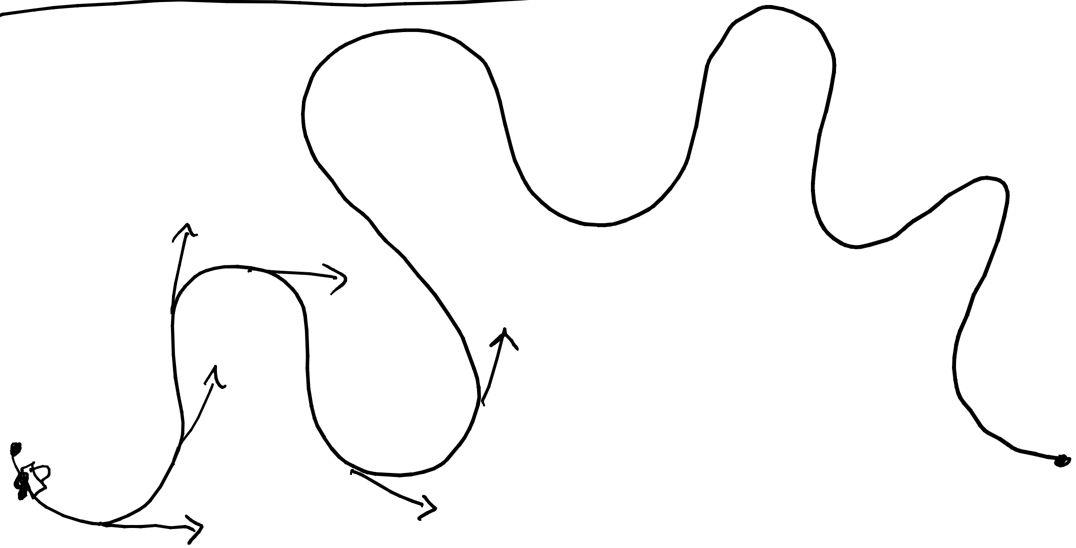


$x = \text{cost}$ $\quad \underline{uv = \text{cost}}$ $\quad T(u,v)$

$\boxed{y = \text{cost}}$ $\quad \frac{u}{v} = \text{cost}$

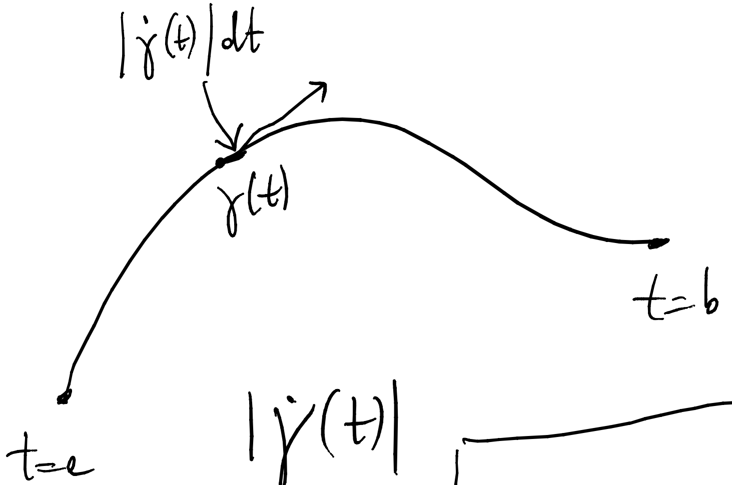
LUNGHEZZA DI UNA CURVA





1 ora $\qquad |v| = \text{cost} = 10\,\text{km/h}$

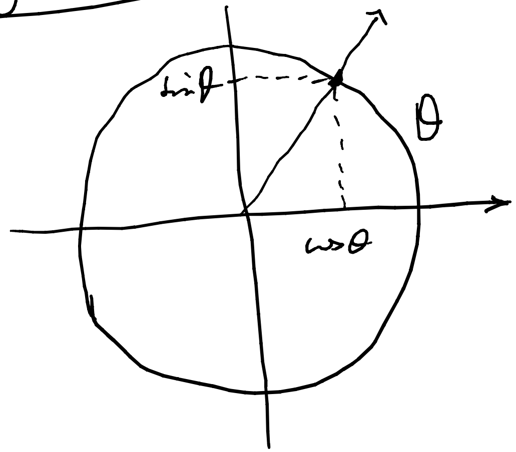
$\Longrightarrow$ lunghezza $= 10\,\text{Km}$

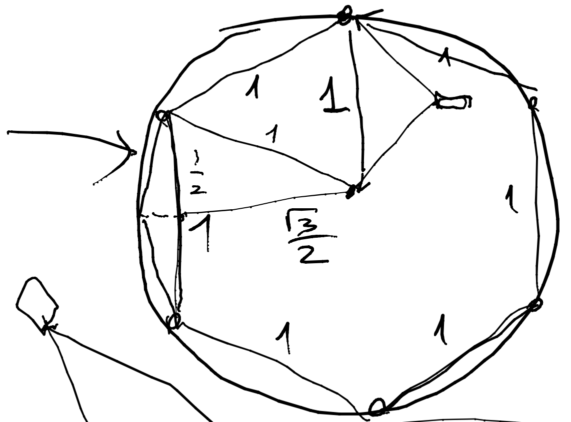


$|\dot\gamma(t)|$

$$\boxed{\Lambda(\gamma) = \int_a^b |\dot\gamma(t)|\,dt}$$

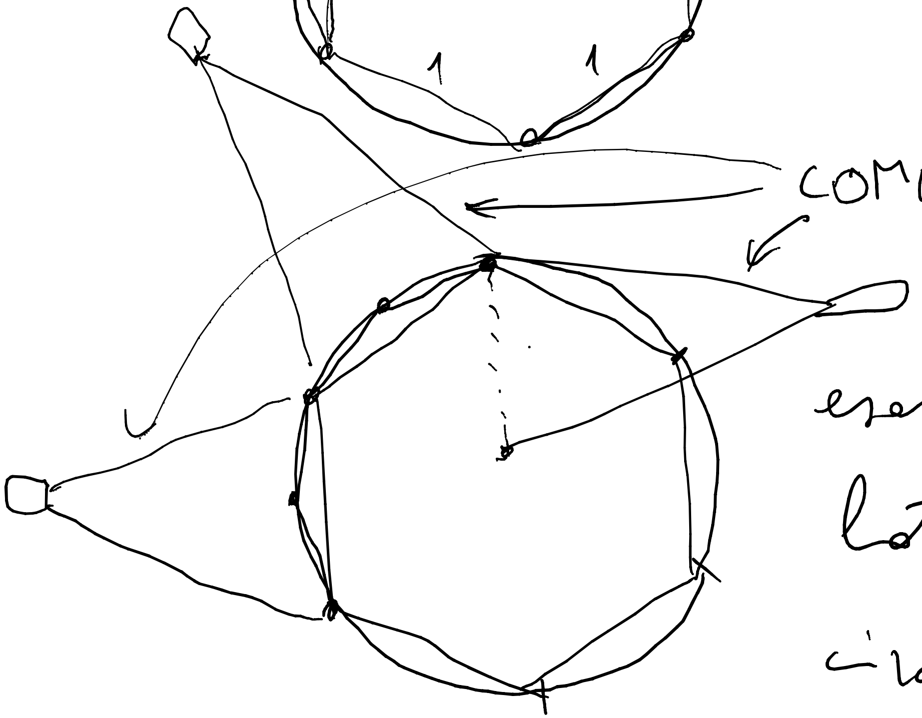
lunghezza





lunghezza (circonf.) $\geq$ perimetro dell'esagono

$2\pi \geq 6 \Rightarrow \underline{\pi \geq 3}$



COMPASSO

esagono regolare

lato = raggio

circonf. circoscritta

<u>IDEA</u>: (Brisone, Antifonte, Eudosso, Archimede)

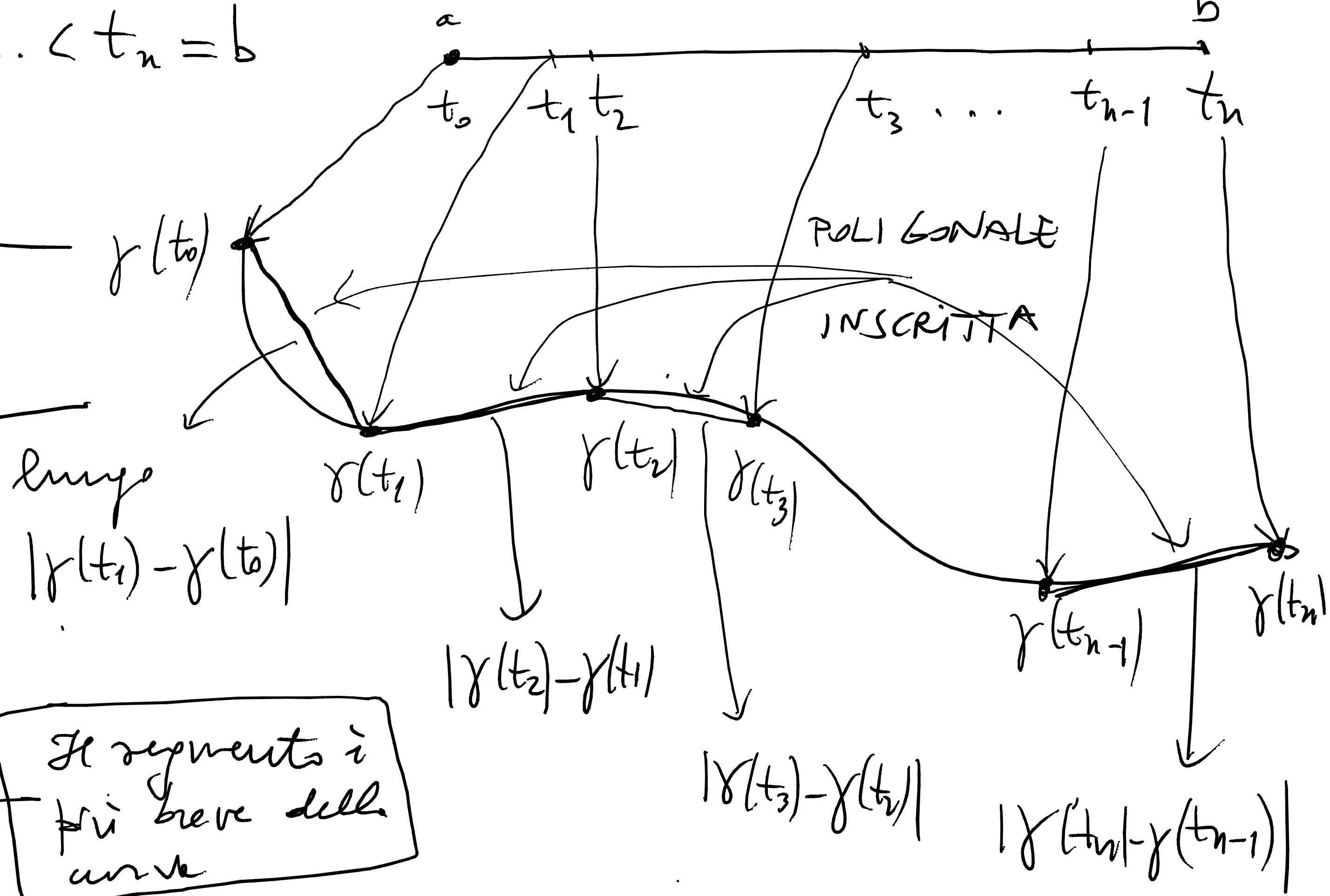
<u>APPROSSIMARE LA CURVA CON UNA SPEZZATA.</u>

RETTIFICABILITA' = LUNGHEZZA FINITA

partizione su $[a,b]$

$$a = t_0 < t_1 < t_2 < \ldots < t_n = b$$

$\Pi = \{t_0, t_1, \ldots, t_n\}$ con



Lunghezza della poligonale inscritta

$$\Lambda(\Pi) = \sum_{i=0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)|$$

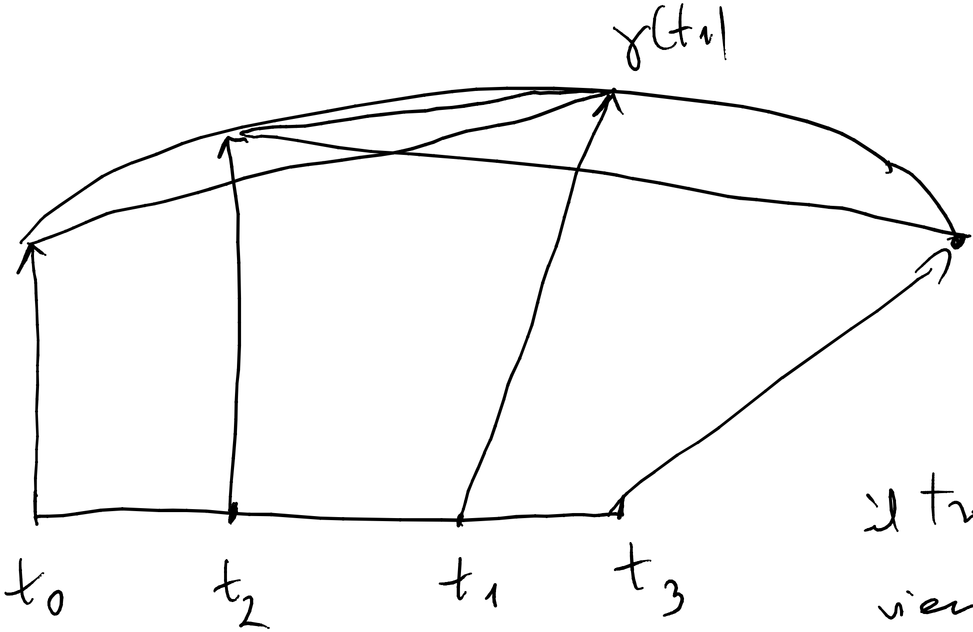
$$\boxed{\Lambda(\Pi) \leq \Lambda(\gamma)} \quad ?$$

Il segmento è più breve della curva

$$\boxed{\Lambda(\gamma) = \sup_{\Pi} \Lambda(\Pi)}$$

DEFINIZIONE

$\sup\{\Lambda(\Pi) : \Pi$ è una partiz. di $[a,b]\}$



se la sequenza $t_0, t_1, \dots, t_n$ non è crescente si ottiene un valore

SCORRETTO

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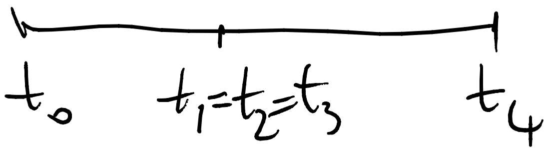
il tratto di curva fra $\gamma(t_2)$ e $\gamma(t_1)$ viene "percorso tre volte", invece di una.

invece $t_0 \le t_1 \dots \le t_n$ non comporta variazioni con il caso $t_0 < t_1 < \dots < t_n$

$$\sum |\gamma(t_{i+1}) - \gamma(t_i)|$$



$\Longrightarrow$

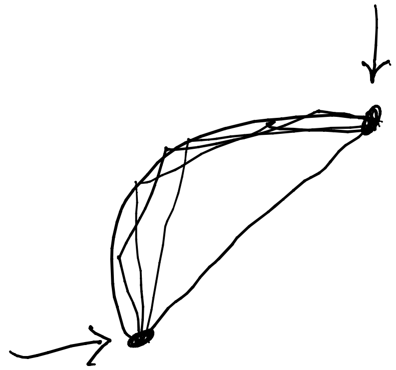
$$|\gamma(t_2) - \gamma(t_1)| = 0$$

$$|\gamma(t_3) - \gamma(t_2)| = 0$$

$\boxed{\gamma \text{ è } \underline{\text{rettificabile}} \text{ se } \Lambda(\gamma) < \infty}$ DEFINIZIONE

$\Pi = \{a, b\}$
$t_0 < t_1$

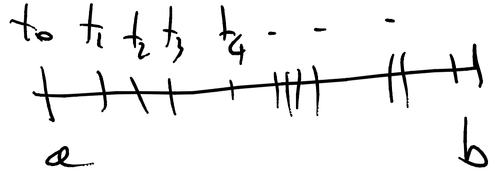
$\Lambda(\gamma) \geq |\gamma(b) - \gamma(a)|$



$\sup \Lambda(\Pi)$
al variare di Π
partiz. di $[a,b]$

Metro da sarto

Modello di curve rettificabili



Teorema Le curve parametriche di classe C^1 sono rettificabili, e inoltre

$$\boxed{\Lambda(\gamma) \leq \int_a^b |\dot{\gamma}(t)|\, dt}$$

$$\gamma \in C^1 \Leftrightarrow \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_N \end{pmatrix} = \gamma \text{ ha componenti } C^1[a,b]$$

→ Sia $a = t_0 < t_1 < \dots < t_n = b$ una partizione arbitraria di $[a,b]$ $\widehat{\Pi}$

→ $\Lambda(\Pi) = \sum_{0}^{n-1} \left| \underbrace{\gamma(t_{i+1}) - \gamma(t_i)} \right| =_{\text{th. fondam. calcolo integrale}}$

$= \sum_{0}^{n-1} \left| \int_{t_i}^{t_{i+1}} \dot\gamma(t)\,dt \right| \leq$

$\leq \sum_{0}^{n-1} \int_{t_i}^{t_{i+1}} \underline{|\dot\gamma(t)|}_{\in C^0}\,dt =$ additività dell'integrale $\cup[t_i, t_{i+1}] = [a,b]$

$= \boxed{\int_a^b |\dot\gamma(t)|\,dt}$ Maggiorante per $\{\Lambda(\Pi) ; \Pi \text{ partiz}\}$

$\left| \int_a^b \right| \leq \int_a^b | \, |$

$a < b$

$\gamma_j(t_{i+1}) - \gamma_j(t_i) =$

$= \int_{t_i}^{t_{i+1}} \dot\gamma_j(t)\,dt$

$\int_{t_i}^{t_{i+1}} \dot\gamma(t) \approx$

$\left| \sum \underbrace{\dot\gamma(\xi)}_{\in \mathbb{R}^N} \underbrace{(t_{i+1} - t_i)}_{\in \mathbb{R}^+} \right| \leq$

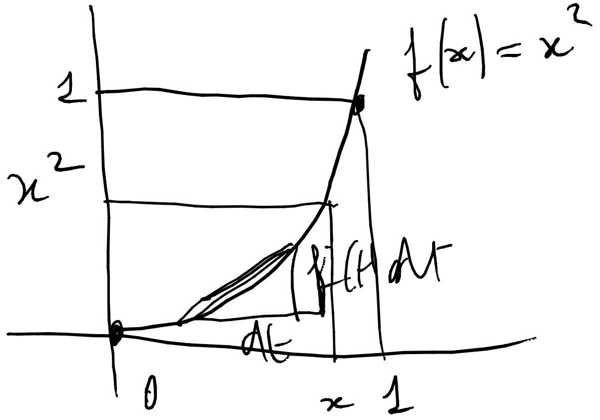
tri disug.

$\leq \sum \left| \dot\gamma(\xi)(t_{i+1} - t_i) \right| =$

$= \sum (t_{i+1} - t_i) \, |\dot\gamma(\xi)|$

$\approx$

$\int_{t_i}^{t_{i+1}} |\dot\gamma(t)|\,dt$



Λ grafico di $f(x) = x^2$ relativo ad $x \in [0,1]$?

$$\gamma(t) = \begin{pmatrix} t \\ f(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$x(t) = t$
$y(t) = f(t)$

$t \in [0,1]$

$$\gamma(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix} \in C^1[0,1] \Rightarrow$$

$$\Rightarrow \Lambda(\gamma) < \infty$$

$$\Lambda(\gamma) = \int_0^1 |\dot\gamma(t)|\,dt =$$

$$\Rightarrow \dot\gamma = \begin{pmatrix} 1 \\ 2t \end{pmatrix}$$

$$= \int_0^1 \sqrt{1 + 4t^2}\,dt = \ldots\ldots$$

$$\Lambda(\mathrm{graph}\, f) = \int_a^b \sqrt{1 + [f'(t)]^2}\,dt$$