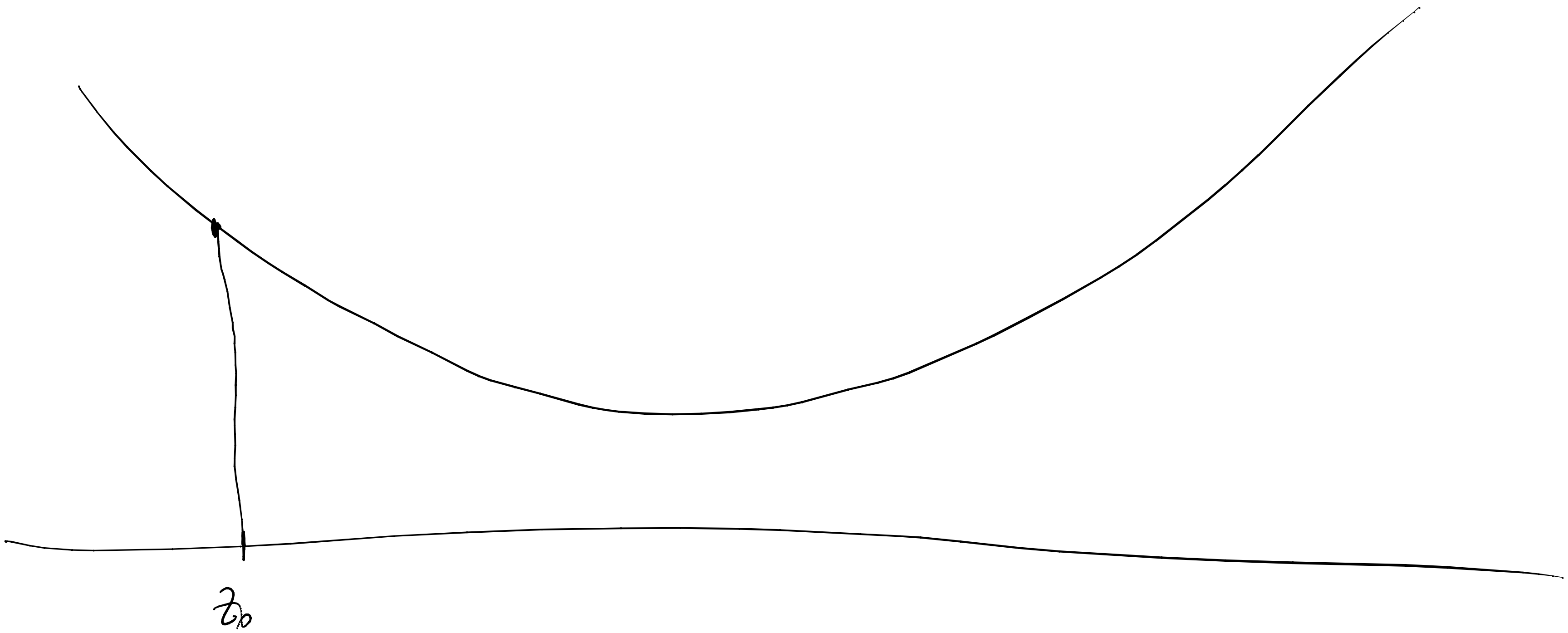
Note Title 4/30/2020



$0 < |f(z_0)|$ altrimenti z_0 è lo zero richiesto di $f(z)$

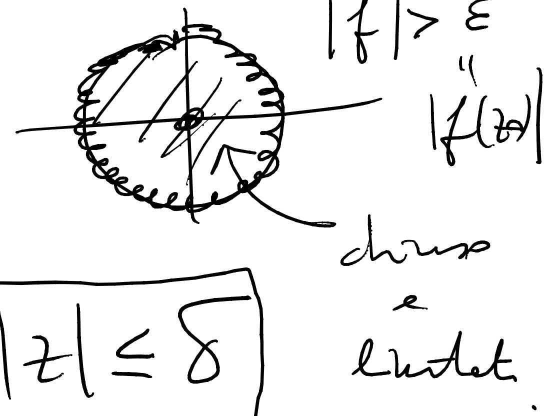
$$\boxed{f : \mathbb{C} \to \mathbb{R}}$$

$|z_0| \le \delta$

$\exists\, \delta : |z| > \delta \qquad |f(z)| > \varepsilon$

Weierstrass (MIN) su $\overline{B_\delta(0)}$ chiuso e limitato

$\exists$ massimo di $|f(z)|$ in z^*



$$|f(z^*)| \le |f(z_0)|$$

$$\boxed{|f(z^*)| \le |f(z)| \quad \forall z : |z| \le \delta}$$

$$|z| > \delta \Rightarrow |f(z)| > \varepsilon = |f(\underset{\in B_\delta(0)}{z_0})| \underset{\text{minimo}}{\ge} |f(z^*)| \qquad \underline{z_0 \in B_\delta(0)}$$

$\boxed{\text{TS}}$

$$\forall z \in \mathbb{C} \quad |f(z)| \ge |f(z^*)|$$

retta tangente $\gamma(t) = (t\cos t,\ t\sin t,\ t)$ $\quad t \in [0, 2\pi]$

$(-\pi, 0, \pi)$

$$\exists\, t \in [0,2\pi] \quad \begin{pmatrix} t\cos t \\ t\sin t \\ t \end{pmatrix} = \begin{pmatrix} -\pi \\ 0 \\ \pi \end{pmatrix}$$

$$\boxed{t = \pi}$$

$\Downarrow$

$\pi \sin \pi = 0$

$\pi \cos \pi = -\pi$

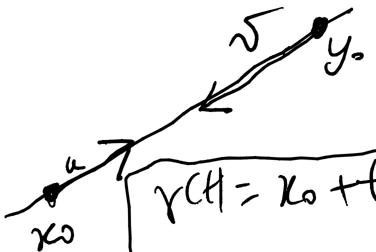
è assunto da γ in $t = \pi$

$$\dot\gamma(t) = \begin{pmatrix} \cos t - t\sin t \\ \sin t + t\cos t \\ 1 \end{pmatrix} \qquad \dot\gamma(\pi) = \begin{pmatrix} -1 \\ -\pi \\ 1 \end{pmatrix}$$

$\dot\gamma \neq 0\ \forall t \Rightarrow \gamma$ regolare

$$\boxed{\varphi(t) = \begin{pmatrix} -\pi \\ 0 \\ \pi \end{pmatrix} + t \begin{pmatrix} -1 \\ -\pi \\ 1 \end{pmatrix}}$$

retta parametrica per $(-\pi, 0, \pi)$ diretta come $(-1, -\pi, 1)$



$\tau(u) = \langle v \rangle$

$\boxed{\gamma(t) = x_0 + tu}$ $\quad \boxed{y_0 + sv = \sigma(s)}$

$\gamma(t)$

tangente in $\gamma(t_0)$ è $\boxed{\varphi(t) = \gamma(t_0) + t\,\dot{\gamma}(t_0)}$

p polinomio non costante e $p(z_0) \neq 0$ allora

$$\exists\, z^* : \ |p(z^*)| < |p(z_0)| \longleftarrow$$

DIM.

$$q(w) = \frac{p(z_0 + w)}{p(z_0)} =$$

1) $q(0) = 1$

2) $\deg q = \deg p$

$$= \underline{\underline{1}} + 0 + 0 + \dots + 0 + \underbrace{\alpha_k}_{\neq 0} w^k + \underline{w^{k+1} \tilde{q}(w)}$$

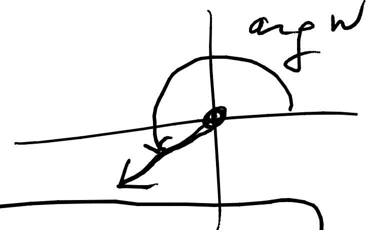
α_k è il coeff. di q non nullo di grado minimo > 0

La tesi è $\exists\, w' : \underbrace{|q(w')|}_{} < 1$

$$\frac{|p(z_0 + w')|}{|p(z_0)|} < 1 \qquad \boxed{z^* = z_0 + w'}$$

Scegliere w in modo da avere $\alpha_k w^k$ reale, negativo e in modulo < 1.

$$\underline{|\alpha_k w^k| < 1} \qquad \alpha_k \neq 0 \qquad |w| < \frac{1}{|\alpha_k|^{\frac{1}{k}}}$$



$$\arg \alpha_k w^k \overset{?}{=} \pi$$

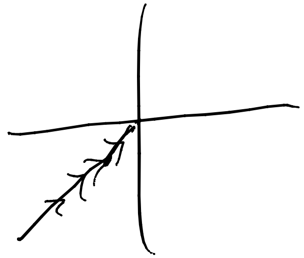
$$\arg \alpha_k + \arg w^k = \arg \alpha_k + k \arg w$$

$$\boxed{\arg w = \frac{\pi - \arg \alpha_k}{k}}$$

$$|q(w)| \leq |1 + \underbrace{\alpha_k w^k}_{\in \mathbb{R}^-}| + |w|^{k+1} |\tilde{q}(w)| = \underline{\text{dis. triangolare}}$$

$$= 1 - |\alpha_k w^k| + |w|^{k+1} |\tilde{q}(w)| = 1 - |w|^k \Big[\underset{>0}{|\alpha_k|} + |w| |\tilde{q}(w)| \Big]$$

per $w \to 0$ $\quad \arg w = \dfrac{\pi - \arg \alpha_k}{k}$



$$[\quad] \longrightarrow |\alpha_k| > 0$$

Term. segno $[\quad] > 0$ per $|w|$ abbast. piccolo

$$|q(w)| = 1 - \underbrace{[\quad]}_{>0} < 1$$

$$x^4 + y^4 - xy = k \qquad f(x,y) = k$$

$$\boxed{\nabla f = \begin{pmatrix} 0 \\ 0 \end{pmatrix} ?}$$

$$f_x = 4x^3 - y$$

$$f_y = 4y^3 - x$$

$$\begin{cases} y = 4x^3 \\ x = 4y^3 \end{cases}$$

$$\boxed{x \neq 0 \quad y \neq 0}$$

$$\frac{y}{x} = \left(\frac{x}{y}\right)^3$$

$(0,0)$ è critico

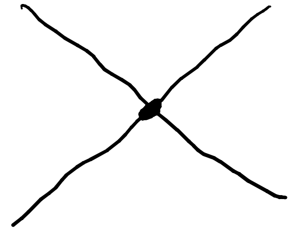
$$\left(\frac{x}{y}\right)^4 = 1$$

$$\frac{x}{y} = 1 \qquad \frac{x}{y} = -1$$

$$\boxed{y = x}$$

$$\boxed{y = -x}$$

$$x=0 \Rightarrow \text{II eq. } y=0$$
$$y=0 \Rightarrow \text{I eq. } x=0$$
$$\Rightarrow (0,0)$$



$y=4x^3 \qquad x=4y^3 \qquad y=x$ oppure $y=-x$

$\boxed{y=x}$ $\quad x=4x^3 \qquad x(4x^2-1)=0 \qquad x=0 \Rightarrow y=0 \qquad \left(\frac{1}{2},\frac{1}{2}\right)$

$x=\pm\frac{1}{2} \Rightarrow y=\pm\frac{1}{2} \qquad \left(-\frac{1}{2},-\frac{1}{2}\right)$

$\boxed{y=-x}$ $\quad -x=4x^3 \qquad x(4x^2+1)=0 \qquad x=0 \quad y=0$

$$x^4+y^4-xy=k$$

$\rightarrow (0,0) \quad \left(\frac{1}{2},\frac{1}{2}\right) \quad \left(-\frac{1}{2},-\frac{1}{2}\right) \qquad f(0,0)=0 \ (=k)$

$$f\left(\frac{1}{2},\frac{1}{2}\right)=\frac{1}{16}+\frac{1}{16}-\frac{1}{4}=-\frac{1}{8}$$

$$f\left(-\frac{1}{2}, -\frac{1}{2}\right) = \frac{1}{16} + \frac{1}{16} - \frac{1}{4} = -\frac{1}{8}$$

La curva di livello $f(x,y) = k$ è sempre esplicita.

salvo che per $\underline{k = 0}$ e $\underline{k = -\frac{1}{8}}$

$$\boxed{y = F(x)}$$

$$\begin{cases} y_1 = F_1(x_1 \dots x_n) \\ \vdots \\ y_n = F_n(x_1, \dots, x_n) \end{cases}$$

$$F : \mathbb{R}^n \to \mathbb{R}$$

$$F(x) - y = 0$$

$$\Updownarrow$$

$$\boxed{\begin{cases} F_1(x_1 \dots x_n) - y_1 = 0 \\ \vdots \\ F_n(x_1 \dots x_n) - y_n = 0 \end{cases}}$$

$$\boxed{\begin{cases} G_1(x_1 \dots x_n, y_1 \dots y_n) = F_1(x_1 \dots x_n) - y_1 \\ \vdots \\ G_n(x_1 \dots x_n, y_1 \dots y_n) = F_n(x_1 \dots x_n) - y_n \end{cases}}$$

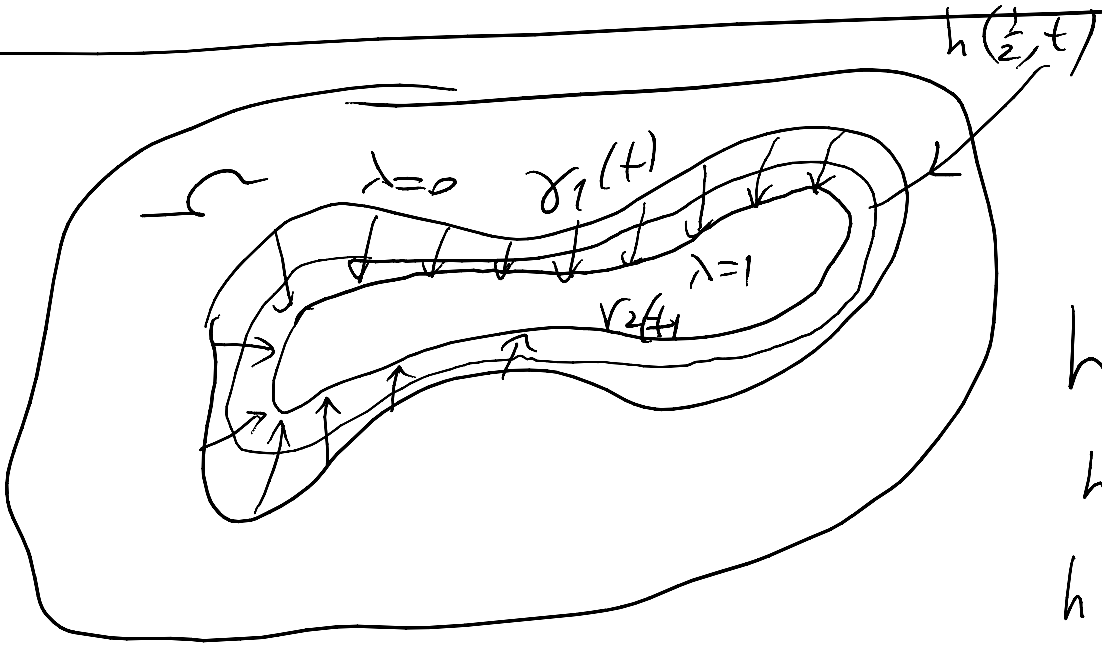
si può applicare il th. funzioni implicite?

$$\frac{\partial(G_1 \dots G_n)}{\partial(x_1 \dots x_n)} = \frac{\partial(F_1 \dots F_n)}{\partial(x_1 \dots x_n)}$$

$$\det \frac{\partial(F_1 \dots F_n)}{\partial(x_1 \dots x_n)} \neq 0$$

$$\frac{\partial(\varphi_1\cdots\varphi_n)}{\partial(y_1\cdots y_n)} = -\left[\frac{\partial(F_1\cdots F_n)}{\partial(x_1,\dots x_n)}\right]^{-1}\underbrace{\boxed{\frac{\partial(G_1\cdots G_n)}{\partial(y_1\cdots y_n)}}}_{-I} =$$

$$= \left[\frac{\partial(F_1\cdots F_n)}{\partial(x_1,\dots x_n)}\right]^{-1}$$



$\gamma_1 : [0,1] \to \Omega$ continue

$\gamma_2 : [0,1] \to \Omega$

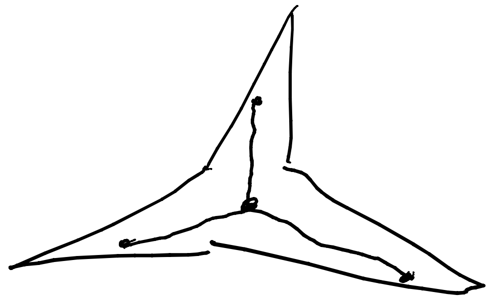
$h : [0,1]\times[0,1] \to \boxed{\Omega}$

$h(0,t) = \gamma_1(t)$

$t \mapsto h(\lambda,t) \quad \lambda \in]0,1[$

$h(1,t) = \gamma_2$

Ω stella vuol dire che $\quad x \in \Omega \quad \lambda x \in \Omega \quad \forall \lambda \in [0,1]$
(rispetto a 0)

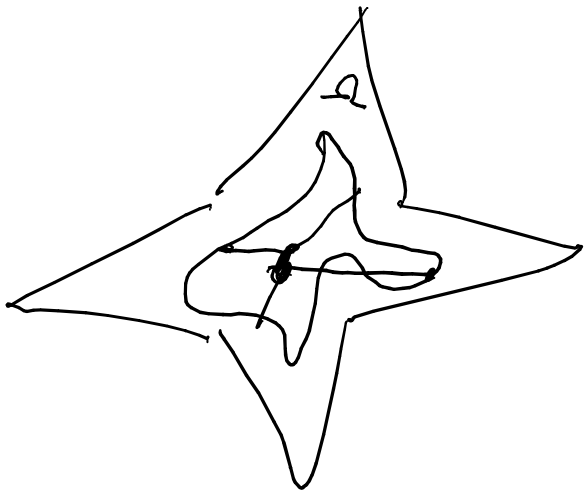


Ogni curva chiusa γ è omòtopa ad una curva costante

$\Leftarrow$ $\gamma(0)=\gamma(1)$ γ chiusa

$$h(\lambda,t) = \underbrace{(1-\lambda)\,\gamma(t)}$$

$\in$ segmento di estremi 0 e $\gamma(t)$



$\lambda=0$

$$h(0,t)=\gamma(t)$$

$\lambda=1$

$$h(1,t)\equiv 0$$

$\sigma(t)\equiv 0$ costante

CAMPI E FORME