Note Title 4/30/2020

Def 1 $\forall w \in X \oplus Y \quad \exists \boxed{\text{unici}}\ x \in X,\ y \in Y :$

$w = x + y$

$\Updownarrow$

Alternative def 2 $\forall\ x \in X,\ y \in Y : \quad x + y = 0 \Rightarrow \begin{matrix} x = 0 \\ y = 0 \end{matrix}$

EQUIVALENTI

Def 1 $\Rightarrow$ def 2

$$\begin{matrix} x & + & y & = & 0 \\ \| & & \| & & \\ 0 & + & 0 & = & 0 \end{matrix} \Rightarrow \begin{matrix} x = 0 \\ y = 0 \end{matrix}$$

Def 2 $\quad x+y=0 \Rightarrow x=0,\ y=0$

$w \in X+Y$

$$w = x+y$$
$$w = x'+y'$$

Sottraendo

$$0 = \underbrace{(x-x')}_{u} + \underbrace{(y-y')}_{v}$$

$u+v=0 \Rightarrow u=0 \quad v=0$

$\Downarrow$

$x-x'=0 \quad y-y'=0$

$x=x' \quad y=y'$

$A: X \to Y$ è iniettiva $\Longleftrightarrow \underbrace{\operatorname{Ker} A = \{0\}}_{A(x)=0 \text{ solo se } x=0}$.

$\Longrightarrow$ A iniettiva $\quad A(0)=0 \Rightarrow A(x)=0$

$$A(x) = A(0) \Longrightarrow x=0$$

$\Longleftarrow$ $A(x) = 0 \Rightarrow x = 0$

iniettiva

$\underline{A(x_1) = A(x_2)} \Leftrightarrow A(x_1) - A(x_2) = 0 \Leftrightarrow$

$A(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow \underline{x_1 = x_2}$

iniettiva

$A : X \to Y$ ad ogni $x \in X$ associa uno e uno solo

$A(x) \in Y$

$$A : X \to Y \qquad \dim X < \infty$$

$$\underbrace{\dim \operatorname{Ker} A + \dim \operatorname{Im} A = \dim X}$$

$$\Downarrow$$

$$\dim \operatorname{Im} A \leq \dim X$$

$$A : X \to Y \qquad \dim X < \dim Y$$

allora $\dim \operatorname{Im} A \leq \dim X < \dim Y$

$$\to \boxed{\forall y \in Y \ \exists x \in X : A(x) = y} \quad \operatorname{Im} A = Y$$

$\dim X > \dim Y \Rightarrow$

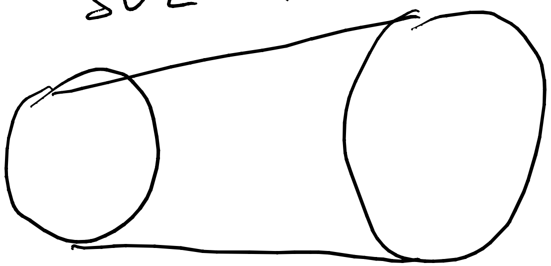
$\dim \mathrm{Im} A \le \dim Y < \dim X$

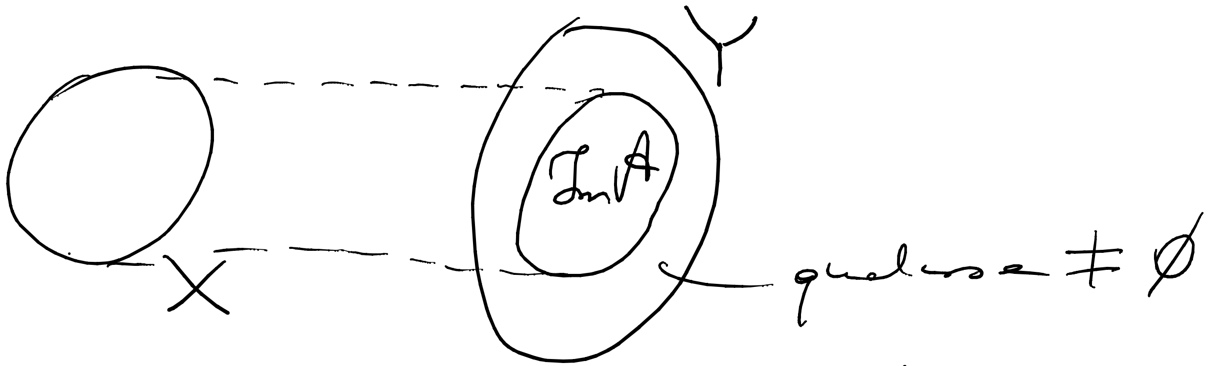
Th. Grassmann. $\dim \mathrm{Ker} + \underline{\dim \mathrm{Im}} = \underline{\dim X}$

$\dim \mathrm{Ker} A > 0 \Rightarrow$ A NON E' INIETTIVA

$A : X \to Y$

suriettiva





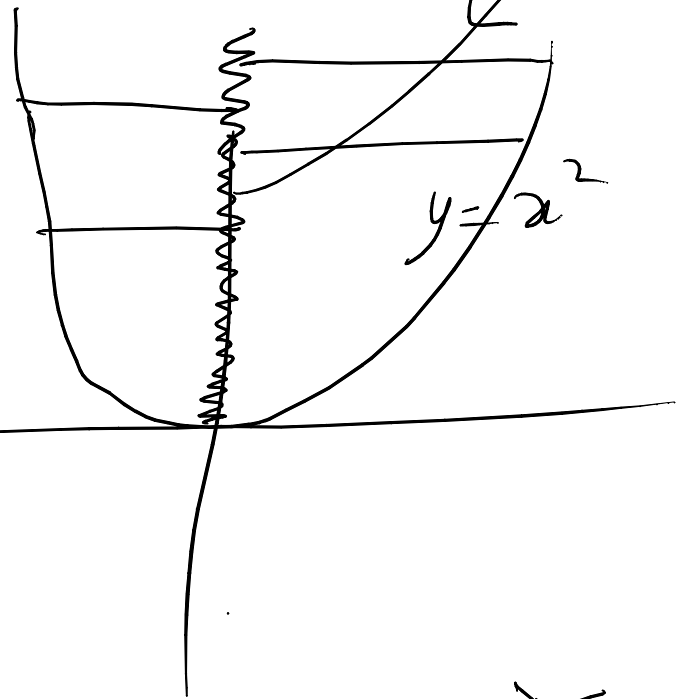
$\dim \mathrm{Im} A \le \dim X < \dim Y$

A suriettiva $Y = \mathrm{Im} A$

$f(x) = x^2$ $\qquad$ $f: \mathbb{R} \to \mathbb{R}$

per ogni $x \in \mathbb{R}$ $\quad \underline{\underline{x^2}} \in \mathbb{R}$ $\quad \mathrm{Im}\, f$

$\mathrm{Im}\, f = \{ y \geq 0 \}$



$$\boxed{\mathbb{R} \text{ codominio } \supset \mathrm{Im}\, f}$$

<u>Th. Cramer</u> $\quad A: X \to Y \quad \dim X = \dim Y$

Se è suriettiva $\quad \mathrm{Im}\, A = Y \Rightarrow \dim \mathrm{Ker} + \underline{\dim \mathrm{Im}} = \dim X$

$\dim \mathrm{Im} = \dim Y$

$\boxed{\dim \mathrm{Ker} = 0}$