ESERCIZI SUL TEOREMA DELLE FUNZIONI IMPLICITE

Note Title 4/29/2020

$x^3y - x = 1$ $\Gamma = \{(x,y) \in \mathbb{R}^2 : x^3y - x - 1 = 0\}$

$x = 0 \Rightarrow$ FALSA $\underline{\underline{0 = 1}}$

$y = 0 \Rightarrow -x = 1$ $(-1,0) \in \Gamma$

$x + y = 1$

$x = 0 \Rightarrow y = 1$

$(0,1)$ è un punto della retta $x+y=1$

$$\begin{cases} f_x = 3x^2y - 1 \\ f_y = x^3 \end{cases}$$

$f_x = 0 \iff x \neq 0 \quad y = \frac{1}{3x^2}$

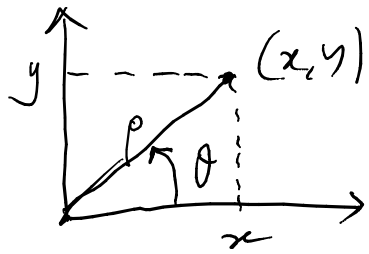
$f_y = 0 \iff x = 0$

$x \neq 0 \Rightarrow f_y \neq 0$ e quindi $\exists$ localmente $\varphi : y = \varphi(x)$ $\rightarrow \nabla f$ non si annulla MAI

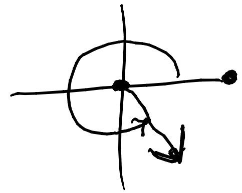
$\boxed{x = 0 \Rightarrow f_x = -1 \neq 0}$ $\exists$ localmente $\psi : x = \psi(y)$

COORDINATE POLARI (PIANE) (in $\mathbb{R}^2$)

$$x = \rho\cos\theta \qquad y = \rho\sin\theta \qquad \rho \in [0,+\infty[\qquad \theta \in [0, 2\pi[$$



$$\boxed{\rho = \sqrt{x^2+y^2}}$$



$$(0,\theta) \xrightarrow{\forall\theta} (0,0)$$

$$(\rho, 0) = (\rho, 2\pi) \quad \forall \rho \geq 0$$

$$T(\rho,\theta) \to (\underbrace{\rho\cos\theta}_{=x}, \underbrace{\rho\sin\theta}_{=y})$$

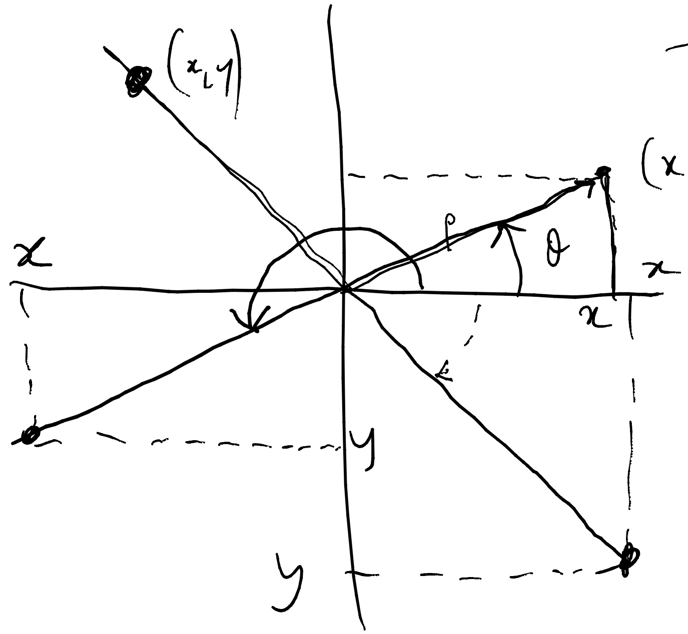
$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

$$\det\begin{pmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \det\begin{pmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{pmatrix} = \rho$$

lo jacobiano è nullo solo se

$$\boxed{\rho = 0}$$

$$\exists S : (\rho,\theta) = S(x,y)$$



$\cancel{[0, 2\pi]}$ $[-\pi, \pi]$

$$\frac{y}{x} = \operatorname{tg}\theta \overset{?}{\Longleftrightarrow} \theta = \operatorname{arctg}\frac{y}{x}$$

$$\operatorname{Im}\operatorname{arctg} = \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

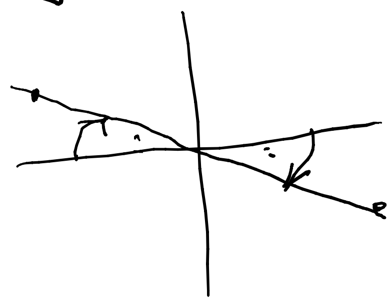
$$\boxed{\theta = \operatorname{arctg}\frac{y}{x}}$$ vero per tutti i θ

$$\theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

$$\boxed{x > 0}$$

$x = 0$ (asse y) $\quad \theta = \begin{cases} \frac{\pi}{2} & y > 0 \\ -\frac{\pi}{2} & y < 0 \end{cases}$

$x < 0$ $\quad \pi + \operatorname{arctg}\boxed{\frac{y}{x}}$ $\quad$ III quadrante, IV quadrante



$$\rho = \sqrt{x^2+y^2}$$

$$\theta = \begin{cases} \arctan\dfrac{y}{x} & \text{se } x>0 \\[2mm] \dfrac{\pi}{2} & \text{se } x=0 \quad y>0 \\[2mm] \pi+\arctan\dfrac{y}{x} & \text{se } x<0 \\[2mm] -\dfrac{\pi}{2} & \text{se } x=0 \quad y<0 \end{cases}$$

$\boxed{\text{atan2}}$

(FORTRAN)

$$\frac{x}{\sqrt{x^2+y^2}} = \cos\theta$$

$\Downarrow$??

$$\boxed{\theta = \arccos\frac{x}{\sqrt{x^2+y^2}}}$$

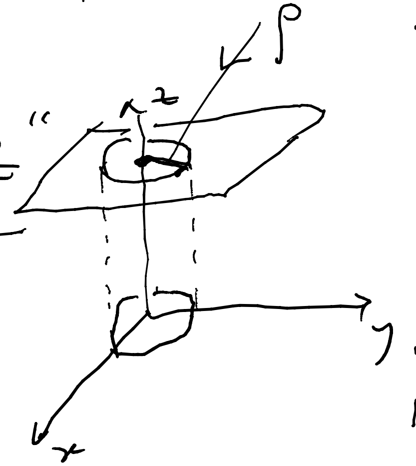
non ha senso solo in $(0,0)$

COORDINATE (POLARI) CILINDRICHE

$\boxed{T}$ $x(\rho,\theta,z) = \rho\cos\theta$ $\quad y(\rho,\theta,z) = \rho\sin\theta$ $\quad$ "$z(\rho,\theta,z) = z$"

$$\rho = \sqrt{x^2+y^2}$$

x, y, z



$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,z)} = \left(\begin{array}{cc|c} \cos\theta & -\rho\sin\theta & 0 \\ \sin\theta & \rho\cos\theta & 0 \\ \hline 0 & 0 & 1 \end{array}\right) \qquad \det(T') = \rho$$

Le coordinate polari cilindriche sono localmente invertibili se $\rho \neq 0 \Longrightarrow$ fuori dell'asse z

$\rho = 0$?

$(\rho,\theta,\zeta) \longrightarrow (\rho\cos\theta, \rho\sin\theta, \zeta)$ $\qquad z(\rho,\theta,\zeta) = \zeta$

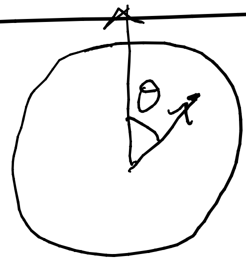
COORDINATE SFERICHE $\qquad \rho = \sqrt{x^2+y^2+y^2} \geq 0$

$x = \rho \sin\theta\cos\varphi \qquad y = \rho\sin\theta\sin\varphi \qquad z = \rho\cos\theta$

$\theta \in [0,\pi]$

$\varphi \in [0,2\pi]$

$T(\rho,\theta,\varphi) = (x,y,z)$



$$\begin{vmatrix} \sin\theta\cos\varphi & \rho\cos\theta\cos\varphi & -\rho\sin\theta\sin\varphi \\ \sin\theta\sin\varphi & \rho\cos\theta\sin\varphi & \rho\sin\theta\cos\varphi \\ \cos\theta & -\rho\sin\theta & 0 \end{vmatrix} =$$

$$= \underline{\rho^2\cos^2\theta\sin\theta\cos^2\varphi} + \rho^2\sin^3\theta\sin^2\varphi -$$

$$-\left(-\underline{\rho^2\cos^2\theta\sin\theta\sin^2\varphi} - \rho^2\sin^3\theta\cos^2\varphi\right) =$$

$$= \rho^2\cos^2\theta\sin\theta + \rho^2\sin^3\theta = \boxed{\rho^2\sin\theta} = 0 \text{ se } \begin{array}{l} \rho = 0 \text{ oppure} \\ \text{se } \theta = 0 \text{ o } \theta = \pi \end{array}$$

NON SI PUÒ APPLICARE IL TEOREMA DI INVERSIONE LOCALE in $\rho=0$ (e cioè $(0,0,0)$) ovvero in $\theta=0$ o $\theta=\pi$ (e cioè in $(0,0,z)$, $z\in\mathbb{R}$).

$\boxed{f(x) = \arccos(\cos x)}$ $\boxed{f(x) = \sqrt{x^2}}$ STUDIARE!

$T(u, v) = \left(\underset{\underset{x}{\shortparallel}}{uv}, \underset{\underset{y}{\shortparallel}}{\frac{u}{v}} \right)$

I quadrante

E' localmente invertibile?