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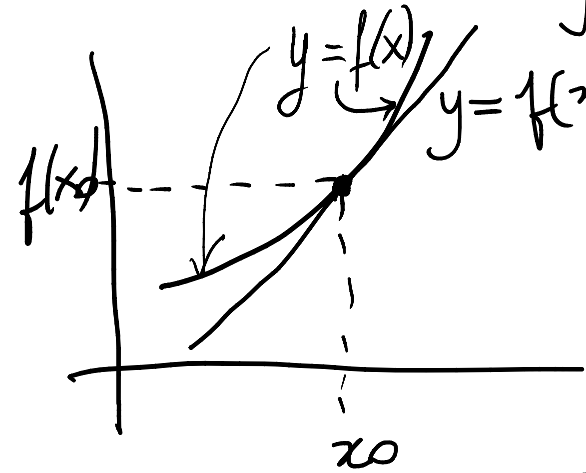
Rette e piani tangenti

1) $f:\mathbb{R}\to\mathbb{R}$ $\quad y=f(x)$ $\quad \{(x,y)\in\mathbb{R}\times\mathbb{R} : y=f(x)\} = \text{graph}\, f$

$f:\mathbb{R}^n\to\mathbb{R}$ $\quad z=f(x)$ $\quad \{(x,z)\in\mathbb{R}^n\times\mathbb{R} : z=f(x)\}=\text{graph}\, f$

1) retta tangente al graph f nel punto $(x_0, \underbrace{f(x_0)}_{y_0})$ è

$$y = \underset{y_0}{\underset{\shortparallel}{f(x_0)}} + f'(x_0)(x-x_0)$$



$$y=f(x_0)+\underbrace{f'(x_0)(x-x_0)}_{df(x_0,\, x-x_0)}$$

2) piano tangente

$$\boxed{z = \underbrace{f(x_0,y_0)}_{z_0} + \underbrace{\nabla f(x_0,y_0)\begin{pmatrix}x-x_0\\ y-y_0\end{pmatrix}}_{df((x_0,y_0);\,(x-x_0,\,y-y_0))}}$$

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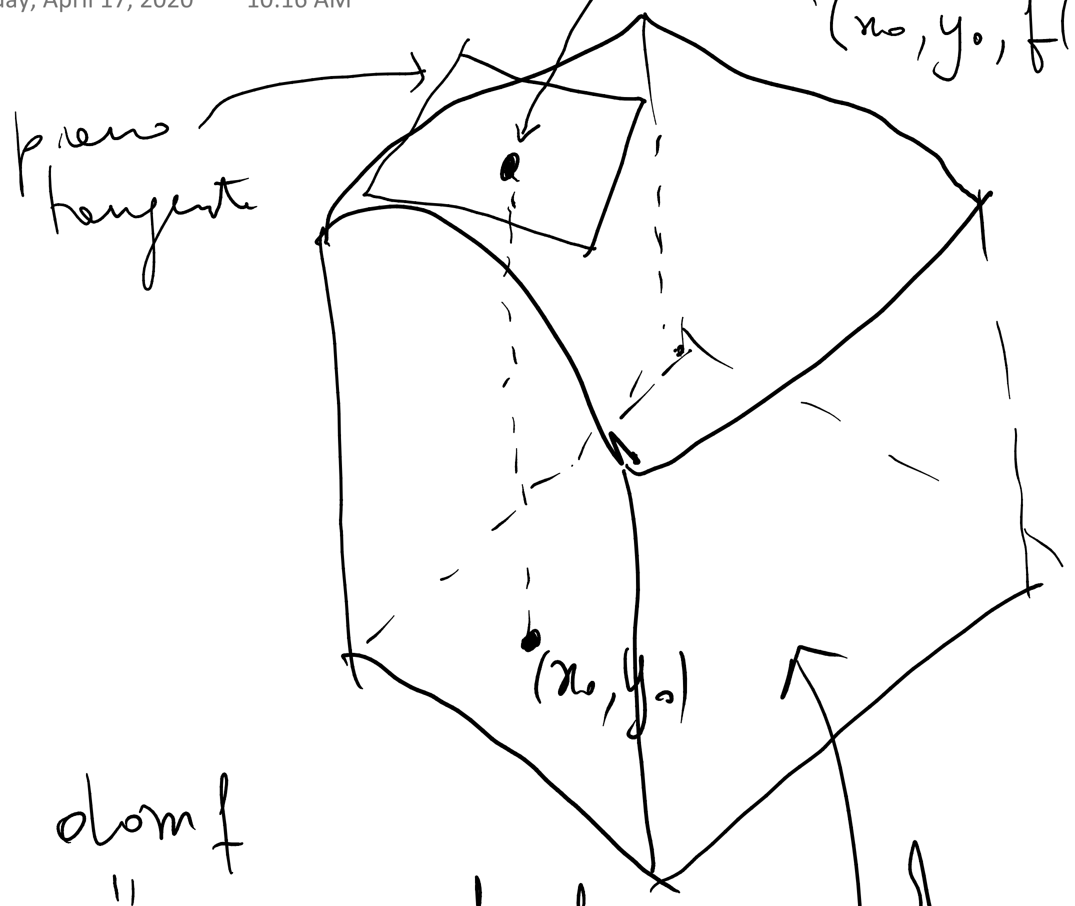
$(x_0, y_0, f(x_0,y_0)) \in$ graph f



$$z - f(x_0,y_0) = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

z_0

$$(\underbrace{x, y}_{\in \text{dom} f}, \underbrace{f(x,y)}_{\in \text{codom} f})$$

$$\underset{x}{\overset{\text{dom} f}{\overset{\shortparallel}{\mathbb{R}^n}}} \times \underset{z}{\overset{\text{codom} f}{\overset{\shortparallel}{\mathbb{R}}}}$$

dom f

$$\boxed{\{(x,y,z) \in \mathbb{R}^3 : z = \sqrt{1-x^2-y^2}\}}$$

$$(\underbrace{x, y}, f(x,y))$$

$$\boxed{x^2+y^2+z^2=1}$$

$$\left\| \left(\underset{x_0}{\frac{1}{\sqrt{3}}}, \underset{y_0}{\frac{1}{\sqrt{3}}}, \underset{z_0}{\frac{1}{\sqrt{3}}} \right) \right\| = 1$$

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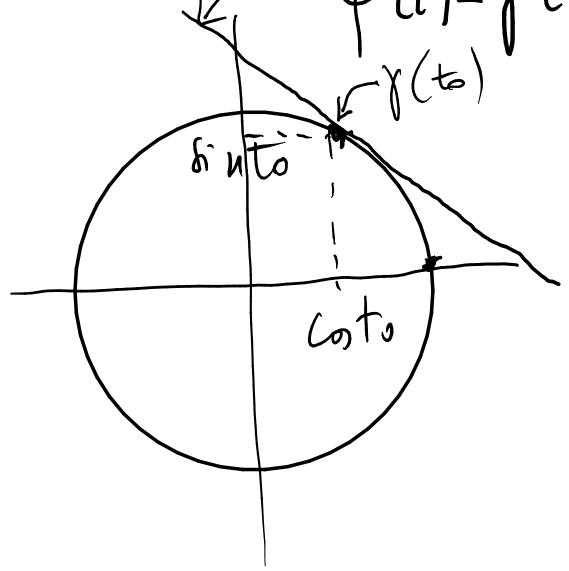
$\gamma(t) \quad \gamma: \mathbb{R} \to \mathbb{R}^n \quad n>1$

Tangente a γ in $\gamma(t_0)$ è la retta per $\gamma(t_0)$ in direzione $\dot{\gamma}(t_0)$

$\varphi(t) = \gamma(t_0) + t\dot{\gamma}(t_0)$



$\gamma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

$t \in [0, 2\pi]$

$\gamma([0,2\pi]) \subseteq \mathbb{R}^2$

$\gamma([0,2\pi]) = \{(x,y) : x^2+y^2=1\}$

$$\boxed{\varphi(t) = \gamma(t_0) + t\dot{\gamma}(t_0)}$$

Equazione parametrica della tangente

$\varphi(0) = \gamma(t_0)$

$\psi(t) = \gamma(t_0) + (t-t_0)\dot{\gamma}(t_0)$

$\psi(t_0) = \gamma(t_0)$

formulazione alternativa: passa per $\gamma(t_0)$ al tempo t_0, invece che per $t=0$

$$\frac{\gamma(t) - \gamma(t_0) - (t-t_0)\dot{\gamma}(t_0)}{|t|} \xrightarrow[t \to t_0]{} 0$$

$$\frac{\gamma(t) - \varphi(t)}{|t|} \longrightarrow 0$$

φ è la retta parametrica che approssima "meglio" γ in vicinanza di $\gamma(t_0)$

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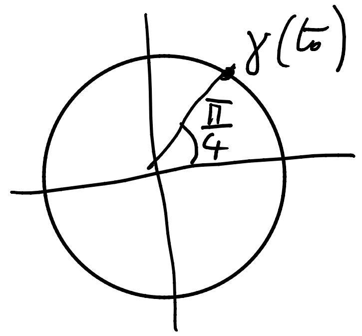
La retta tangente in $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ al sostegno della (alla) curva parametrica $\gamma(t)=\begin{pmatrix}\cos t\\ \sin t\end{pmatrix}$ è

$t\in[0,2\pi]$

$$\varphi(t)=\underbrace{\gamma(t_0)}_{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}+t\,\dot\gamma(t_0)$$

$$\dot\gamma(t)=\begin{pmatrix}-\sin t\\ \cos t\end{pmatrix}$$

$$\gamma(t_0)=\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} \qquad \begin{cases}\cos t_0=\frac{1}{\sqrt{2}}\\ \sin t_0=\frac{1}{\sqrt{2}}\end{cases} \quad \text{cioè} \quad \boxed{t_0=\frac{\pi}{4}}$$



$$\boxed{\varphi(t)=\underbrace{\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}}_{=\gamma\left(\frac{\pi}{4}\right)}+t\underbrace{\begin{pmatrix}-1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}}_{=\dot\gamma\left(\frac{\pi}{4}\right)}}$$

eq. param. tangente al sostegno di γ nel punto $\gamma\left(\frac{\pi}{4}\right)=\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$